

Computer algebra independent integration tests

7-Inverse-hyperbolic-functions/7.3-Inverse-hyperbolic-tangent/7.3.5-u-a+b-arctanh-c+d-x-^p

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3.40	$\int (e + fx) (a + b \tanh^{-1}(c + dx))^2 dx$	260
3.41	$\int (a + b \tanh^{-1}(c + dx))^2 dx$	267
3.42	$\int \frac{(a+b \tanh^{-1}(c+dx))^2}{e+fx} dx$	271
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3.44	$\int \frac{(a+b \tanh^{-1}(c+dx))^2}{(e+fx)^3} dx$	284
3.45	$\int (e + fx)^2 (a + b \tanh^{-1}(c + dx))^3 dx$	293
3.46	$\int (e + fx) (a + b \tanh^{-1}(c + dx))^3 dx$	302
3.47	$\int (a + b \tanh^{-1}(c + dx))^3 dx$	308
3.48	$\int \frac{(a+b \tanh^{-1}(c+dx))^3}{e+fx} dx$	313

3.49	$\int \frac{(a+b \tanh^{-1}(c+dx))^3}{(e+fx)^2} dx$	318
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Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from <https://rulebasedintegration.org>

The number of integrals in this report is [62]. This is test number [195].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.3 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12.1 on windows 10.
3. Maple 2021.1 (64 bit) on windows 10.
4. Maxima 5.44 on Linux. (via sagemath 9.3)
5. Fricas 1.3.7 on Linux (via sagemath 9.3)
6. Giac/Xcas 1.7 on Linux. (via sagemath 9.3)
7. Sympy 1.8 under Python 3.8.8 using Anaconda distribution on Ubuntu.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 under windows 10 (64 bit)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or Hypergeometric₂F₁ functions. RootSum and RootOf are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100.00 (62)	% 0.00 (0)
Mathematica	% 96.77 (60)	% 3.23 (2)
Maple	% 98.39 (61)	% 1.61 (1)
Maxima	% 54.84 (34)	% 45.16 (28)
Fricas	% 27.42 (17)	% 72.58 (45)
Sympy	% 22.58 (14)	% 77.42 (48)
Giac	% 27.42 (17)	% 72.58 (45)
Mupad	% 27.42 (17)	% 72.58 (45)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons <ol style="list-style-type: none"> 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

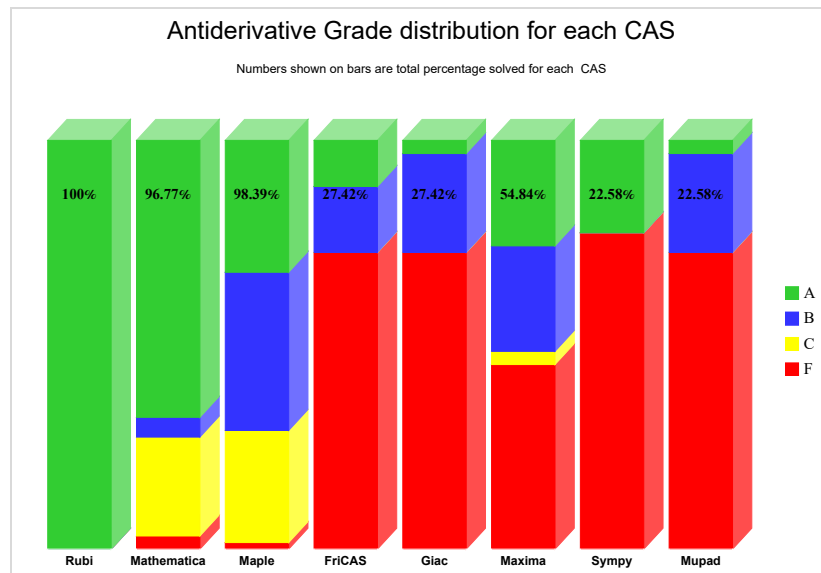
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

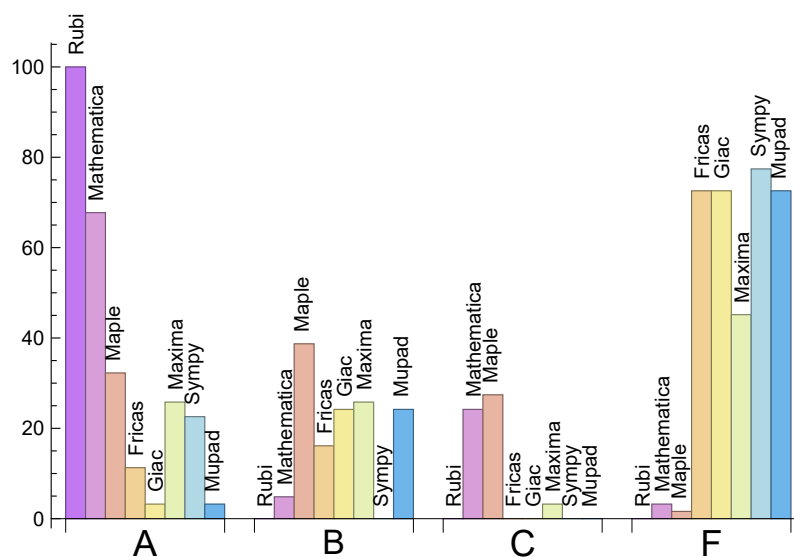
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.00	0.00	0.00	0.00
Mathematica	67.74	4.84	24.19	3.23
Maple	32.26	38.71	27.42	1.61
Maxima	25.81	25.81	3.23	45.16
Fricas	11.29	16.13	0.00	72.58
Sympy	22.58	0.00	0.00	77.42
Giac	3.23	24.19	0.00	72.58
Mupad	3.23	24.19	0.00	72.58

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This is the typical normal failure **F**.

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage time-out failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	2	100.00 %	0.00 %	0.00 %
Maple	1	100.00 %	0.00 %	0.00 %
Maxima	28	96.43 %	0.00 %	3.57 %
Fricas	45	100.00 %	0.00 %	0.00 %
Sympy	48	79.17 %	20.83 %	0.00 %
Giac	45	77.78 %	22.22 %	0.00 %
Mupad	45	100.00 %	0.00 %	0.00 %

Table 1.4: Time and leaf size performance for each CAS

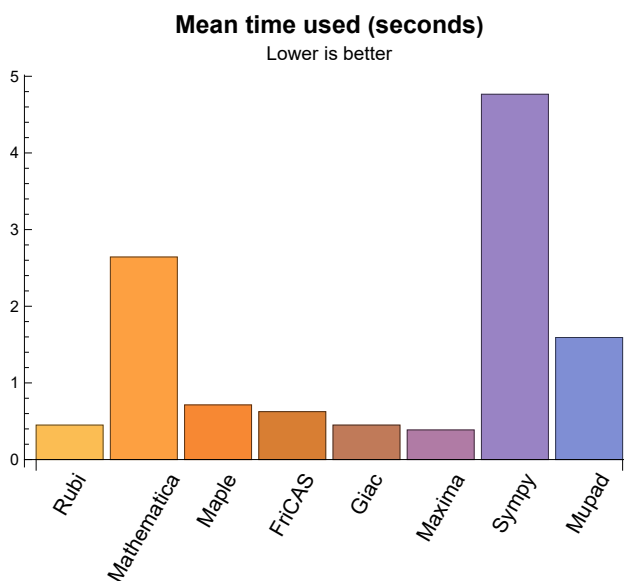
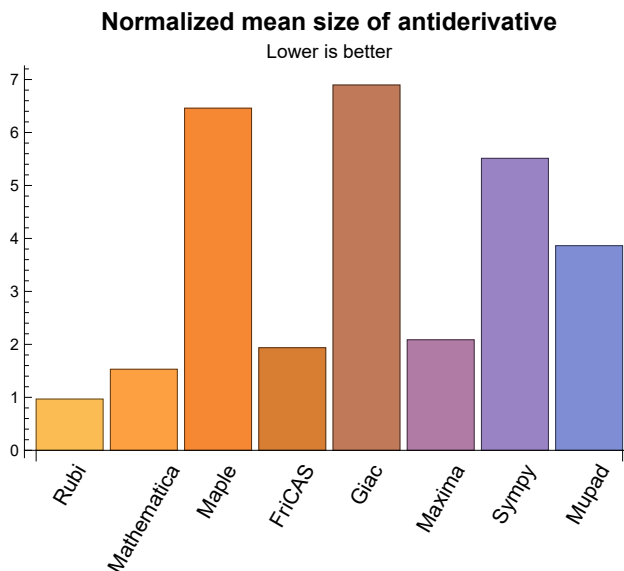
1.3 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.45	243.21	0.97	166.50	1.00
Mathematica	2.64	399.90	1.53	210.50	1.25
Maple	0.71	1873.15	6.46	492.00	2.70
Maxima	0.39	331.44	2.09	251.50	1.73
Fricas	0.63	227.06	1.94	159.00	2.02
Sympy	4.77	672.57	5.51	291.50	3.43
Giac	0.45	905.12	6.90	364.00	4.41
Mupad	1.59	499.06	3.86	237.00	2.50

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used columns from the above table.



1.4 list of integrals that has no closed form antiderivative

{50,51}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {1, 2, 3, 4, 5, 6, 7, 8, 16, 18, 19, 21, 23, 24, 25, 26, 27, 28, 35, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 49, 55, 56, 57, 58, 61, 62}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime')
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using `sagemath` (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. The exception message will indicate of the error is due to the interactive question being asked or not.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See <https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/> for reference.

1.9.2 Important note about FriCAS and Giac/X-CAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the builtin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy and Giac antiderivatives is determined using the following function, thanks to user `slelievre` at <https://>

ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which is called directly from Python, the following code is used to obtain the leafsize of its result

```
try:
    # 1.7 is a fudge factor since it is low side from actual leaf count
    leafCount = round(1.7*count_ops(anti))

except Exception as ee:
    leafCount = 1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative, Maple was used to determine the leaf size of Mupad output by post processing.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine, 'cos(x)*sin(x)')
the_variable = evalin(symengine, 'x')
anti = int(integrand, the_variable)
```

Which gives $\sin(x)^2/2$

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. This is description of each record

1. integer, the problem number.
 2. integer. 0 for failed, 1 for passed, -1 for timeout, -2 for CAS specific exception. (this is not the grade field)
 3. integer. Leaf size of result.
 4. integer. Leaf size of the optimal antiderivative.
 5. number. CPU time used to solve this integral. 0 if failed.
 6. string. The integral in Latex format
 7. string. The input used in CAS own syntax.
 8. string. The result (antiderivative) produced by CAS in Latex format
 9. string. The optimal antiderivative in Latex format.
 10. integer. 0 or 1. Indicates if problem has known antiderivative or not
 11. String. The result (antiderivative) in CAS own syntax.
 12. String. The grade of the antiderivative. Can be "A", "B", "C", or "F"
- The following field present only in Rubi and Mathematica Tables*
13. integer. 1 if result was verified or 0 if not verified.
- The following fields present only in Rubi Tables*
14. integer. Number of rules used.
 15. integer. Integrand leaf size.
 16. real number. Ratio of field 14 over field 15
 17. integer. 1 if result was verified or 0 if not verified.
 18. String of form "{n,n,..}" which is list of the rules used by Rubi

High level overview of the CAS independent integration test build system

Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 3, 4, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 19, 20, 21, 22, 23, 24, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 40, 41, 46, 47, 50, 51, 53, 54, 55, 59, 60, 61, 62 }

B grade: { 2, 39, 45 }

C grade: { 5, 6, 7, 18, 25, 26, 27, 28, 42, 43, 44, 49, 56, 57, 58 }

F grade: { 48, 52 }

2.1.3 Maple

A grade: { 4, 6, 7, 12, 13, 14, 29, 34, 35, 36, 37, 41, 43, 44, 50, 51, 55, 56, 59, 60 }

B grade: { 1, 2, 3, 9, 10, 11, 15, 16, 17, 19, 20, 21, 22, 30, 31, 32, 33, 38, 39, 40, 47, 54, 61, 62 }

C grade: { 5, 8, 18, 23, 24, 25, 26, 27, 28, 42, 45, 46, 48, 49, 53, 57, 58 }

F grade: { 52 }

2.1.4 Maxima

A grade: { 1, 2, 3, 4, 6, 7, 13, 32, 33, 34, 36, 37, 50, 51, 55, 56 }

B grade: { 9, 10, 11, 14, 15, 16, 17, 20, 22, 24, 29, 30, 31, 38, 39, 40 }

C grade: { 54, 57 }

F grade: { 5, 8, 12, 18, 19, 21, 23, 25, 26, 27, 28, 35, 41, 42, 43, 44, 45, 46, 47, 48, 49, 52, 53, 58, 59, 60, 61, 62 }

2.1.5 FriCAS

A grade: { 11, 13, 14, 33, 34, 50, 51 }

B grade: { 9, 10, 15, 17, 20, 22, 31, 32, 36, 37 }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 12, 16, 18, 19, 21, 23, 24, 25, 26, 27, 28, 29, 30, 35, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62 }

2.1.6 Sympy

A grade: { 9, 10, 11, 13, 14, 15, 17, 20, 22, 31, 32, 33, 34, 36 }

B grade: { }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 12, 16, 18, 19, 21, 23, 24, 25, 26, 27, 28, 29, 30, 35, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62 }

2.1.7 Giac

A grade: { 50, 51 }

B grade: { 9, 10, 11, 13, 14, 15, 17, 20, 22, 31, 32, 33, 34, 36, 37 }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 12, 16, 18, 19, 21, 23, 24, 25, 26, 27, 28, 29, 30, 35, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62 }

2.1.8 Mupad

A grade: { 50, 51 }

B grade: { 9, 10, 11, 13, 14, 15, 17, 20, 22, 31, 32, 33, 34, 36, 37 }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 12, 16, 18, 19, 21, 23, 24, 25, 26, 27, 28, 29, 30, 35, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table, the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	263	263	187	967	320	0	0	0	-1
normalized size	1	1.00	0.71	3.68	1.22	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.339	1.260	0.060	0.327	0.477	0.000	0.000	0.000
Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	204	204	463	729	259	0	0	0	-1
normalized size	1	1.00	2.27	3.57	1.27	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.269	2.359	0.061	0.332	0.448	0.000	0.000	0.000
Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	98	365	202	0	0	0	-1
normalized size	1	1.00	0.72	2.68	1.49	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.200	0.281	0.057	0.336	0.549	0.000	0.000	0.000

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	55	103	139	0	0	0	-1
normalized size	1	1.00	0.68	1.27	1.72	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.086	0.068	0.275	0.324	0.449	0.000	0.000	0.000

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	472	1022	0	0	0	0	-1
normalized size	1	1.00	3.19	6.91	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.090	2.645	0.854	0.000	0.433	0.000	0.000	0.000

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	251	251	208	342	244	0	0	0	-1
normalized size	1	1.00	0.83	1.36	0.97	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.699	1.437	0.069	0.339	0.410	0.000	0.000	0.000

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	370	370	271	467	360	0	0	0	-1
normalized size	1	1.00	0.73	1.26	0.97	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.801	2.350	0.077	0.347	0.569	0.000	0.000	0.000

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	75	160	0	0	0	0	-1
normalized size	1	1.00	1.34	2.86	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.128	0.090	0.447	0.000	0.469	0.000	0.000	0.000

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	78	242	357	159	231	282	414
normalized size	1	1.00	1.08	3.36	4.96	2.21	3.21	3.92	5.75
time (sec)	N/A	0.067	0.104	0.033	0.330	0.556	4.272	3.077	1.421

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	59	174	225	144	180	364	237
normalized size	1	1.00	0.86	2.52	3.26	2.09	2.61	5.28	3.43
time (sec)	N/A	0.065	0.075	0.033	0.321	0.497	3.033	0.343	0.618

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	77	107	113	73	95	137	73
normalized size	1	1.00	1.60	2.23	2.35	1.52	1.98	2.85	1.52
time (sec)	N/A	0.034	0.049	0.033	0.314	0.557	1.664	0.394	1.842

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	54	89	0	0	0	0	-1
normalized size	1	1.00	1.00	1.65	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.045	0.025	0.048	0.000	0.516	0.000	0.000	0.000

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	69	86	95	85	270	137	122
normalized size	1	1.00	1.10	1.37	1.51	1.35	4.29	2.17	1.94
time (sec)	N/A	0.059	0.071	0.038	0.315	0.819	2.834	0.184	1.369

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	100	88	131	88	313	134	67
normalized size	1	1.00	1.59	1.40	2.08	1.40	4.97	2.13	1.06
time (sec)	N/A	0.049	0.063	0.037	0.321	0.572	3.546	0.333	1.732

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	148	732	827	383	581	791	1730
normalized size	1	1.00	0.93	4.60	5.20	2.41	3.65	4.97	10.88
time (sec)	N/A	0.248	0.192	0.067	0.622	0.739	8.073	0.251	2.124

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	150	583	619	0	0	0	-1
normalized size	1	1.00	0.84	3.26	3.46	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.237	0.470	0.062	0.628	0.663	0.000	0.000	0.000

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	134	390	316	191	238	369	432
normalized size	1	1.00	1.41	4.11	3.33	2.01	2.51	3.88	4.55
time (sec)	N/A	0.135	0.103	0.063	0.605	0.472	3.227	0.347	1.563

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	424	893	0	0	0	0	-1
normalized size	1	1.00	2.52	5.32	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.303	0.393	0.215	0.000	0.511	0.000	0.000	0.000

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	126	396	0	0	0	0	-1
normalized size	1	1.00	1.21	3.81	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.181	0.254	0.071	0.000	0.496	0.000	0.000	0.000

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	136	371	329	270	1102	356	776
normalized size	1	1.00	1.14	3.12	2.76	2.27	9.26	2.99	6.52
time (sec)	N/A	0.174	0.168	0.077	0.359	0.486	4.310	0.183	2.418

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	180	180	218	492	0	0	0	0	-1
normalized size	1	1.00	1.21	2.73	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.270	0.538	0.079	0.000	0.477	0.000	0.000	0.000

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	218	431	613	547	3516	758	2746
normalized size	1	1.00	1.27	2.51	3.56	3.18	20.44	4.41	15.97
time (sec)	N/A	0.262	0.263	0.077	0.377	0.523	10.589	0.199	3.424

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	263	263	336	1768	0	0	0	0	-1
normalized size	1	1.00	1.28	6.72	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.467	0.743	0.858	0.000	0.651	0.000	0.000	0.000

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	213	6834	629	0	0	0	-1
normalized size	1	1.00	1.33	42.71	3.93	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.263	1.135	0.612	0.637	0.464	0.000	0.000	0.000

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	257	257	599	1842	0	0	0	0	-1
normalized size	1	1.00	2.33	7.17	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.492	0.511	0.183	0.000	0.438	0.000	0.000	0.000

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	248	2001	0	0	0	0	-1
normalized size	1	1.00	1.73	13.99	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.304	0.605	0.511	0.000	0.501	0.000	0.000	0.000

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	335	5796	0	0	0	0	-1
normalized size	1	1.00	2.02	34.92	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.335	1.178	0.681	0.000	0.488	0.000	0.000	0.000

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	269	269	393	2247	0	0	0	0	-1
normalized size	1	1.00	1.46	8.35	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.514	1.334	0.985	0.000	0.523	0.000	0.000	0.000

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	31	34	58	0	0	0	-1
normalized size	1	1.00	1.48	1.62	2.76	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.022	0.003	0.046	0.317	0.456	0.000	0.000	0.000

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	52	59	132	0	0	0	-1
normalized size	1	1.00	1.62	1.84	4.12	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.031	0.006	0.046	0.337	0.459	0.000	0.000	0.000

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	270	786	333	386	644	3990	737
normalized size	1	1.00	1.61	4.68	1.98	2.30	3.83	23.75	4.39
time (sec)	N/A	0.331	0.281	0.043	0.363	0.506	7.640	0.634	1.724

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	174	477	207	242	369	1717	381
normalized size	1	1.00	1.45	3.98	1.72	2.02	3.08	14.31	3.18
time (sec)	N/A	0.204	0.168	0.038	0.319	0.479	4.567	0.296	1.387

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	138	184	109	134	173	536	136
normalized size	1	1.00	1.42	1.90	1.12	1.38	1.78	5.53	1.40
time (sec)	N/A	0.169	0.045	0.036	0.324	0.520	2.348	0.207	1.337

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	48	44	36	61	46	200	48
normalized size	1	1.00	1.20	1.10	0.90	1.52	1.15	5.00	1.20
time (sec)	N/A	0.024	0.016	0.027	0.318	0.456	0.602	0.518	1.440

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	126	202	0	0	0	0	-1
normalized size	1	1.00	0.97	1.55	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.136	0.076	0.123	0.000	0.494	0.000	0.000	0.000

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	125	141	121	263	1658	896	170
normalized size	1	1.00	1.09	1.23	1.05	2.29	14.42	7.79	1.48
time (sec)	N/A	0.169	0.181	0.038	0.322	0.613	10.012	0.273	1.638

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	174	236	291	834	0	4720	417
normalized size	1	1.00	1.04	1.41	1.74	4.99	0.00	28.26	2.50
time (sec)	N/A	0.254	0.320	0.045	0.336	1.602	0.000	0.412	3.034

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	562	562	1082	4401	1363	0	0	0	-1
normalized size	1	1.00	1.93	7.83	2.43	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.041	7.481	0.087	0.647	0.593	0.000	0.000	0.000

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	374	374	795	2694	806	0	0	0	-1
normalized size	1	1.00	2.13	7.20	2.16	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.640	3.809	0.071	0.624	0.569	0.000	0.000	0.000

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	221	220	271	857	415	0	0	0	-1
normalized size	1	1.00	1.23	3.88	1.88	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.462	0.541	0.064	0.617	0.515	0.000	0.000	0.000

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	107	174	0	0	0	0	-1
normalized size	1	1.00	1.10	1.79	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.121	0.189	0.227	0.000	0.420	0.000	0.000	0.000

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	214	214	1757	1984	0	0	0	0	-1
normalized size	1	1.00	8.21	9.27	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.154	24.276	0.967	0.000	0.588	0.000	0.000	0.000

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	480	485	425	783	0	0	0	0	-1
normalized size	1	1.01	0.89	1.63	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.713	7.526	0.080	0.000	0.509	0.000	0.000	0.000

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	750	750	1968	1428	0	0	0	0	-1
normalized size	1	1.00	2.62	1.90	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.126	14.789	0.094	0.000	0.551	0.000	0.000	0.000

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	546	546	1868	12111	0	0	0	0	-1
normalized size	1	1.00	3.42	22.18	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.046	10.161	22.267	0.000	0.705	0.000	0.000	0.000

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	326	325	566	20255	0	0	0	0	-1
normalized size	1	1.00	1.74	62.13	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.726	1.155	1.684	0.000	0.660	0.000	0.000	0.000

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	205	346	0	0	0	0	-1
normalized size	1	1.00	1.55	2.62	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.230	0.272	0.286	0.000	0.521	0.000	0.000	0.000

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	C	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	308	308	0	4064	0	0	0	0	-1
normalized size	1	1.00	0.00	13.19	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.193	22.912	0.868	0.000	0.568	0.000	0.000	0.000

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	1089	1094	1946	5728	0	0	0	0	-1
normalized size	1	1.00	1.79	5.26	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.827	23.211	1.003	0.000	0.905	0.000	0.000	0.000

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.068	4.509	1.619	0.000	0.696	0.000	0.000	0.000

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.069	0.303	1.401	0.000	0.533	0.000	0.000	0.000

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.257	0.081	1.417	0.000	0.696	0.000	0.000	0.000

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	780	780	623	587	0	0	0	0	-1
normalized size	1	1.00	0.80	0.75	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.395	0.758	0.913	0.000	0.547	0.000	0.000	0.000

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	481	481	365	1300	589	0	0	0	-1
normalized size	1	1.00	0.76	2.70	1.22	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.606	0.307	0.742	0.586	0.515	0.000	0.000	0.000

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	138	176	192	0	0	0	-1
normalized size	1	1.00	1.15	1.47	1.60	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.130	0.013	0.122	0.331	0.483	0.000	0.000	0.000

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	186	186	759	297	192	0	0	0	-1
normalized size	1	1.00	4.08	1.60	1.03	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.237	4.146	0.115	0.321	0.528	0.000	0.000	0.000

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	545	545	1456	20505	647	0	0	0	-1
normalized size	1	1.00	2.67	37.62	1.19	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.911	25.322	1.814	0.550	1.122	0.000	0.000	0.000

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	832	832	917	650	0	0	0	0	-1
normalized size	1	1.00	1.10	0.78	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.430	8.063	0.890	0.000	0.563	0.000	0.000	0.000

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	585	585	549	738	0	0	0	0	-1
normalized size	1	1.00	0.94	1.26	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.013	0.539	0.100	0.000	0.507	0.000	0.000	0.000

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	661	661	598	970	0	0	0	0	-1
normalized size	1	1.00	0.90	1.47	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.072	0.638	0.104	0.000	0.726	0.000	0.000	0.000

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	335	335	403	2140	0	0	0	0	-1
normalized size	1	1.00	1.20	6.39	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.721	0.572	0.825	0.000	0.613	0.000	0.000	0.000

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	114	194	0	0	0	0	-1
normalized size	1	1.00	1.37	2.34	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.263	0.121	0.061	0.000	0.786	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules**

column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [7] had the largest ratio of [1.333]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	19	15	1.00	12	1.250
2	A	15	13	1.00	12	1.083
3	A	12	10	1.00	10	1.000
4	A	6	6	1.00	8	0.750
5	A	2	2	1.00	12	0.167
6	A	17	15	1.00	12	1.250
7	A	21	16	1.00	12	1.333
8	A	4	5	1.00	12	0.417
9	A	6	5	1.00	21	0.238
10	A	6	5	1.00	21	0.238
11	A	5	5	1.00	19	0.263
12	A	3	3	1.00	21	0.143
13	A	7	7	1.00	21	0.333
14	A	5	5	1.00	21	0.238
15	A	13	9	1.00	23	0.391
16	A	11	10	1.00	23	0.435
17	A	8	7	1.00	21	0.333
18	A	8	7	1.00	23	0.304
19	A	6	6	1.00	23	0.261
20	A	10	9	1.00	23	0.391
21	A	10	9	1.00	23	0.391
22	A	15	10	1.00	23	0.435

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
23	A	14	11	1.00	23	0.478
24	A	10	10	1.00	21	0.476
25	A	10	8	1.00	23	0.348
26	A	7	8	1.00	23	0.348
27	A	9	8	1.00	23	0.348
28	A	16	13	1.00	23	0.565
29	A	3	3	1.00	12	0.250
30	A	3	3	1.00	19	0.158
31	A	7	5	1.00	18	0.278
32	A	7	5	1.00	18	0.278
33	A	7	5	1.00	16	0.312
34	A	4	3	1.00	10	0.300
35	A	5	5	1.00	18	0.278
36	A	7	5	1.00	18	0.278
37	A	5	4	1.00	18	0.222
38	A	20	15	1.00	20	0.750
39	A	16	13	1.00	20	0.650
40	A	13	10	1.00	18	0.556
41	A	6	6	1.00	12	0.500
42	A	2	2	1.00	20	0.100
43	A	21	19	1.01	20	0.950
44	A	26	18	1.00	20	0.900
45	A	21	14	1.00	20	0.700
46	A	15	11	1.00	18	0.611
47	A	6	7	1.00	12	0.583
48	A	2	2	1.00	20	0.100

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
49	A	30	18	1.00	20	0.900
50	A	0	0	0.00	0	0.000
51	A	0	0	0.00	0	0.000
52	A	6	4	1.00	18	0.222
53	A	23	5	1.00	16	0.312
54	A	17	5	1.00	16	0.312
55	A	5	5	1.00	14	0.357
56	A	15	7	1.00	16	0.438
57	A	25	7	1.00	16	0.438
58	A	31	7	1.00	16	0.438
59	A	31	13	1.00	18	0.722
60	A	37	16	1.00	18	0.889
61	A	12	8	1.00	19	0.421
62	A	6	5	1.00	32	0.156

Chapter 3

Listing of integrals

3.1 $\int x^3 \tanh^{-1}(a + bx)^2 dx$

Optimal. Leaf size=263

$$\frac{a(a^2 + 1) \operatorname{Li}_2\left(-\frac{a+bx+1}{-a-bx+1}\right)}{b^4} + \frac{(6a^2 + 1) \log(1 - (a + bx)^2)}{4b^4} - \frac{a(a^2 + 1) \tanh^{-1}(a + bx)^2}{b^4} + \frac{(6a^2 + 1)(a + bx) \tanh^{-1}(a + bx)}{2b^4}$$

[Out] $-a*x/b^3+1/12*(b*x+a)^2/b^4+a*\operatorname{arctanh}(b*x+a)/b^4+1/2*(6*a^2+1)*(b*x+a)*\operatorname{arctanh}(b*x+a)/b^4-a*(b*x+a)^2*\operatorname{arctanh}(b*x+a)/b^4+1/6*(b*x+a)^3*\operatorname{arctanh}(b*x+a)/b^4-a*(a^2+1)*\operatorname{arctanh}(b*x+a)^2/b^4-1/4*(a^4+6*a^2+1)*\operatorname{arctanh}(b*x+a)^2/b^4+1/4*x^4*\operatorname{arctanh}(b*x+a)^2+2*a*(a^2+1)*\operatorname{arctanh}(b*x+a)*\ln(2/(-b*x-a+1))/b^4+1/12*\ln(1-(b*x+a)^2)/b^4+1/4*(6*a^2+1)*\ln(1-(b*x+a)^2)/b^4+a*(a^2+1)*\operatorname{polylog}(2,(-b*x-a-1)/(-b*x-a+1))/b^4$

Rubi [A] time = 0.34, antiderivative size = 263, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 15, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 1.250$, Rules used = {6111, 5928, 5910, 260, 5916, 321, 206, 266, 43, 6048, 5948, 5984, 5918, 2402, 2315}

$$\frac{a(a^2 + 1) \operatorname{PolyLog}\left(2, -\frac{a+bx+1}{-a-bx+1}\right)}{b^4} + \frac{(6a^2 + 1) \log(1 - (a + bx)^2)}{4b^4} - \frac{a(a^2 + 1) \tanh^{-1}(a + bx)^2}{b^4} - \frac{(a^4 + 6a^2 + 1) \tanh^{-1}(a + bx)}{4b^4}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[x^3*\operatorname{ArcTanh}[a + b*x]^2, x]$

[Out] $-((a*x)/b^3) + (a + b*x)^2/(12*b^4) + (a*\operatorname{ArcTanh}[a + b*x])/b^4 + ((1 + 6*a^2)*(a + b*x)*\operatorname{ArcTanh}[a + b*x])/(2*b^4) - (a*(a + b*x)^2*\operatorname{ArcTanh}[a + b*x])/b^4 + ((a + b*x)^3*\operatorname{ArcTanh}[a + b*x])/(6*b^4) - (a*(1 + a^2)*\operatorname{ArcTanh}[a + b*x])$

$$\frac{\text{^2}}{b^4} - \frac{((1 + 6a^2 + a^4) \text{ArcTanh}[a + b*x]^2)/(4*b^4) + (x^4 \text{ArcTanh}[a + b*x]^2)/4 + (2*a*(1 + a^2) \text{ArcTanh}[a + b*x] \text{Log}[2/(1 - a - b*x)])}{b^4} + \text{Log}[1 - (a + b*x)^2]/(12*b^4) + \frac{((1 + 6*a^2) \text{Log}[1 - (a + b*x)^2])}{(4*b^4)} + \frac{(a*(1 + a^2) \text{PolyLog}[2, -(1 + a + b*x)/(1 - a - b*x)])}{b^4}$$
Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] := Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^p, x_Symbol] := Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 321

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^p, x_Symbol] := Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
+ 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] := -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] := -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
```

c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]

Rule 5910

Int[((a_.) + ArcTanh[(c_.)*(x_)])*(b_.))^(p_.), x_Symbol] := Simp[x*(a + b*ArcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]

Rule 5916

Int[((a_.) + ArcTanh[(c_.)*(x_)])*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol] := Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || IntegerQ[m]) && NeQ[m, -1]

Rule 5918

Int[((a_.) + ArcTanh[(c_.)*(x_)])*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol] := -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/((1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0]

Rule 5928

Int[((a_.) + ArcTanh[(c_.)*(x_)])*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.), x_Symbol] := Simp[((d + e*x)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(e*(q + 1)), x] - Dist[(b*c*p)/(e*(q + 1)), Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^(p - 1), (d + e*x)^(q + 1)/(1 - c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]

Rule 5948

Int[((a_.) + ArcTanh[(c_.)*(x_)])*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbol] := Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]

Rule 5984

Int[((a_.) + ArcTanh[(c_.)*(x_)])*(b_.))^(p_.)*(x_)/((d_) + (e_.)*(x_)^2), x_Symbol] := Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]

Rule 6048

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.))/(
(d_) + (e_.)*(x_)^2), x_Symbol] := Int[ExpandIntegrand[(a + b*ArcTanh[c*x])
^p/(d + e*x^2), (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && I
GtQ[p, 0] && EqQ[c^2*d + e, 0] && IGtQ[m, 0]
```

Rule 6111

```
Int[((a_.) + ArcTanh[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] := Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcTanh[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGt
Q[p, 0]
```

Rubi steps

$$\begin{aligned}
\int x^3 \tanh^{-1}(a + bx)^2 dx &= \frac{\text{Subst}\left(\int \left(-\frac{a}{b} + \frac{x}{b}\right)^3 \tanh^{-1}(x)^2 dx, x, a + bx\right)}{b} \\
&= \frac{1}{4}x^4 \tanh^{-1}(a + bx)^2 - \frac{1}{2} \text{Subst}\left(\int \left(-\frac{(1 + 6a^2) \tanh^{-1}(x)}{b^4} + \frac{4ax \tanh^{-1}(x)}{b^4} - \frac{x^2 \tanh^{-1}(x)}{b^4}\right) dx, x, a + bx\right) \\
&= \frac{1}{4}x^4 \tanh^{-1}(a + bx)^2 + \frac{\text{Subst}\left(\int x^2 \tanh^{-1}(x) dx, x, a + bx\right)}{2b^4} - \frac{\text{Subst}\left(\int \frac{(1+6a^2+a^4-4a^2x^2) \tanh^{-1}(x)}{b^4} dx, x, a + bx\right)}{b^4} \\
&= \frac{(1 + 6a^2)(a + bx) \tanh^{-1}(a + bx)}{2b^4} - \frac{a(a + bx)^2 \tanh^{-1}(a + bx)}{b^4} + \frac{(a + bx)^3 \tanh^{-1}(a + bx)}{6b^4} \\
&= -\frac{ax}{b^3} + \frac{(1 + 6a^2)(a + bx) \tanh^{-1}(a + bx)}{2b^4} - \frac{a(a + bx)^2 \tanh^{-1}(a + bx)}{b^4} + \frac{(a + bx)^3 \tanh^{-1}(a + bx)}{6b^4} \\
&= -\frac{ax}{b^3} + \frac{a \tanh^{-1}(a + bx)}{b^4} + \frac{(1 + 6a^2)(a + bx) \tanh^{-1}(a + bx)}{2b^4} - \frac{a(a + bx)^2 \tanh^{-1}(a + bx)}{b^4} \\
&= -\frac{ax}{b^3} + \frac{(a + bx)^2}{12b^4} + \frac{a \tanh^{-1}(a + bx)}{b^4} + \frac{(1 + 6a^2)(a + bx) \tanh^{-1}(a + bx)}{2b^4} - \frac{a(a + bx)^2 \tanh^{-1}(a + bx)}{b^4} \\
&= -\frac{ax}{b^3} + \frac{(a + bx)^2}{12b^4} + \frac{a \tanh^{-1}(a + bx)}{b^4} + \frac{(1 + 6a^2)(a + bx) \tanh^{-1}(a + bx)}{2b^4} - \frac{a(a + bx)^2 \tanh^{-1}(a + bx)}{b^4} \\
&= -\frac{ax}{b^3} + \frac{(a + bx)^2}{12b^4} + \frac{a \tanh^{-1}(a + bx)}{b^4} + \frac{(1 + 6a^2)(a + bx) \tanh^{-1}(a + bx)}{2b^4} - \frac{a(a + bx)^2 \tanh^{-1}(a + bx)}{b^4}
\end{aligned}$$

Mathematica [A] time = 1.26, size = 187, normalized size = 0.71

$$12(a^3 + a) \text{Li}_2\left(-e^{-2 \tanh^{-1}(a+bx)}\right) + 36a^2 \log\left(\frac{1}{\sqrt{1-(a+bx)^2}}\right) + 11a^2 - 2 \tanh^{-1}(a + bx) \left(12(a^3 + a) \log\left(e^{-2 \tanh^{-1}(a+bx)}\right) + \dots\right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[x^3*ArcTanh[a + b*x]^2,x]

[Out] -1/12*(1 + 11*a^2 + 10*a*b*x - b^2*x^2 + 3*(1 - 4*a + 6*a^2 - 4*a^3 + a^4 - b^4*x^4)*ArcTanh[a + b*x]^2 - 2*ArcTanh[a + b*x]*(9*a + 13*a^3 + 3*b*x + 9*a^2*b*x - 3*a*b^2*x^2 + b^3*x^3 + 12*(a + a^3)*Log[1 + E^(-2*ArcTanh[a + b*x])])

*x)))] + 8*Log[1/Sqrt[1 - (a + b*x)^2]] + 36*a^2*Log[1/Sqrt[1 - (a + b*x)^2]] + 12*(a + a^3)*PolyLog[2, -E^(-2*ArcTanh[a + b*x]))]/b^4

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}(x^3 \operatorname{artanh}(bx + a)^2, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*arctanh(b*x+a)^2,x, algorithm="fricas")

[Out] integral(x^3*arctanh(b*x + a)^2, x)

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*arctanh(b*x+a)^2,x, algorithm="giac")

[Out] Timed out

maple [B] time = 0.06, size = 967, normalized size = 3.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*arctanh(b*x+a)^2,x)

[Out] 1/2/b^4*ln(-1/2*b*x-1/2*a+1/2)*ln(1/2+1/2*b*x+1/2*a)*a^3+3/4/b^4*ln(-1/2*b*x-1/2*a+1/2)*ln(1/2+1/2*b*x+1/2*a)*a^2-3/4/b^4*ln(b*x+a+1)*ln(-1/2*b*x-1/2*a+1/2)*a^2-1/b^4*arctanh(b*x+a)*ln(b*x+a-1)*a+1/2/b^4*ln(-1/2*b*x-1/2*a+1/2)*ln(1/2+1/2*b*x+1/2*a)*a-1/8/b^4*ln(b*x+a+1)*ln(-1/2*b*x-1/2*a+1/2)*a^4+1/4/b^4*arctanh(b*x+a)*ln(b*x+a-1)*a^4+1/2/b^4*ln(b*x+a-1)*ln(1/2+1/2*b*x+1/2*a)*a-1/8/b^4*ln(b*x+a-1)*ln(1/2+1/2*b*x+1/2*a)*a^4+3/2/b^4*arctanh(b*x+a)*ln(b*x+a-1)*a^2+1/2/b^4*ln(b*x+a-1)*ln(1/2+1/2*b*x+1/2*a)*a^3-3/4/b^4*ln(b*x+a-1)*ln(1/2+1/2*b*x+1/2*a)*a^2-1/b^4*arctanh(b*x+a)*ln(b*x+a+1)*a^3-3/2/b^4*arctanh(b*x+a)*ln(b*x+a+1)*a^2-1/b^4*arctanh(b*x+a)*ln(b*x+a+1)*a-1/b^4*arctanh(b*x+a)*ln(b*x+a-1)*a^3-1/4/b^4*arctanh(b*x+a)*ln(b*x+a+1)*a^4-1/2/b^4*arctanh(b*x+a)*x^2*a+3/2/b^3*arctanh(b*x+a)*x*a^2-1/2/b^4*ln(b*x+a+1)*ln(-1/2*b*x-1/2*a+1/2)*a^3-1/2/b^4*ln(b*x+a+1)*ln(-1/2*b*x-1/2*a+1/2)*a+1/8/b^4*ln(-1/2*b*x-1/2*a+1/2)*ln(1/2+1/2*b*x+1/2*a)*a^4+1/2*a*arctanh(b*x+a)/b^4-5/6*a*x/b^3-1/4/b^4*arctanh(b*x+a)*ln(b*x+a+1)-1/8/b^4*ln(-1/2*b*x-1/2*a+1/2)*ln(b*x+a+1)+1/8/b^4*ln(-1/2*b*x-1/2*a+1/2)*ln(1/2+1/2*b*x+1/2*a)+13/6/b^4*arctanh(b*x+a)*a^3+3/2/b^4*ln(b*x+a-1)*a^2+3/2/b^4*ln(b*x+a+1)*a^2-1/8/

$b^4 \ln(bx+a-1) \ln(1/2+1/2bx+1/2a) + 1/4/b^4 \operatorname{arctanh}(bx+a) \ln(bx+a-1) + 3/8/b^4 \ln(bx+a+1)^2 a^2 + 1/4/b^4 \ln(bx+a+1)^2 a + 1/16/b^4 \ln(bx+a-1)^2 a^4 - 1/4/b^4 \ln(bx+a-1)^2 a^3 + 3/8/b^4 \ln(bx+a-1)^2 a^2 - 1/4/b^4 \ln(bx+a-1)^2 a + 1/b^4 \operatorname{dilog}(1/2+1/2bx+1/2a) a^3 + 1/b^4 \operatorname{dilog}(1/2+1/2bx+1/2a) a - 1/2/b^4 a \ln(bx+a-1) + 1/2/b^4 a \ln(bx+a+1) + 1/6/b^4 \operatorname{arctanh}(bx+a) x^3 + 1/2/b^4 \operatorname{arctanh}(bx+a) x + 1/16/b^4 \ln(bx+a+1)^2 a^4 + 1/4/b^4 \ln(bx+a+1)^2 a^3 - 11/12/b^4 a^2 + 1/12/b^4 x^2 + 1/3/b^4 \ln(bx+a+1) + 1/3/b^4 \ln(bx+a-1) + 1/16/b^4 \ln(bx+a-1)^2 + 1/16/b^4 \ln(bx+a+1)^2 + 1/4 x^4 \operatorname{arctanh}(bx+a)^2$

maxima [A] time = 0.33, size = 320, normalized size = 1.22

$$\frac{1}{4} x^4 \operatorname{artanh}(bx+a)^2 + \frac{1}{48} b^2 \left(\frac{48(a^3+a) \left(\log(bx+a-1) \log\left(\frac{1}{2}bx + \frac{1}{2}a + \frac{1}{2}\right) + \operatorname{Li}_2\left(-\frac{1}{2}bx - \frac{1}{2}a + \frac{1}{2}\right)\right)}{b^6} + \frac{4(13a^3 + 18a^2 + 9a + 4) \log(bx+a+1)}{b^6} + \frac{(4b^2x^2 - 40abx + 3(a^4 + 4a^3 + 6a^2 + 4a + 1) \log(bx+a+1)^2 - 6(a^4 + 4a^3 + 6a^2 + 4a + 1) \log(bx+a+1) \log(bx+a-1) + 3(a^4 - 4a^3 + 6a^2 - 4a + 1) \log(bx+a-1)^2 - 4(13a^3 - 18a^2 + 9a - 4) \log(bx+a-1))}{b^6} + \frac{1}{12} b (2(b^2x^3 - 3abx^2 + 3(3a^2 + 1)x) / b^4 - 3(a^4 + 4a^3 + 6a^2 + 4a + 1) \log(bx+a+1) / b^5 + 3(a^4 - 4a^3 + 6a^2 - 4a + 1) \log(bx+a-1) / b^5) \operatorname{arctanh}(bx+a) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*arctanh(b*x+a)^2,x, algorithm="maxima")

[Out] 1/4*x^4*arctanh(b*x + a)^2 + 1/48*b^2*(48*(a^3 + a)*(log(b*x + a - 1)*log(1/2*b*x + 1/2*a + 1/2) + dilog(-1/2*b*x - 1/2*a + 1/2))/b^6 + 4*(13*a^3 + 18*a^2 + 9*a + 4)*log(b*x + a + 1)/b^6 + (4*b^2*x^2 - 40*a*b*x + 3*(a^4 + 4*a^3 + 6*a^2 + 4*a + 1)*log(b*x + a + 1)^2 - 6*(a^4 + 4*a^3 + 6*a^2 + 4*a + 1)*log(b*x + a + 1)*log(b*x + a - 1) + 3*(a^4 - 4*a^3 + 6*a^2 - 4*a + 1)*log(b*x + a - 1)^2 - 4*(13*a^3 - 18*a^2 + 9*a - 4)*log(b*x + a - 1))/b^6) + 1/12*b*(2*(b^2*x^3 - 3*a*b*x^2 + 3*(3*a^2 + 1)*x)/b^4 - 3*(a^4 + 4*a^3 + 6*a^2 + 4*a + 1)*log(b*x + a + 1)/b^5 + 3*(a^4 - 4*a^3 + 6*a^2 - 4*a + 1)*log(b*x + a - 1)/b^5)*arctanh(b*x + a)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^3 \operatorname{atanh}(a + bx)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*atanh(a + b*x)^2,x)

[Out] int(x^3*atanh(a + b*x)^2, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \operatorname{atanh}^2(a + bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3*atanh(b*x+a)**2,x)

[Out] Integral(x**3*atanh(a + b*x)**2, x)

3.2 $\int x^2 \tanh^{-1}(a + bx)^2 dx$

Optimal. Leaf size=204

$$-\frac{(3a^2 + 1) \operatorname{Li}_2\left(-\frac{a+bx+1}{-a-bx+1}\right)}{3b^3} + \frac{a(a^2 + 3) \tanh^{-1}(a + bx)^2}{3b^3} + \frac{(3a^2 + 1) \tanh^{-1}(a + bx)^2}{3b^3} - \frac{2(3a^2 + 1) \log\left(\frac{2}{-a-bx+1}\right) \tanh^{-1}(a + bx)}{3b^3}$$

[Out] $1/3*x/b^2-1/3*\operatorname{arctanh}(b*x+a)/b^3-2*a*(b*x+a)*\operatorname{arctanh}(b*x+a)/b^3+1/3*(b*x+a)^2*\operatorname{arctanh}(b*x+a)/b^3+1/3*a*(a^2+3)*\operatorname{arctanh}(b*x+a)^2/b^3+1/3*(3*a^2+1)*\operatorname{arctanh}(b*x+a)^2/b^3+1/3*x^3*\operatorname{arctanh}(b*x+a)^2-2/3*(3*a^2+1)*\operatorname{arctanh}(b*x+a)*\ln(2/(-b*x-a+1))/b^3-a*\ln(1-(b*x+a)^2)/b^3-1/3*(3*a^2+1)*\operatorname{polylog}(2,(-b*x-a-1)/(-b*x-a+1))/b^3$

Rubi [A] time = 0.27, antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 13, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 1.083$, Rules used = {6111, 5928, 5910, 260, 5916, 321, 206, 6048, 5948, 5984, 5918, 2402, 2315}

$$-\frac{(3a^2 + 1) \operatorname{PolyLog}\left(2, -\frac{a+bx+1}{-a-bx+1}\right)}{3b^3} + \frac{a(a^2 + 3) \tanh^{-1}(a + bx)^2}{3b^3} + \frac{(3a^2 + 1) \tanh^{-1}(a + bx)^2}{3b^3} - \frac{2(3a^2 + 1) \log\left(\frac{2}{-a-bx+1}\right) \tanh^{-1}(a + bx)}{3b^3}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[x^2*\operatorname{ArcTanh}[a + b*x]^2, x]$

[Out] $x/(3*b^2) - \operatorname{ArcTanh}[a + b*x]/(3*b^3) - (2*a*(a + b*x)*\operatorname{ArcTanh}[a + b*x])/b^3 + ((a + b*x)^2*\operatorname{ArcTanh}[a + b*x])/((3*b^3) + (a*(3 + a^2)*\operatorname{ArcTanh}[a + b*x]^2)/((3*b^3) + ((1 + 3*a^2)*\operatorname{ArcTanh}[a + b*x]^2)/((3*b^3) + (x^3*\operatorname{ArcTanh}[a + b*x]^2)/3 - (2*(1 + 3*a^2)*\operatorname{ArcTanh}[a + b*x]*\operatorname{Log}[2/(1 - a - b*x)])/((3*b^3) - (a*\operatorname{Log}[1 - (a + b*x)^2])/b^3 - ((1 + 3*a^2)*\operatorname{PolyLog}[2, -((1 + a + b*x)/(1 - a - b*x))])/((3*b^3)$

Rule 206

$\operatorname{Int}[(a_) + (b_.)*(x_)^2)^{-1}, x_Symbol] \rightarrow \operatorname{Simp}[(1*\operatorname{ArcTanh}[\operatorname{Rt}[-b, 2]*x]/\operatorname{Rt}[a, 2])]/(\operatorname{Rt}[a, 2]*\operatorname{Rt}[-b, 2]), x] /;$ $\operatorname{FreeQ}\{a, b\}, x \ \&\& \operatorname{NegQ}[a/b] \ \&\& (\operatorname{Gt} Q[a, 0] \ || \ \operatorname{Lt} Q[b, 0])$

Rule 260

$\operatorname{Int}[(x_)^{(m_.)}/((a_) + (b_.)*(x_)^{(n_.)}), x_Symbol] \rightarrow \operatorname{Simp}[\operatorname{Log}[\operatorname{RemoveContent}[a + b*x^n, x]]]/(b*n), x] /;$ $\operatorname{FreeQ}\{a, b, m, n\}, x \ \&\& \operatorname{Eq} Q[m, n - 1]$

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
+ 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] := -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] := -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] := Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:= Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2
*x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
:= -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/((1 - c^2*x^2)
), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0]
]
```

Rule 5928

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.), x_S
ymbol] := Simp[((d + e*x)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(e*(q + 1)), x] -
Dist[(b*c*p)/(e*(q + 1)), Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^(p - 1)
, (d + e*x)^(q + 1)/(1 - c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x]
```

&& IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]

Rule 5948

Int[((a_.) + ArcTanh[(c_.)*(x_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbol] := Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]

Rule 5984

Int[(((a_.) + ArcTanh[(c_.)*(x_.)]*(b_.))^(p_.)*(x_.))/((d_) + (e_.)*(x_)^2), x_Symbol] := Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]

Rule 6048

Int[(((a_.) + ArcTanh[(c_.)*(x_.)]*(b_.))^(p_.)*((f_) + (g_.)*(x_)^(m_.)))/((d_) + (e_.)*(x_)^2), x_Symbol] := Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p/(d + e*x^2), (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGtQ[p, 0] && EqQ[c^2*d + e, 0] && IGtQ[m, 0]

Rule 6111

Int[((a_.) + ArcTanh[(c_) + (d_.)*(x_.)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_)^(m_.)), x_Symbol] := Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*ArcTanh[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]

Rubi steps

$$\begin{aligned}
\int x^2 \tanh^{-1}(a + bx)^2 dx &= \frac{\text{Subst}\left(\int \left(-\frac{a}{b} + \frac{x}{b}\right)^2 \tanh^{-1}(x)^2 dx, x, a + bx\right)}{b} \\
&= \frac{1}{3}x^3 \tanh^{-1}(a + bx)^2 - \frac{2}{3} \text{Subst}\left(\int \left(\frac{3a \tanh^{-1}(x)}{b^3} - \frac{x \tanh^{-1}(x)}{b^3} - \frac{(a(3+a^2) - (1}{b^3}\right)\right. \\
&= \frac{1}{3}x^3 \tanh^{-1}(a + bx)^2 + \frac{2 \text{Subst}\left(\int x \tanh^{-1}(x) dx, x, a + bx\right)}{3b^3} + \frac{2 \text{Subst}\left(\int \frac{(a(3+a^2) - (1}{b^3}\right)}{3b^3} \\
&= -\frac{2a(a + bx) \tanh^{-1}(a + bx)}{b^3} + \frac{(a + bx)^2 \tanh^{-1}(a + bx)}{3b^3} + \frac{1}{3}x^3 \tanh^{-1}(a + bx)^2 - \frac{S}{3b^3} \\
&= \frac{x}{3b^2} - \frac{2a(a + bx) \tanh^{-1}(a + bx)}{b^3} + \frac{(a + bx)^2 \tanh^{-1}(a + bx)}{3b^3} + \frac{1}{3}x^3 \tanh^{-1}(a + bx)^2 \\
&= \frac{x}{3b^2} - \frac{\tanh^{-1}(a + bx)}{3b^3} - \frac{2a(a + bx) \tanh^{-1}(a + bx)}{b^3} + \frac{(a + bx)^2 \tanh^{-1}(a + bx)}{3b^3} + \frac{a}{3b^3} \\
&= \frac{x}{3b^2} - \frac{\tanh^{-1}(a + bx)}{3b^3} - \frac{2a(a + bx) \tanh^{-1}(a + bx)}{b^3} + \frac{(a + bx)^2 \tanh^{-1}(a + bx)}{3b^3} + \frac{a}{3b^3} \\
&= \frac{x}{3b^2} - \frac{\tanh^{-1}(a + bx)}{3b^3} - \frac{2a(a + bx) \tanh^{-1}(a + bx)}{b^3} + \frac{(a + bx)^2 \tanh^{-1}(a + bx)}{3b^3} + \frac{a}{3b^3} \\
&= \frac{x}{3b^2} - \frac{\tanh^{-1}(a + bx)}{3b^3} - \frac{2a(a + bx) \tanh^{-1}(a + bx)}{b^3} + \frac{(a + bx)^2 \tanh^{-1}(a + bx)}{3b^3} + \frac{a}{3b^3}
\end{aligned}$$

Mathematica [B] time = 2.36, size = 463, normalized size = 2.27

$$(1 - (a + bx)^2)^{3/2} \left(-\frac{4(3a^2+1)\text{Li}_2\left(-e^{-2\tanh^{-1}(a+bx)}\right)}{(1-(a+bx)^2)^{3/2}} - \frac{3a^2(a+bx)\tanh^{-1}(a+bx)^2}{\sqrt{1-(a+bx)^2}} + \frac{3(3a^2-4a+1)\tanh^{-1}(a+bx)^2+2\tanh^{-1}(a+bx)\left((9a^2+1)\right)}{\sqrt{1-(a+bx)^2}} \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[x^2*ArcTanh[a + b*x]^2,x]

[Out] -1/12*((1 - (a + b*x)^2)^(3/2))*(-(a + b*x)/Sqrt[1 - (a + b*x)^2]) + (6*a*(a + b*x)*ArcTanh[a + b*x])/Sqrt[1 - (a + b*x)^2] + (3*(a + b*x)*ArcTanh[a +

$$\frac{b^2 x^2}{\sqrt{1 - (a + bx)^2}} - \frac{(3a^2(a + bx) \operatorname{ArcTanh}[a + bx]^2)}{\sqrt{1 - (a + bx)^2}} + \operatorname{ArcTanh}[a + bx]^2 \operatorname{Cosh}[3 \operatorname{ArcTanh}[a + bx]] + 3a^2 \operatorname{ArcTanh}[a + bx]^2 \operatorname{Cosh}[3 \operatorname{ArcTanh}[a + bx]] + 2 \operatorname{ArcTanh}[a + bx] \operatorname{Cosh}[3 \operatorname{ArcTanh}[a + bx]] \operatorname{Log}[1 + E^{(-2 \operatorname{ArcTanh}[a + bx])}] + 6a^2 \operatorname{ArcTanh}[a + bx] \operatorname{Cosh}[3 \operatorname{ArcTanh}[a + bx]] \operatorname{Log}[1 + E^{(-2 \operatorname{ArcTanh}[a + bx])}] - 6a \operatorname{Cosh}[3 \operatorname{ArcTanh}[a + bx]] \operatorname{Log}[1/\sqrt{1 - (a + bx)^2}] + (3(1 - 4a + 3a^2) \operatorname{ArcTanh}[a + bx]^2 + 2 \operatorname{ArcTanh}[a + bx] (2 + (3 + 9a^2) \operatorname{Log}[1 + E^{(-2 \operatorname{ArcTanh}[a + bx])}])) - 18a \operatorname{Log}[1/\sqrt{1 - (a + bx)^2}]) / \sqrt{1 - (a + bx)^2} - (4(1 + 3a^2) \operatorname{PolyLog}[2, -E^{(-2 \operatorname{ArcTanh}[a + bx])}]) / (1 - (a + bx)^2)^{3/2} - \operatorname{Sinh}[3 \operatorname{ArcTanh}[a + bx]] + 6a \operatorname{ArcTanh}[a + bx] \operatorname{Sinh}[3 \operatorname{ArcTanh}[a + bx]] - \operatorname{ArcTanh}[a + bx]^2 \operatorname{Sinh}[3 \operatorname{ArcTanh}[a + bx]] - 3a^2 \operatorname{ArcTanh}[a + bx]^2 \operatorname{Sinh}[3 \operatorname{ArcTanh}[a + bx]]) / b^3$$

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\operatorname{integral}(x^2 \operatorname{artanh}(bx + a)^2, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*arctanh(b*x+a)^2,x, algorithm="fricas")`

[Out] `integral(x^2*arctanh(b*x + a)^2, x)`

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \operatorname{artanh}(bx + a)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*arctanh(b*x+a)^2,x, algorithm="giac")`

[Out] `integrate(x^2*arctanh(b*x + a)^2, x)`

maple [B] time = 0.06, size = 729, normalized size = 3.57

$$\frac{\operatorname{arctanh}(bx + a) \ln(bx + a + 1) a^2}{b^3} - \frac{4 \operatorname{arctanh}(bx + a) xa}{3b^2} - \frac{\operatorname{arctanh}(bx + a) \ln(bx + a - 1) a}{b^3} - \frac{\ln\left(-\frac{bx}{2} - \frac{a}{2} + \frac{1}{2}\right)}{2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*arctanh(b*x+a)^2,x)`

[Out] `-1/2/b^3*ln(-1/2*b*x-1/2*a+1/2)*ln(1/2+1/2*b*x+1/2*a)*a^2+1/2/b^3*ln(b*x+a+1)*ln(-1/2*b*x-1/2*a+1/2)*a-1/6/b^3*ln(-1/2*b*x-1/2*a+1/2)*ln(1/2+1/2*b*x+1/2*a)*a^3+1/2/b^3*ln(b*x+a-1)*ln(1/2+1/2*b*x+1/2*a)*a-1/2/b^3*ln(-1/2*b*x-1/2*a+1/2)*ln(1/2+1/2*b*x+1/2*a)*a+1/b^3*arctanh(b*x+a)*ln(b*x+a-1)*a^2+1/b^3`

$3 \operatorname{arctanh}(bx+a) \ln(bx+a+1) a^{-1/3} / b^3 \operatorname{arctanh}(bx+a) \ln(bx+a-1) a^{3+1/3} / b^3 \operatorname{arctanh}(bx+a) \ln(bx+a+1) a^{3+1/3} / b^3 \operatorname{arctanh}(bx+a) \ln(bx+a+1) a^{2-4/3} / b^2 \operatorname{arctanh}(bx+a) x a^{-1/2} / b^3 \ln(bx+a-1) \ln(1/2+1/2bx+1/2a) a^{2+1/6} / b^3 \ln(bx+a+1) \ln(-1/2bx-1/2a+1/2) a^{3+1/2} / b^3 \ln(bx+a+1) \ln(-1/2bx-1/2a+1/2) a^{2-1/3} \operatorname{arctanh}(bx+a) \ln(bx+a-1) a^{1/6} / b^3 \ln(bx+a-1) \ln(1/2+1/2bx+1/2a) a^{3+1/3} x / b^2 - 1/b^3 \operatorname{dilog}(1/2+1/2bx+1/2a) a^{2-1/6} / b^3 \ln(bx+a-1) \ln(1/2+1/2bx+1/2a) + 1/3/b^3 \operatorname{arctanh}(bx+a) \ln(bx+a-1) + 1/3/b^3 \operatorname{arctanh}(bx+a) \ln(bx+a+1) + 1/6/b^3 \ln(-1/2bx-1/2a+1/2) \ln(bx+a+1) - 1/6/b^3 \ln(-1/2bx-1/2a+1/2) \ln(1/2+1/2bx+1/2a) - 5/3/b^3 \operatorname{arctanh}(bx+a) a^{2-1/12} / b^3 \ln(bx+a+1)^2 a^{3-1/4} / b^3 \ln(bx+a+1)^2 a^{2-1/4} / b^3 \ln(bx+a+1)^2 a^{-1/b^3} a \ln(bx+a-1) - 1/b^3 a \ln(bx+a+1) - 1/12/b^3 \ln(bx+a-1)^2 a^{3+1/4} / b^3 \ln(bx+a-1)^2 a^{2-1/4} / b^3 \ln(bx+a-1)^2 a^{1/3} / b \operatorname{arctanh}(bx+a) x^2 + 1/3/b^3 a^{-1/6} / b^3 \ln(bx+a+1) + 1/6/b^3 \ln(bx+a-1) - 1/3/b^3 \operatorname{dilog}(1/2+1/2bx+1/2a) + 1/12/b^3 \ln(bx+a-1)^2 - 1/12/b^3 \ln(bx+a+1)^2 + 1/3 x^3 \operatorname{arctanh}(bx+a)^2$

maxima [A] time = 0.33, size = 259, normalized size = 1.27

$$\frac{1}{3} x^3 \operatorname{arctanh}(bx+a)^2 - \frac{1}{12} b^2 \left(\frac{4(3a^2+1) \left(\log(bx+a-1) \log\left(\frac{1}{2}bx + \frac{1}{2}a + \frac{1}{2}\right) + \operatorname{Li}_2\left(-\frac{1}{2}bx - \frac{1}{2}a + \frac{1}{2}\right) \right)}{b^5} + \frac{2(5}{3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*arctanh(b*x+a)^2,x, algorithm="maxima")

[Out] $1/3 x^3 \operatorname{arctanh}(bx+a)^2 - 1/12 b^2 (4(3a^2+1) (\log(bx+a-1) \log(1/2bx + 1/2a + 1/2) + \operatorname{dilog}(-1/2bx - 1/2a + 1/2)) / b^5 + 2(5a^2 + 6a + 1) \log(bx+a+1) / b^5 + ((a^3 + 3a^2 + 3a + 1) \log(bx+a+1)^2 - 2(a^3 + 3a^2 + 3a + 1) \log(bx+a+1) \log(bx+a-1) + (a^3 - 3a^2 + 3a - 1) \log(bx+a-1)^2 - 4bx - 2(5a^2 - 6a + 1) \log(bx+a-1)) / b^5) + 1/3 b ((bx^2 - 4ax) / b^3 + (a^3 + 3a^2 + 3a + 1) \log(bx+a+1) / b^4 - (a^3 - 3a^2 + 3a - 1) \log(bx+a-1) / b^4) \operatorname{arctanh}(bx+a)$

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^2 \operatorname{atanh}(a+bx)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*atanh(a+b*x)^2,x)

[Out] int(x^2*atanh(a+b*x)^2, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \operatorname{atanh}^2(a+bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atanh(b*x+a)**2,x)
```

```
[Out] Integral(x**2*atanh(a + b*x)**2, x)
```

3.3 $\int x \tanh^{-1}(a + bx)^2 dx$

Optimal. Leaf size=136

$$-\frac{(a^2 + 1) \tanh^{-1}(a + bx)^2}{2b^2} + \frac{a \operatorname{Li}_2\left(-\frac{a+bx+1}{-a-bx+1}\right)}{b^2} + \frac{\log(1 - (a + bx)^2)}{2b^2} - \frac{a \tanh^{-1}(a + bx)^2}{b^2} + \frac{(a + bx) \tanh^{-1}(a + bx)}{b^2}$$

[Out] (b*x+a)*arctanh(b*x+a)/b^2-a*arctanh(b*x+a)^2/b^2-1/2*(a^2+1)*arctanh(b*x+a)^2/b^2+1/2*x^2*arctanh(b*x+a)^2+2*a*arctanh(b*x+a)*ln(2/(-b*x-a+1))/b^2+1/2*ln(1-(b*x+a)^2)/b^2+a*polylog(2,(-b*x-a-1)/(-b*x-a+1))/b^2

Rubi [A] time = 0.20, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {6111, 5928, 5910, 260, 6048, 5948, 5984, 5918, 2402, 2315}

$$\frac{a \operatorname{PolyLog}\left(2, -\frac{a+bx+1}{-a-bx+1}\right)}{b^2} - \frac{(a^2 + 1) \tanh^{-1}(a + bx)^2}{2b^2} + \frac{\log(1 - (a + bx)^2)}{2b^2} - \frac{a \tanh^{-1}(a + bx)^2}{b^2} + \frac{(a + bx) \tanh^{-1}(a + bx)}{b^2}$$

Antiderivative was successfully verified.

[In] Int[x*ArcTanh[a + b*x]^2,x]

[Out] ((a + b*x)*ArcTanh[a + b*x])/b^2 - (a*ArcTanh[a + b*x]^2)/b^2 - ((1 + a^2)*ArcTanh[a + b*x]^2)/(2*b^2) + (x^2*ArcTanh[a + b*x]^2)/2 + (2*a*ArcTanh[a + b*x]*Log[2/(1 - a - b*x)])/b^2 + Log[1 - (a + b*x)^2]/(2*b^2) + (a*PolyLog[2, -((1 + a + b*x)/(1 - a - b*x))])/b^2

Rule 260

Int[(x_)^(m_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 2315

Int[Log[(c_)*(x_)]/((d_) + (e_)*(x_)), x_Symbol] := -Simp[PolyLog[2, 1 - c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]

Rule 2402

Int[Log[(c_)/((d_) + (e_)*(x_))]/((f_) + (g_)*(x_)^2), x_Symbol] := -Dist[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]

Rule 5910

Int[((a_.) + ArcTanh[(c_.)*(x_)])*(b_.))^(p_.), x_Symbol] := Simp[x*(a + b*ArcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]

Rule 5918

Int[((a_.) + ArcTanh[(c_.)*(x_)])*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol] := -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/((1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0]

Rule 5928

Int[((a_.) + ArcTanh[(c_.)*(x_)])*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.), x_Symbol] := Simp[((d + e*x)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(e*(q + 1)), x] - Dist[(b*c*p)/(e*(q + 1)), Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^(p - 1), (d + e*x)^(q + 1)/(1 - c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]

Rule 5948

Int[((a_.) + ArcTanh[(c_.)*(x_)])*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbol] := Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]

Rule 5984

Int[(((a_.) + ArcTanh[(c_.)*(x_)])*(b_.))^(p_.)*(x_)/((d_) + (e_.)*(x_)^2), x_Symbol] := Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]

Rule 6048

Int[(((a_.) + ArcTanh[(c_.)*(x_)])*(b_.))^(p_.)*((f_) + (g_.)*(x_))^(m_.)/((d_) + (e_.)*(x_)^2), x_Symbol] := Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p/(d + e*x^2), (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGtQ[p, 0] && EqQ[c^2*d + e, 0] && IGtQ[m, 0]

Rule 6111

Int[((a_.) + ArcTanh[(c_) + (d_.)*(x_)])*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(

```
m_.), x_Symbol] := Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*ArcTanh[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int x \tanh^{-1}(a + bx)^2 dx &= \frac{\text{Subst}\left(\int \left(-\frac{a}{b} + \frac{x}{b}\right) \tanh^{-1}(x)^2 dx, x, a + bx\right)}{b} \\
&= \frac{1}{2}x^2 \tanh^{-1}(a + bx)^2 - \text{Subst}\left(\int \left(-\frac{\tanh^{-1}(x)}{b^2} + \frac{(1 + a^2 - 2ax) \tanh^{-1}(x)}{b^2(1 - x^2)}\right) dx, x, a + bx\right) \\
&= \frac{1}{2}x^2 \tanh^{-1}(a + bx)^2 + \frac{\text{Subst}\left(\int \tanh^{-1}(x) dx, x, a + bx\right)}{b^2} - \frac{\text{Subst}\left(\int \frac{(1 + a^2 - 2ax) \tanh^{-1}(x)}{1 - x^2} dx, x, a + bx\right)}{b^2} \\
&= \frac{(a + bx) \tanh^{-1}(a + bx)}{b^2} + \frac{1}{2}x^2 \tanh^{-1}(a + bx)^2 - \frac{\text{Subst}\left(\int \frac{x}{1 - x^2} dx, x, a + bx\right)}{b^2} - \frac{\text{Subst}\left(\int \frac{1}{1 - x^2} dx, x, a + bx\right)}{b^2} \\
&= \frac{(a + bx) \tanh^{-1}(a + bx)}{b^2} + \frac{1}{2}x^2 \tanh^{-1}(a + bx)^2 + \frac{\log(1 - (a + bx)^2)}{2b^2} + \frac{(2a) \text{Subst}\left(\int \frac{1}{1 - x^2} dx, x, a + bx\right)}{b^2} \\
&= \frac{(a + bx) \tanh^{-1}(a + bx)}{b^2} - \frac{a \tanh^{-1}(a + bx)^2}{b^2} - \frac{(1 + a^2) \tanh^{-1}(a + bx)^2}{2b^2} + \frac{1}{2}x^2 \tanh^{-1}(a + bx)^2 \\
&= \frac{(a + bx) \tanh^{-1}(a + bx)}{b^2} - \frac{a \tanh^{-1}(a + bx)^2}{b^2} - \frac{(1 + a^2) \tanh^{-1}(a + bx)^2}{2b^2} + \frac{1}{2}x^2 \tanh^{-1}(a + bx)^2 \\
&= \frac{(a + bx) \tanh^{-1}(a + bx)}{b^2} - \frac{a \tanh^{-1}(a + bx)^2}{b^2} - \frac{(1 + a^2) \tanh^{-1}(a + bx)^2}{2b^2} + \frac{1}{2}x^2 \tanh^{-1}(a + bx)^2 \\
&= \frac{(a + bx) \tanh^{-1}(a + bx)}{b^2} - \frac{a \tanh^{-1}(a + bx)^2}{b^2} - \frac{(1 + a^2) \tanh^{-1}(a + bx)^2}{2b^2} + \frac{1}{2}x^2 \tanh^{-1}(a + bx)^2
\end{aligned}$$

Mathematica [A] time = 0.28, size = 98, normalized size = 0.72

$$\frac{(-a^2 + 2a + b^2x^2 - 1) \tanh^{-1}(a + bx)^2 - 2a \text{Li}_2\left(-e^{-2 \tanh^{-1}(a + bx)}\right) - 2 \log\left(\frac{1}{\sqrt{1 - (a + bx)^2}}\right) + 2 \tanh^{-1}(a + bx) \left(2a \log\left(\frac{1}{\sqrt{1 - (a + bx)^2}}\right) + \tanh^{-1}(a + bx)\right)}{2b^2}$$

Warning: Unable to verify antiderivative.

[In] Integrate[x*ArcTanh[a + b*x]^2,x]

[Out] $((-1 + 2*a - a^2 + b^2*x^2)*ArcTanh[a + b*x]^2 + 2*ArcTanh[a + b*x]*(a + b*x + 2*a*Log[1 + E^{(-2*ArcTanh[a + b*x])}])) - 2*Log[1/Sqrt[1 - (a + b*x)^2]] - 2*a*PolyLog[2, -E^{(-2*ArcTanh[a + b*x])}])/(2*b^2)$

fricas [F] time = 0.55, size = 0, normalized size = 0.00

$$\text{integral}(x \operatorname{artanh}(bx + a)^2, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*arctanh(b*x+a)^2,x, algorithm="fricas")

[Out] integral(x*arctanh(b*x + a)^2, x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \operatorname{artanh}(bx + a)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*arctanh(b*x+a)^2,x, algorithm="giac")

[Out] integrate(x*arctanh(b*x + a)^2, x)

maple [B] time = 0.06, size = 365, normalized size = 2.68

$$\frac{x^2 \operatorname{arctanh}(bx + a)^2}{2} - \frac{\operatorname{arctanh}(bx + a)^2 a^2}{2b^2} + \frac{\operatorname{arctanh}(bx + a)x}{b} + \frac{\operatorname{arctanh}(bx + a)a}{b^2} - \frac{\operatorname{arctanh}(bx + a) \ln(bx + a)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*arctanh(b*x+a)^2,x)

[Out] $1/2*x^2*arctanh(b*x+a)^2 - 1/2/b^2*arctanh(b*x+a)^2*a^2 + 1/b*arctanh(b*x+a)*x + 1/b^2*arctanh(b*x+a)*a - 1/b^2*arctanh(b*x+a)*ln(b*x+a-1)*a + 1/2/b^2*arctanh(b*x+a)*ln(b*x+a-1) - 1/b^2*arctanh(b*x+a)*ln(b*x+a+1)*a - 1/2/b^2*arctanh(b*x+a)*ln(b*x+a+1) + 1/2/b^2*ln(b*x+a-1) + 1/2/b^2*ln(b*x+a+1) - 1/4/b^2*ln(b*x+a-1)^2*a + 1/b^2*dilog(1/2+1/2*b*x+1/2*a)*a + 1/2/b^2*ln(b*x+a-1)*ln(1/2+1/2*b*x+1/2*a) + 1/8/b^2*ln(b*x+a-1)^2 - 1/4/b^2*ln(b*x+a-1)*ln(1/2+1/2*b*x+1/2*a) + 1/4/b^2*ln(b*x+a+1)^2*a - 1/2/b^2*ln(b*x+a+1)*ln(-1/2*b*x-1/2*a+1/2)*a + 1/2/b^2*ln(-1/2*b*x-1/2*a+1/2)*ln(1/2+1/2*b*x+1/2*a)*a + 1/8/b^2*ln(b*x+a+1)^2 - 1/4/b^2*ln(-1/2*b*x-1/2*a+1/2)*ln(b*x+a+1) + 1/4/b^2*ln(-1/2*b*x-1/2*a+1/2)*ln(1/2+1/2*b*x+1/2*a)$

maxima [A] time = 0.34, size = 202, normalized size = 1.49

$$\frac{1}{2} x^2 \operatorname{artanh}(bx + a)^2 + \frac{1}{8} b^2 \left(\frac{8 \left(\log(bx + a - 1) \log\left(\frac{1}{2} bx + \frac{1}{2} a + \frac{1}{2}\right) + \operatorname{Li}_2\left(-\frac{1}{2} bx - \frac{1}{2} a + \frac{1}{2}\right)\right) a}{b^4} + \frac{4(a+1) \log(\dots)}{b^4} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*arctanh(b*x+a)^2,x, algorithm="maxima")

[Out] 1/2*x^2*arctanh(b*x + a)^2 + 1/8*b^2*(8*(log(b*x + a - 1)*log(1/2*b*x + 1/2*a + 1/2) + dilog(-1/2*b*x - 1/2*a + 1/2))*a/b^4 + 4*(a + 1)*log(b*x + a + 1)/b^4 + ((a^2 + 2*a + 1)*log(b*x + a + 1)^2 - 2*(a^2 + 2*a + 1)*log(b*x + a + 1)*log(b*x + a - 1) + (a^2 - 2*a + 1)*log(b*x + a - 1)^2 - 4*(a - 1)*log(b*x + a - 1))/b^4 + 1/2*b*(2*x/b^2 - (a^2 + 2*a + 1)*log(b*x + a + 1)/b^3 + (a^2 - 2*a + 1)*log(b*x + a - 1)/b^3)*arctanh(b*x + a)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x \operatorname{atanh}(a + bx)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*atanh(a + b*x)^2,x)

[Out] int(x*atanh(a + b*x)^2, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \operatorname{atanh}^2(a + bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*atanh(b*x+a)**2,x)

[Out] Integral(x*atanh(a + b*x)**2, x)

3.4 $\int \tanh^{-1}(a + bx)^2 dx$

Optimal. Leaf size=81

$$-\frac{\operatorname{Li}_2\left(-\frac{a+bx+1}{-a-bx+1}\right)}{b} + \frac{(a+bx)\tanh^{-1}(a+bx)^2}{b} + \frac{\tanh^{-1}(a+bx)^2}{b} - \frac{2\log\left(\frac{2}{-a-bx+1}\right)\tanh^{-1}(a+bx)}{b}$$

[Out] $\operatorname{arctanh}(b*x+a)^2/b + (b*x+a)*\operatorname{arctanh}(b*x+a)^2/b - 2*\operatorname{arctanh}(b*x+a)*\ln(2/(-b*x-a+1))/b - \operatorname{polylog}(2, (-b*x-a-1)/(-b*x-a+1))/b$

Rubi [A] time = 0.09, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {6103, 5910, 5984, 5918, 2402, 2315}

$$-\frac{\operatorname{PolyLog}\left(2, -\frac{a+bx+1}{-a-bx+1}\right)}{b} + \frac{(a+bx)\tanh^{-1}(a+bx)^2}{b} + \frac{\tanh^{-1}(a+bx)^2}{b} - \frac{2\log\left(\frac{2}{-a-bx+1}\right)\tanh^{-1}(a+bx)}{b}$$

Antiderivative was successfully verified.

[In] `Int[ArcTanh[a + b*x]^2, x]`

[Out] $\operatorname{ArcTanh}[a + b*x]^2/b + ((a + b*x)*\operatorname{ArcTanh}[a + b*x]^2)/b - (2*\operatorname{ArcTanh}[a + b*x]*\operatorname{Log}[2/(1 - a - b*x)])/b - \operatorname{PolyLog}[2, -((1 + a + b*x)/(1 - a - b*x))]/b$

Rule 2315

`Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] := -Simp[PolyLog[2, 1 - c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]`

Rule 2402

`Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] := -Dist[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]`

Rule 5910

`Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^p, x_Symbol] := Simp[x*(a + b*ArcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]`

Rule 5918

`Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^p/((d_) + (e_.)*(x_)), x_Symbol] := -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*`

p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0]

Rule 5984

Int[((a_.) + ArcTanh[(c_.)*(x_.)]*(b_.))^(p_.)*(x_.)/((d_.) + (e_.)*(x_.)^2), x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]

Rule 6103

Int[((a_.) + ArcTanh[(c_.) + (d_.)*(x_.)]*(b_.))^(p_.), x_Symbol] :> Dist[1/d, Subst[Int[(a + b*ArcTanh[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[p, 0]

Rubi steps

$$\begin{aligned}
 \int \tanh^{-1}(a + bx)^2 dx &= \frac{\text{Subst}\left(\int \tanh^{-1}(x)^2 dx, x, a + bx\right)}{b} \\
 &= \frac{(a + bx) \tanh^{-1}(a + bx)^2}{b} - \frac{2 \text{Subst}\left(\int \frac{x \tanh^{-1}(x)}{1-x^2} dx, x, a + bx\right)}{b} \\
 &= \frac{\tanh^{-1}(a + bx)^2}{b} + \frac{(a + bx) \tanh^{-1}(a + bx)^2}{b} - \frac{2 \text{Subst}\left(\int \frac{\tanh^{-1}(x)}{1-x} dx, x, a + bx\right)}{b} \\
 &= \frac{\tanh^{-1}(a + bx)^2}{b} + \frac{(a + bx) \tanh^{-1}(a + bx)^2}{b} - \frac{2 \tanh^{-1}(a + bx) \log\left(\frac{2}{1-a-bx}\right)}{b} + \frac{2 \text{Subst}\left(\int \frac{1}{1-x} dx, x, a + bx\right)}{b} \\
 &= \frac{\tanh^{-1}(a + bx)^2}{b} + \frac{(a + bx) \tanh^{-1}(a + bx)^2}{b} - \frac{2 \tanh^{-1}(a + bx) \log\left(\frac{2}{1-a-bx}\right)}{b} - \frac{2 \text{Subst}\left(\int \frac{1}{1-x} dx, x, a + bx\right)}{b} \\
 &= \frac{\tanh^{-1}(a + bx)^2}{b} + \frac{(a + bx) \tanh^{-1}(a + bx)^2}{b} - \frac{2 \tanh^{-1}(a + bx) \log\left(\frac{2}{1-a-bx}\right)}{b} - \frac{\text{Li}_2\left(1 - \frac{2}{1-a-bx}\right)}{b}
 \end{aligned}$$

Mathematica [A] time = 0.07, size = 55, normalized size = 0.68

$$\frac{\text{Li}_2\left(-e^{-2 \tanh^{-1}(a+bx)}\right) + \tanh^{-1}(a + bx) \left((a + bx - 1) \tanh^{-1}(a + bx) - 2 \log\left(e^{-2 \tanh^{-1}(a+bx)} + 1\right) \right)}{b}$$

Warning: Unable to verify antiderivative.

[In] Integrate[ArcTanh[a + b*x]^2, x]

[Out] (ArcTanh[a + b*x]*((-1 + a + b*x)*ArcTanh[a + b*x] - 2*Log[1 + E^(-2*ArcTanh[a + b*x])]) + PolyLog[2, -E^(-2*ArcTanh[a + b*x])])/b

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}(\text{artanh}(bx + a)^2, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctanh(b*x+a)^2,x, algorithm="fricas")

[Out] integral(arctanh(b*x + a)^2, x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \text{artanh}(bx + a)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctanh(b*x+a)^2,x, algorithm="giac")

[Out] integrate(arctanh(b*x + a)^2, x)

maple [A] time = 0.28, size = 103, normalized size = 1.27

$$x \text{arctanh}(bx + a)^2 + \frac{\text{arctanh}(bx + a)^2 a}{b} + \frac{\text{arctanh}(bx + a)^2}{b} - \frac{2 \text{arctanh}(bx + a) \ln\left(1 + \frac{(bx+a+1)^2}{1-(bx+a)^2}\right)}{b} - \frac{\text{polylog}\left(2, \frac{(bx+a+1)^2}{1-(bx+a)^2}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arctanh(b*x+a)^2,x)

[Out] x*arctanh(b*x+a)^2+1/b*arctanh(b*x+a)^2*a+arctanh(b*x+a)^2/b-2/b*arctanh(b*x+a)*ln(1+(b*x+a+1)^2/(1-(b*x+a)^2))-1/b*polylog(2,-(b*x+a+1)^2/(1-(b*x+a)^2))

maxima [A] time = 0.32, size = 139, normalized size = 1.72

$$-\frac{1}{4} b^2 \left(\frac{(a+1) \log(bx+a+1)^2 - 2(a+1) \log(bx+a+1) \log(bx+a-1) + (a-1) \log(bx+a-1)^2}{b^3} + \frac{4 \left(\log(bx+a+1) \right)}{b} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctanh(b*x+a)^2,x, algorithm="maxima")

[Out]
$$-1/4*b^2*((a + 1)*\log(b*x + a + 1)^2 - 2*(a + 1)*\log(b*x + a + 1)*\log(b*x + a - 1) + (a - 1)*\log(b*x + a - 1)^2)/b^3 + 4*(\log(b*x + a - 1)*\log(1/2*b*x + 1/2*a + 1/2) + \operatorname{dilog}(-1/2*b*x - 1/2*a + 1/2))/b^3 + b*((a + 1)*\log(b*x + a + 1)/b^2 - (a - 1)*\log(b*x + a - 1)/b^2)*\operatorname{arctanh}(b*x + a) + x*\operatorname{arctanh}(b*x + a)^2$$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \operatorname{atanh}(a + bx)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(atanh(a + b*x)^2,x)

[Out] int(atanh(a + b*x)^2, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{atanh}^2(a + bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(atanh(b*x+a)**2,x)

[Out] Integral(atanh(a + b*x)**2, x)

3.5 $\int \frac{\tanh^{-1}(a+bx)^2}{x} dx$

Optimal. Leaf size=148

$$\frac{1}{2}\text{Li}_3\left(1 - \frac{2}{a+bx+1}\right) - \frac{1}{2}\text{Li}_3\left(1 - \frac{2bx}{(1-a)(a+bx+1)}\right) + \text{Li}_2\left(1 - \frac{2}{a+bx+1}\right) \tanh^{-1}(a+bx) - \text{Li}_2\left(1 - \frac{2bx}{(1-a)(a+bx+1)}\right)$$

[Out] $-\text{arctanh}(b*x+a)^2*\ln(2/(b*x+a+1))+\text{arctanh}(b*x+a)^2*\ln(2*b*x/(1-a)/(b*x+a+1))+\text{arctanh}(b*x+a)*\text{polylog}(2,1-2/(b*x+a+1))-\text{arctanh}(b*x+a)*\text{polylog}(2,1-2*b*x/(1-a)/(b*x+a+1))+1/2*\text{polylog}(3,1-2/(b*x+a+1))-1/2*\text{polylog}(3,1-2*b*x/(1-a)/(b*x+a+1))$

Rubi [A] time = 0.09, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {6111, 5922}

$$\frac{1}{2}\text{PolyLog}\left(3,1 - \frac{2}{a+bx+1}\right) - \frac{1}{2}\text{PolyLog}\left(3,1 - \frac{2bx}{(1-a)(a+bx+1)}\right) + \tanh^{-1}(a+bx)\text{PolyLog}\left(2,1 - \frac{2}{a+bx+1}\right)$$

Antiderivative was successfully verified.

[In] Int[ArcTanh[a + b*x]^2/x, x]

[Out] $-(\text{ArcTanh}[a + b*x]^2*\text{Log}[2/(1 + a + b*x)]) + \text{ArcTanh}[a + b*x]^2*\text{Log}[(2*b*x)/((1 - a)*(1 + a + b*x))] + \text{ArcTanh}[a + b*x]*\text{PolyLog}[2, 1 - 2/(1 + a + b*x)] - \text{ArcTanh}[a + b*x]*\text{PolyLog}[2, 1 - (2*b*x)/((1 - a)*(1 + a + b*x))] + \text{PolyLog}[3, 1 - 2/(1 + a + b*x)]/2 - \text{PolyLog}[3, 1 - (2*b*x)/((1 - a)*(1 + a + b*x))]/2$

Rule 5922

Int[((a_.) + ArcTanh[(c_.)*(x_.)]*(b_.))^2/((d_.) + (e_.)*(x_.)), x_Symbol] :>
 $-\text{Simp}[(a + b*\text{ArcTanh}[c*x])^2*\text{Log}[2/(1 + c*x)]/e, x] + (\text{Simp}[(a + b*\text{ArcTanh}[c*x])^2*\text{Log}[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e, x] + \text{Simp}[(b*(a + b*\text{ArcTanh}[c*x])*PolyLog[2, 1 - 2/(1 + c*x)])/e, x] - \text{Simp}[(b*(a + b*\text{ArcTanh}[c*x])*PolyLog[2, 1 - (2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e, x] + \text{Simp}[(b^2*PolyLog[3, 1 - 2/(1 + c*x)])/(2*e), x] - \text{Simp}[(b^2*PolyLog[3, 1 - (2*c*(d + e*x))/((c*d + e)*(1 + c*x)))/(2*e), x]) /; FreeQ[{a, b, c, d, e}, x] && NeQ[c^2*d^2 - e^2, 0]$

Rule 6111

Int[((a_.) + ArcTanh[(c_.) + (d_.)*(x_.)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_.))^(m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*ArcTanh[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGt

Q[p, 0]

Rubi steps

$$\int \frac{\tanh^{-1}(a+bx)^2}{x} dx = \frac{\text{Subst}\left(\int \frac{\tanh^{-1}(x)^2}{-\frac{a}{b}+\frac{x}{b}} dx, x, a+bx\right)}{b}$$

$$= -\tanh^{-1}(a+bx)^2 \log\left(\frac{2}{1+a+bx}\right) + \tanh^{-1}(a+bx)^2 \log\left(\frac{2bx}{(1-a)(1+a+bx)}\right) + \tan$$

Mathematica [C] time = 2.64, size = 472, normalized size = 3.19

$$\frac{2\sqrt{1-a^2} e^{\tanh^{-1}(a)} \tanh^{-1}(a+bx)^3}{3a} + \tanh^{-1}(a+bx) \text{Li}_2\left(-e^{-2\tanh^{-1}(a+bx)}\right) + 2 \tanh^{-1}(a+bx) \text{Li}_2\left(-e^{\tanh^{-1}(a+bx)-\tanh^{-1}(a)}\right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[ArcTanh[a + b*x]^2/x,x]

[Out] $(-4*\text{ArcTanh}[a + b*x]^3)/3 - (2*\text{ArcTanh}[a + b*x]^3)/(3*a) + (2*\text{Sqrt}[1 - a^2] * E^{\text{ArcTanh}[a]} * \text{ArcTanh}[a + b*x]^3)/(3*a) - \text{ArcTanh}[a + b*x]^2 * \text{Log}[1 + E^{(-2*\text{ArcTanh}[a + b*x])}] - I*\text{Pi} * \text{ArcTanh}[a + b*x] * \text{Log}[(E^{(-\text{ArcTanh}[a + b*x])} + E^{\text{ArcTanh}[a + b*x]})/2] + \text{ArcTanh}[a + b*x]^2 * \text{Log}[(1 + a - E^{(2*\text{ArcTanh}[a + b*x])}) + a * E^{(2*\text{ArcTanh}[a + b*x])}] / (2 * E^{\text{ArcTanh}[a + b*x]})] + \text{ArcTanh}[a + b*x]^2 * \text{Log}[1 - E^{(-\text{ArcTanh}[a] + \text{ArcTanh}[a + b*x])}] + \text{ArcTanh}[a + b*x]^2 * \text{Log}[1 + E^{(-\text{ArcTanh}[a] + \text{ArcTanh}[a + b*x])}] - 2*\text{ArcTanh}[a] * \text{ArcTanh}[a + b*x] * \text{Log}[(I/2) * (-E^{\text{ArcTanh}[a]} - \text{ArcTanh}[a + b*x]) + E^{(-\text{ArcTanh}[a] + \text{ArcTanh}[a + b*x])}] + I*\text{Pi} * \text{ArcTanh}[a + b*x] * \text{Log}[1/\text{Sqrt}[1 - (a + b*x)^2]] - \text{ArcTanh}[a + b*x]^2 * \text{Log}[-((b*x)/\text{Sqrt}[1 - (a + b*x)^2])] + 2*\text{ArcTanh}[a] * \text{ArcTanh}[a + b*x] * \text{Log}[(-I) * \text{Sinh}[\text{ArcTanh}[a] - \text{ArcTanh}[a + b*x]]] + \text{ArcTanh}[a + b*x] * \text{PolyLog}[2, -E^{(-2*\text{ArcTanh}[a + b*x])}] + 2*\text{ArcTanh}[a + b*x] * \text{PolyLog}[2, -E^{(-\text{ArcTanh}[a] + \text{ArcTanh}[a + b*x])}] + 2*\text{ArcTanh}[a + b*x] * \text{PolyLog}[2, E^{(-\text{ArcTanh}[a] + \text{ArcTanh}[a + b*x])}] + \text{PolyLog}[3, -E^{(-2*\text{ArcTanh}[a + b*x])}] / 2 - 2*\text{PolyLog}[3, -E^{(-\text{ArcTanh}[a] + \text{ArcTanh}[a + b*x])}] - 2*\text{PolyLog}[3, E^{(-\text{ArcTanh}[a] + \text{ArcTanh}[a + b*x])}]$

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\text{artanh}(bx+a)^2}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctanh(b*x+a)^2/x,x, algorithm="fricas")

[Out] integral(arctanh(b*x + a)^2/x, x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{artanh}(bx+a)^2}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctanh(b*x+a)^2/x,x, algorithm="giac")

[Out] integrate(arctanh(b*x + a)^2/x, x)

maple [C] time = 0.85, size = 1022, normalized size = 6.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arctanh(b*x+a)^2/x,x)

[Out] ln(b*x)*arctanh(b*x+a)^2-arctanh(b*x+a)^2*ln(a*(1+(b*x+a+1)^2/(1-(b*x+a)^2))-(b*x+a+1)^2/(1-(b*x+a)^2)+1)+1/2*I*Pi*(csgn(I/(1+(b*x+a+1)^2/(1-(b*x+a)^2)))*csgn(I*(a*(1+(b*x+a+1)^2/(1-(b*x+a)^2))-(b*x+a+1)^2/(1-(b*x+a)^2)+1))*csgn(I*(a*(1+(b*x+a+1)^2/(1-(b*x+a)^2))-(b*x+a+1)^2/(1-(b*x+a)^2)+1)/(1+(b*x+a+1)^2/(1-(b*x+a)^2)))-csgn(I/(1+(b*x+a+1)^2/(1-(b*x+a)^2)))*csgn(I*(a*(1+(b*x+a+1)^2/(1-(b*x+a)^2))-(b*x+a+1)^2/(1-(b*x+a)^2)+1)/(1+(b*x+a+1)^2/(1-(b*x+a)^2)))^2+2*csgn(I*(a*(1+(b*x+a+1)^2/(1-(b*x+a)^2))-(b*x+a+1)^2/(1-(b*x+a)^2)))-csgn(I*(a*(1+(b*x+a+1)^2/(1-(b*x+a)^2))-(b*x+a+1)^2/(1-(b*x+a)^2)+1))*csgn(I*(a*(1+(b*x+a+1)^2/(1-(b*x+a)^2))-(b*x+a+1)^2/(1-(b*x+a)^2)+1)/(1+(b*x+a+1)^2/(1-(b*x+a)^2)))^2-csgn(I*(a*(1+(b*x+a+1)^2/(1-(b*x+a)^2))-(b*x+a+1)^2/(1-(b*x+a)^2)+1))*csgn(I*(a*(1+(b*x+a+1)^2/(1-(b*x+a)^2))-(b*x+a+1)^2/(1-(b*x+a)^2)+1)/(1+(b*x+a+1)^2/(1-(b*x+a)^2)))^3-2)*arctanh(b*x+a)^2-arctanh(b*x+a)*polylog(2, -(b*x+a+1)^2/(1-(b*x+a)^2))+1/2*polylog(3, -(b*x+a+1)^2/(1-(b*x+a)^2))+a/(a-1)*arctanh(b*x+a)^2*ln(1-(a-1)*(b*x+a+1)^2/(1-(b*x+a)^2)/(-1-a))+a/(a-1)*arctanh(b*x+a)*polylog(2, (a-1)*(b*x+a+1)^2/(1-(b*x+a)^2)/(-1-a))-1/2*a/(a-1)*polylog(3, (a-1)*(b*x+a+1)^2/(1-(b*x+a)^2)/(-1-a))-1/(a-1)*arctanh(b*x+a)^2*ln(1-(a-1)*(b*x+a+1)^2/(1-(b*x+a)^2)/(-1-a))-1/(a-1)*arctanh(b*x+a)*polylog(2, (a-1)*(b*x+a+1)^2/(1-(b*x+a)^2)/(-1-a))+1/2/(a-1)*polylog(3, (a-1)*(b*x+a+1)^2/(1-(b*x+a)^2)/(-1-a))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{artanh}(bx+a)^2}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctanh(b*x+a)^2/x,x, algorithm="maxima")

[Out] integrate(arctanh(b*x + a)^2/x, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{atanh}(a + bx)^2}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(atanh(a + b*x)^2/x,x)

[Out] int(atanh(a + b*x)^2/x, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}^2(a + bx)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(atanh(b*x+a)**2/x,x)

[Out] Integral(atanh(a + b*x)**2/x, x)

3.6 $\int \frac{\tanh^{-1}(a+bx)^2}{x^2} dx$

Optimal. Leaf size=251

$$\frac{b\text{Li}_2\left(1 - \frac{2}{a+bx+1}\right)}{1-a^2} - \frac{b\text{Li}_2\left(1 - \frac{2bx}{(1-a)(a+bx+1)}\right)}{1-a^2} - \frac{2b \log\left(\frac{2}{a+bx+1}\right) \tanh^{-1}(a+bx)}{1-a^2} + \frac{2b \log\left(\frac{2bx}{(1-a)(a+bx+1)}\right) \tanh^{-1}(a+bx)}{1-a^2}$$

[Out] $-\text{arctanh}(b*x+a)^2/x+b*\text{arctanh}(b*x+a)*\ln(2/(-b*x-a+1))/(1-a)+b*\text{arctanh}(b*x+a)*\ln(2/(b*x+a+1))/(1+a)-2*b*\text{arctanh}(b*x+a)*\ln(2/(b*x+a+1))/(-a^2+1)+2*b*\text{arctanh}(b*x+a)*\ln(2*b*x/(1-a)/(b*x+a+1))/(-a^2+1)+1/2*b*\text{polylog}(2,(-b*x-a-1)/(-b*x-a+1))/(1-a)-1/2*b*\text{polylog}(2,1-2/(b*x+a+1))/(1+a)+b*\text{polylog}(2,1-2/(b*x+a+1))/(-a^2+1)-b*\text{polylog}(2,1-2*b*x/(1-a)/(b*x+a+1))/(-a^2+1)$

Rubi [A] time = 0.70, antiderivative size = 251, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 15, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 1.250$, Rules used = {6109, 371, 706, 31, 633, 6741, 6121, 6688, 12, 6725, 5920, 2402, 2315, 2447, 5918}

$$\frac{b\text{PolyLog}\left(2, 1 - \frac{2}{a+bx+1}\right)}{1-a^2} - \frac{b\text{PolyLog}\left(2, 1 - \frac{2bx}{(1-a)(a+bx+1)}\right)}{1-a^2} + \frac{b\text{PolyLog}\left(2, -\frac{a+bx+1}{-a-bx+1}\right)}{2(1-a)} - \frac{b\text{PolyLog}\left(2, 1 - \frac{2}{a+bx+1}\right)}{2(a+1)}$$

Antiderivative was successfully verified.

[In] Int[ArcTanh[a + b*x]^2/x^2, x]

[Out] $-(\text{ArcTanh}[a + b*x]^2/x) + (b*\text{ArcTanh}[a + b*x]*\text{Log}[2/(1 - a - b*x)])/(1 - a) + (b*\text{ArcTanh}[a + b*x]*\text{Log}[2/(1 + a + b*x)])/(1 + a) - (2*b*\text{ArcTanh}[a + b*x]*\text{Log}[2/(1 + a + b*x)])/(1 - a^2) + (2*b*\text{ArcTanh}[a + b*x]*\text{Log}[(2*b*x)/((1 - a)*(1 + a + b*x))])/(1 - a^2) + (b*\text{PolyLog}[2, -((1 + a + b*x)/(1 - a - b*x))])/(2*(1 - a)) - (b*\text{PolyLog}[2, 1 - 2/(1 + a + b*x)])/(2*(1 + a)) + (b*\text{PolyLog}[2, 1 - 2/(1 + a + b*x)])/(1 - a^2) - (b*\text{PolyLog}[2, 1 - (2*b*x)/((1 - a)*(1 + a + b*x))])/(1 - a^2)$

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 31

Int[((a_) + (b_.)*(x_))^-1, x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 371

```
Int[((a_) + (b_)*(v_)^(n_))^(p_)*(x_)^(m_), x_Symbol] := With[{c = Coefficient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[SimplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /; FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 633

```
Int[((d_) + (e_)*(x_))/((a_) + (c_)*(x_)^2), x_Symbol] := With[{q = Rt[-(a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x), x], x] + Dist[e/2 - (c*d)/(2*q), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[-(a*c)]
```

Rule 706

```
Int[1/(((d_) + (e_)*(x_))*((a_) + (c_)*(x_)^2)), x_Symbol] := Dist[e^2/(c*d^2 + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d^2 + a*e^2), Int[(c*d - c*e*x)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0]
```

Rule 2315

```
Int[Log[(c_)*(x_)]/((d_) + (e_)*(x_)), x_Symbol] := -Simp[PolyLog[2, 1 - c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2402

```
Int[Log[(c_)/((d_) + (e_)*(x_))]/((f_) + (g_)*(x_)^2), x_Symbol] := -Dist[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2447

```
Int[Log[u]*(Pq_)^(m_), x_Symbol] := With[{C = FullSimplify[(Pq^m*(1 - u))/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x] /; IntegerQ[m] && PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u, x][[2]], Expon[Pq, x]]
```

Rule 5918

```
Int[((a_) + ArcTanh[(c_)*(x_)]*(b_))^(p_)/((d_) + (e_)*(x_)), x_Symbol] := -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/((1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0]
```

]

Rule 5920

```
Int[((a_.) + ArcTanh[(c_.)*(x_.)]*(b_.))/((d_.) + (e_.)*(x_.)), x_Symbol] :> -
Simp[((a + b*ArcTanh[c*x])*Log[2/(1 + c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x))
/((c*d + e)*(1 + c*x))]/(1 - c^2*x^2), x], x] + Simp[((a + b*ArcTanh[c*x])*
Log[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))]/e, x]) /; FreeQ[{a, b, c, d, e}
, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rule 6109

```
Int[((a_.) + ArcTanh[(c_) + (d_.)*(x_.)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_.))^(
m_), x_Symbol] :> Simp[((e + f*x)^(m + 1)*(a + b*ArcTanh[c + d*x])^p)/(f*(m
+ 1)), x] - Dist[(b*d*p)/(f*(m + 1)), Int[((e + f*x)^(m + 1)*(a + b*ArcTan
h[c + d*x])^(p - 1))/(1 - (c + d*x)^2), x], x] /; FreeQ[{a, b, c, d, e, f},
x] && IGtQ[p, 0] && ILtQ[m, -1]
```

Rule 6121

```
Int[((a_.) + ArcTanh[(c_) + (d_.)*(x_.)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_.))^(
m_.)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)^(q_.), x_Symbol] :> Dist[1/d, Subs
t[Int[((d*e - c*f)/d + (f*x)/d)^m*(-(C/d^2) + (C*x^2)/d^2)^q*(a + b*ArcTanh
[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m, p, q},
x] && EqQ[B*(1 - c^2) + 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

Rule 6688

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 6741

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v !=
= u]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tanh^{-1}(a+bx)^2}{x^2} dx &= -\frac{\tanh^{-1}(a+bx)^2}{x} + (2b) \int \frac{\tanh^{-1}(a+bx)}{x(1-(a+bx)^2)} dx \\
&= -\frac{\tanh^{-1}(a+bx)^2}{x} + (2b) \int \frac{\tanh^{-1}(a+bx)}{x(1-a^2-2abx-b^2x^2)} dx \\
&= -\frac{\tanh^{-1}(a+bx)^2}{x} + 2 \text{Subst} \left(\int \frac{\tanh^{-1}(x)}{\left(-\frac{a}{b} + \frac{x}{b}\right)(1-x^2)} dx, x, a+bx \right) \\
&= -\frac{\tanh^{-1}(a+bx)^2}{x} + 2 \text{Subst} \left(\int \frac{b \tanh^{-1}(x)}{(-a+x)(1-x^2)} dx, x, a+bx \right) \\
&= -\frac{\tanh^{-1}(a+bx)^2}{x} + (2b) \text{Subst} \left(\int \frac{\tanh^{-1}(x)}{(-a+x)(1-x^2)} dx, x, a+bx \right) \\
&= -\frac{\tanh^{-1}(a+bx)^2}{x} + (2b) \text{Subst} \left(\int \left(\frac{\tanh^{-1}(x)}{(-1+a^2)(a-x)} + \frac{\tanh^{-1}(x)}{2(-1+a)(-1+x)} - \frac{\tanh^{-1}(x)}{2(1+a)} \right) dx, x, a+bx \right) \\
&= -\frac{\tanh^{-1}(a+bx)^2}{x} - \frac{b \text{Subst} \left(\int \frac{\tanh^{-1}(x)}{-1+x} dx, x, a+bx \right)}{1-a} - \frac{b \text{Subst} \left(\int \frac{\tanh^{-1}(x)}{1+x} dx, x, a+bx \right)}{1+a} \\
&= -\frac{\tanh^{-1}(a+bx)^2}{x} + \frac{b \tanh^{-1}(a+bx) \log\left(\frac{2}{1-a-bx}\right)}{1-a} + \frac{b \tanh^{-1}(a+bx) \log\left(\frac{2}{1+a+bx}\right)}{1+a} \\
&= -\frac{\tanh^{-1}(a+bx)^2}{x} + \frac{b \tanh^{-1}(a+bx) \log\left(\frac{2}{1-a-bx}\right)}{1-a} + \frac{b \tanh^{-1}(a+bx) \log\left(\frac{2}{1+a+bx}\right)}{1+a} \\
&= -\frac{\tanh^{-1}(a+bx)^2}{x} + \frac{b \tanh^{-1}(a+bx) \log\left(\frac{2}{1-a-bx}\right)}{1-a} + \frac{b \tanh^{-1}(a+bx) \log\left(\frac{2}{1+a+bx}\right)}{1+a}
\end{aligned}$$

Mathematica [C] time = 1.44, size = 208, normalized size = 0.83

$$-\left(\left(a^3 + a^2bx + bx\left(\sqrt{1-a^2} e^{\tanh^{-1}(a)} - 1\right) - a\right) \tanh^{-1}(a+bx)^2\right) + abx \text{Li}_2\left(e^{2 \tanh^{-1}(a)-2 \tanh^{-1}(a+bx)}\right) + abx \tanh^{-1}(a+bx)$$

Warning: Unable to verify antiderivative.

[In] Integrate[ArcTanh[a + b*x]^2/x^2,x]

[Out] (-((-a + a^3 + a^2*b*x + b*(-1 + Sqrt[1 - a^2])*E^ArcTanh[a])*x)*ArcTanh[a + b*x]^2) + a*b*x*ArcTanh[a + b*x]*((-I)*Pi + 2*ArcTanh[a] - 2*Log[1 - E^(2*

$\text{ArcTanh}[a] - 2*\text{ArcTanh}[a + b*x])]) + a*b*x*(I*\text{Pi}*(\text{Log}[1 + E^{(2*\text{ArcTanh}[a + b*x])}] - \text{Log}[1/\text{Sqrt}[1 - (a + b*x)^2]]) + 2*\text{ArcTanh}[a]*(\text{Log}[1 - E^{(2*\text{ArcTanh}[a] - 2*\text{ArcTanh}[a + b*x])}] - \text{Log}[(-I)*\text{Sinh}[\text{ArcTanh}[a] - \text{ArcTanh}[a + b*x]])]) + a*b*x*\text{PolyLog}[2, E^{(2*\text{ArcTanh}[a] - 2*\text{ArcTanh}[a + b*x])})]/(a*(-1 + a^2)*x)$

fricas [F] time = 0.41, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\text{artanh}(bx + a)^2}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctanh(b*x+a)^2/x^2,x, algorithm="fricas")

[Out] integral(arctanh(b*x + a)^2/x^2, x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{artanh}(bx + a)^2}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctanh(b*x+a)^2/x^2,x, algorithm="giac")

[Out] integrate(arctanh(b*x + a)^2/x^2, x)

maple [A] time = 0.07, size = 342, normalized size = 1.36

$$-\frac{\text{arctanh}(bx + a)^2}{x} + \frac{2b \text{arctanh}(bx + a) \ln(bx + a - 1)}{2a - 2} - \frac{2b \text{arctanh}(bx + a) \ln(bx + a + 1)}{2 + 2a} - \frac{2b \text{arctanh}(bx + a)}{(a - 1)(1 + a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arctanh(b*x+a)^2/x^2,x)

[Out] $-\text{arctanh}(b*x+a)^2/x + 2*b*\text{arctanh}(b*x+a)/(2*a-2)*\ln(b*x+a-1) - 2*b*\text{arctanh}(b*x+a)/(2+2*a)*\ln(b*x+a+1) - 2*b*\text{arctanh}(b*x+a)/(a-1)/(1+a)*\ln(b*x) - b/(a-1)/(1+a)*\text{dilog}((b*x+a-1)/(a-1)) - b/(a-1)/(1+a)*\ln(b*x)*\ln((b*x+a-1)/(a-1)) + b/(a-1)/(1+a)*\text{dilog}((b*x+a+1)/(1+a)) + b/(a-1)/(1+a)*\ln(b*x)*\ln((b*x+a+1)/(1+a)) + 1/4*b/(1+a)*\ln(b*x+a+1)^2 - 1/2*b/(1+a)*\ln(-1/2*b*x-1/2*a+1/2)*\ln(b*x+a+1) + 1/2*b/(1+a)*\ln(-1/2*b*x-1/2*a+1/2)*\ln(1/2+1/2*b*x+1/2*a) + 1/2*b/(1+a)*\text{dilog}(1/2+1/2*b*x+1/2*a) + 1/4*b/(a-1)*\ln(b*x+a-1)^2 - 1/2*b/(a-1)*\text{dilog}(1/2+1/2*b*x+1/2*a) - 1/2*b/(a-1)*\ln(b*x+a-1)*\ln(1/2+1/2*b*x+1/2*a)$

maxima [A] time = 0.34, size = 244, normalized size = 0.97

$$\frac{1}{4} b^2 \left(\frac{(a-1) \log(bx+a+1)^2 - 2(a-1) \log(bx+a+1) \log(bx+a-1) + (a+1) \log(bx+a-1)^2}{a^2 b - b} - \frac{4 \left(\log(bx+a-1) \log(bx+a+1) + \log(bx+a-1) \log(x) + \text{dilog}(-b*x/(a+1)) \right)}{a^2 b - b} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctanh(b*x+a)^2/x^2,x, algorithm="maxima")

[Out] 1/4*b^2*((a-1)*log(b*x+a+1)^2 - 2*(a-1)*log(b*x+a+1)*log(b*x+a-1) + (a+1)*log(b*x+a-1)^2)/(a^2*b-b) - 4*(log(b*x+a-1)*log(1/2*b*x+1/2*a+1/2) + dilog(-1/2*b*x-1/2*a+1/2))/(a^2*b-b) + 4*(log(b*x/(a+1)+1)*log(x) + dilog(-b*x/(a+1)))/(a^2*b-b) - 4*(log(b*x/(a-1)+1)*log(x) + dilog(-b*x/(a-1)))/(a^2*b-b) - b*(log(b*x+a+1)/(a+1) - log(b*x+a-1)/(a-1) + 2*log(x)/(a^2-1))*arctanh(b*x+a) - arctanh(b*x+a)^2/x

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\text{atanh}(a+bx)^2}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(atanh(a+b*x)^2/x^2,x)

[Out] int(atanh(a+b*x)^2/x^2, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{atanh}^2(a+bx)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(atanh(b*x+a)**2/x**2,x)

[Out] Integral(atanh(a+b*x)**2/x**2, x)

3.7 $\int \frac{\tanh^{-1}(a+bx)^2}{x^3} dx$

Optimal. Leaf size=370

$$\frac{ab^2 \operatorname{Li}_2\left(1 - \frac{2}{a+bx+1}\right)}{(1-a^2)^2} - \frac{ab^2 \operatorname{Li}_2\left(1 - \frac{2bx}{(1-a)(a+bx+1)}\right)}{(1-a^2)^2} + \frac{b^2 \log(x)}{(1-a^2)^2} - \frac{2ab^2 \log\left(\frac{2}{a+bx+1}\right) \tanh^{-1}(a+bx)}{(1-a^2)^2} + \frac{2ab^2 \log\left(\frac{2bx}{(1-a)(a+bx+1)}\right)}{(1-a^2)^2}$$

[Out] $-b \operatorname{arctanh}(b*x+a)/(-a^2+1)/x - 1/2 \operatorname{arctanh}(b*x+a)^2/x^2 + b^2 \ln(x)/(-a^2+1)^2 + 1/2 b^2 \operatorname{arctanh}(b*x+a) \ln(2/(-b*x-a+1))/(1-a)^2 - 1/2 b^2 \ln(-b*x-a+1)/(1-a)^2 + 1/2 b^2 \operatorname{arctanh}(b*x+a) \ln(2/(b*x+a+1))/(1+a)^2 - 2 a b^2 \operatorname{arctanh}(b*x+a) \ln(2/(b*x+a+1))/(-a^2+1)^2 + 2 a b^2 \operatorname{arctanh}(b*x+a) \ln(2 b*x/(1-a)/(b*x+a+1))/(-a^2+1)^2 - 1/2 b^2 \ln(b*x+a+1)/(1-a)/(1+a)^2 + 1/4 b^2 \operatorname{polylog}(2, (-b*x-a-1)/(-b*x-a+1))/(1-a)^2 + 1/4 b^2 \operatorname{polylog}(2, 1-2/(b*x+a+1))/(1+a)^2 + a b^2 \operatorname{polylog}(2, 1-2/(b*x+a+1))/(-a^2+1)^2 - a b^2 \operatorname{polylog}(2, 1-2 b*x/(1-a)/(b*x+a+1))/(-a^2+1)^2$

Rubi [A] time = 0.80, antiderivative size = 370, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 16, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 1.333$, Rules used = {6109, 371, 710, 801, 6741, 6121, 6725, 5926, 706, 31, 633, 5920, 2402, 2315, 2447, 5918}

$$\frac{ab^2 \operatorname{PolyLog}\left(2, 1 - \frac{2}{a+bx+1}\right)}{(1-a^2)^2} - \frac{ab^2 \operatorname{PolyLog}\left(2, 1 - \frac{2bx}{(1-a)(a+bx+1)}\right)}{(1-a^2)^2} + \frac{b^2 \operatorname{PolyLog}\left(2, -\frac{a+bx+1}{-a-bx+1}\right)}{4(1-a)^2} + \frac{b^2 \operatorname{PolyLog}\left(2, 1 - \frac{2bx}{(1-a)(a+bx+1)}\right)}{4(a+1)^2}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\operatorname{ArcTanh}[a + b*x]^2/x^3, x]$

[Out] $-(b \operatorname{ArcTanh}[a + b*x])/((1 - a^2)*x) - \operatorname{ArcTanh}[a + b*x]^2/(2*x^2) + (b^2 \operatorname{Log}[x])/((1 - a^2)^2 + (b^2 \operatorname{ArcTanh}[a + b*x] \operatorname{Log}[2/(1 - a - b*x)])/(2*(1 - a)^2) - (b^2 \operatorname{Log}[1 - a - b*x])/(2*(1 - a)^2*(1 + a)) - (b^2 \operatorname{ArcTanh}[a + b*x] \operatorname{Log}[2/(1 + a + b*x)])/(2*(1 + a)^2) - (2*a*b^2 \operatorname{ArcTanh}[a + b*x] \operatorname{Log}[2/(1 + a + b*x)])/(1 - a^2)^2 + (2*a*b^2 \operatorname{ArcTanh}[a + b*x] \operatorname{Log}[(2*b*x)/((1 - a)*(1 + a + b*x))])/(1 - a^2)^2 - (b^2 \operatorname{Log}[1 + a + b*x])/(2*(1 - a)*(1 + a)^2) + (b^2 \operatorname{PolyLog}[2, -((1 + a + b*x)/(1 - a - b*x))])/(4*(1 - a)^2) + (b^2 \operatorname{PolyLog}[2, 1 - 2/(1 + a + b*x)])/(4*(1 + a)^2) + (a*b^2 \operatorname{PolyLog}[2, 1 - 2/(1 + a + b*x)])/(1 - a^2)^2 - (a*b^2 \operatorname{PolyLog}[2, 1 - (2*b*x)/((1 - a)*(1 + a + b*x))])/(1 - a^2)^2$

Rule 31

$\operatorname{Int}[(a + (b*x)^n)^{-1}, x, \text{Symbol}] \rightarrow \operatorname{Simp}[\operatorname{Log}[\operatorname{RemoveContent}[a + b*x, x]]/b, x] /; \operatorname{FreeQ}\{a, b\}, x]$

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] := With[{c = Coefficient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[SimplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /; FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 633

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] := With[{q = Rt[-(a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x), x], x] + Dist[e/2 - (c*d)/(2*q), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[-(a*c)]
```

Rule 706

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)), x_Symbol] := Dist[e^2/(c*d^2 + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d^2 + a*e^2), Int[(c*d - c*e*x)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0]
```

Rule 710

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] := Simp[(e*(d + e*x)^(m + 1))/((m + 1)*(c*d^2 + a*e^2)), x] + Dist[c/(c*d^2 + a*e^2), Int[(d + e*x)^(m + 1)*(d - e*x)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2), x_Symbol] := Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] := -Simp[PolyLog[2, 1 - c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_.) + (g_.)*(x_)^2), x_Symbol] := -Dist[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_), x_Symbol] := With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
x][[2]], Expon[Pq, x]]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_.)]*(b_.))^((d_.) + (e_.)*(x_.)), x_Symbol
] := -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)]/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 5920

```
Int[((a_.) + ArcTanh[(c_.)*(x_.)]*(b_.))/((d_.) + (e_.)*(x_.)), x_Symbol] := -
Simp[((a + b*ArcTanh[c*x])*Log[2/(1 + c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x))
/((c*d + e)*(1 + c*x))]/(1 - c^2*x^2), x], x] + Simp[((a + b*ArcTanh[c*x])*
Log[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e, x]) /; FreeQ[{a, b, c, d, e}
, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rule 5926

```
Int[((a_.) + ArcTanh[(c_.)*(x_.)]*(b_.))*((d_.) + (e_.)*(x_.))^(q_.), x_Symbol
] := Simp[((d + e*x)^(q + 1)*(a + b*ArcTanh[c*x]))/(e*(q + 1)), x] - Dist[(
b*c)/(e*(q + 1)), Int[(d + e*x)^(q + 1)/(1 - c^2*x^2), x], x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[q, -1]
```

Rule 6109

```
Int[((a_.) + ArcTanh[(c_.) + (d_.)*(x_.)]*(b_.))^((p_.)*((e_.) + (f_.)*(x_.))^(
m_.), x_Symbol] := Simp[((e + f*x)^(m + 1)*(a + b*ArcTanh[c + d*x])^p)/(f*(m
+ 1)), x] - Dist[(b*d*p)/(f*(m + 1)), Int[((e + f*x)^(m + 1)*(a + b*ArcTan
h[c + d*x])^(p - 1))/(1 - (c + d*x)^2), x], x] /; FreeQ[{a, b, c, d, e, f},
x] && IGtQ[p, 0] && ILtQ[m, -1]
```

Rule 6121

```
Int[((a_.) + ArcTanh[(c_.) + (d_.)*(x_.)]*(b_.))^((p_.)*((e_.) + (f_.)*(x_.))^(
m_.)*((A_.) + (B_.)*(x_.) + (C_.)*(x_)^2)^(q_.), x_Symbol] := Dist[1/d, Subs
t[Int[((d*e - c*f)/d + (f*x)/d)^m*(-(C/d^2) + (C*x^2)/d^2)^q*(a + b*ArcTanh
[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m, p, q},
```


$x] \ \&\& \ \text{EqQ}[B*(1 - c^2) + 2*A*c*d, 0] \ \&\& \ \text{EqQ}[2*c*C - B*d, 0]$

Rule 6725

$\text{Int}[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] \ :> \ \text{With}[\{v = \text{RationalFunctionE}\text{x}\text{pand}[u/(a + b*x^n), x]\}, \ \text{Int}[v, x] \ /; \ \text{SumQ}[v]] \ /; \ \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{IGtQ}[n, 0]$

Rule 6741

$\text{Int}[u_, x_Symbol] \ :> \ \text{With}[\{v = \text{NormalizeIntegrand}[u, x]\}, \ \text{Int}[v, x] \ /; \ v \neq u]$

Rubi steps

$$\begin{aligned}
\int \frac{\tanh^{-1}(a+bx)^2}{x^3} dx &= -\frac{\tanh^{-1}(a+bx)^2}{2x^2} + b \int \frac{\tanh^{-1}(a+bx)}{x^2(1-(a+bx)^2)} dx \\
&= -\frac{\tanh^{-1}(a+bx)^2}{2x^2} + b \int \frac{\tanh^{-1}(a+bx)}{x^2(1-a^2-2abx-b^2x^2)} dx \\
&= -\frac{\tanh^{-1}(a+bx)^2}{2x^2} + \text{Subst} \left(\int \frac{\tanh^{-1}(x)}{\left(-\frac{a}{b} + \frac{x}{b}\right)^2(1-x^2)} dx, x, a+bx \right) \\
&= -\frac{\tanh^{-1}(a+bx)^2}{2x^2} + \text{Subst} \left(\int \left(-\frac{b^2 \tanh^{-1}(x)}{(-1+a^2)(a-x)^2} - \frac{2ab^2 \tanh^{-1}(x)}{(-1+a^2)^2(a-x)} - \frac{b^2 \tanh^{-1}(x)}{2(-1+a^2)^2} \right) dx, x, a+bx \right) \\
&= -\frac{\tanh^{-1}(a+bx)^2}{2x^2} - \frac{b^2 \text{Subst} \left(\int \frac{\tanh^{-1}(x)}{-1+x} dx, x, a+bx \right)}{2(1-a)^2} + \frac{b^2 \text{Subst} \left(\int \frac{\tanh^{-1}(x)}{1+x} dx, x, a+bx \right)}{2(1+a)^2} \\
&= -\frac{b \tanh^{-1}(a+bx)}{(1-a^2)x} - \frac{\tanh^{-1}(a+bx)^2}{2x^2} + \frac{b^2 \tanh^{-1}(a+bx) \log\left(\frac{2}{1-a-bx}\right)}{2(1-a)^2} - \frac{b^2 \tanh^{-1}(a+bx) \log\left(\frac{2}{1+a-bx}\right)}{2(1+a)^2} \\
&= -\frac{b \tanh^{-1}(a+bx)}{(1-a^2)x} - \frac{\tanh^{-1}(a+bx)^2}{2x^2} + \frac{b^2 \tanh^{-1}(a+bx) \log\left(\frac{2}{1-a-bx}\right)}{2(1-a)^2} - \frac{b^2 \tanh^{-1}(a+bx) \log\left(\frac{2}{1+a-bx}\right)}{2(1+a)^2} \\
&= -\frac{b \tanh^{-1}(a+bx)}{(1-a^2)x} - \frac{\tanh^{-1}(a+bx)^2}{2x^2} + \frac{b^2 \log(x)}{(1-a^2)^2} + \frac{b^2 \tanh^{-1}(a+bx) \log\left(\frac{2}{1-a-bx}\right)}{2(1-a)^2} - \frac{b^2 \tanh^{-1}(a+bx) \log\left(\frac{2}{1+a-bx}\right)}{2(1+a)^2} \\
&= -\frac{b \tanh^{-1}(a+bx)}{(1-a^2)x} - \frac{\tanh^{-1}(a+bx)^2}{2x^2} + \frac{b^2 \log(x)}{(1-a^2)^2} + \frac{b^2 \tanh^{-1}(a+bx) \log\left(\frac{2}{1-a-bx}\right)}{2(1-a)^2} - \frac{b^2 \tanh^{-1}(a+bx) \log\left(\frac{2}{1+a-bx}\right)}{2(1+a)^2}
\end{aligned}$$

Mathematica [C] time = 2.35, size = 271, normalized size = 0.73

$$2bx \tanh^{-1}(a+bx) \left(a^2 + abx + i\pi abx - 2abx \tanh^{-1}(a) + 2abx \log\left(1 - e^{2 \tanh^{-1}(a) - 2 \tanh^{-1}(a+bx)}\right) - 1 \right) - (a^4 - a^2)$$

Warning: Unable to verify antiderivative.

[In] Integrate[ArcTanh[a + b*x]^2/x^3,x]

```
[Out] (-((1 + a^4 - b^2*(-1 + 2*sqrt[1 - a^2]*E^ArcTanh[a])*x^2 - a^2*(2 + b^2*x^2))*ArcTanh[a + b*x]^2) + 2*b*x*ArcTanh[a + b*x]*(-1 + a^2 + a*b*x + I*a*b*Pi*x - 2*a*b*x*ArcTanh[a] + 2*a*b*x*Log[1 - E^(2*ArcTanh[a] - 2*ArcTanh[a + b*x])]) + 2*b^2*x^2*((-I)*a*Pi*Log[1 + E^(2*ArcTanh[a + b*x])] + I*a*Pi*Log[1/Sqrt[1 - (a + b*x)^2]] + Log[-((b*x)/Sqrt[1 - (a + b*x)^2]]) - 2*a*ArcTanh[a]*(Log[1 - E^(2*ArcTanh[a] - 2*ArcTanh[a + b*x])] - Log[(-I)*Sinh[ArcTanh[a] - ArcTanh[a + b*x]]])) - 2*a*b^2*x^2*PolyLog[2, E^(2*ArcTanh[a] - 2*ArcTanh[a + b*x])])/(2*(-1 + a^2)^2*x^2)
```

fricas [F] time = 0.57, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\text{artanh}(bx+a)^2}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(b*x+a)^2/x^3,x, algorithm="fricas")
```

```
[Out] integral(arctanh(b*x + a)^2/x^3, x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(b*x+a)^2/x^3,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.08, size = 467, normalized size = 1.26

$$\frac{\text{arctanh}(bx+a)^2}{2x^2} - \frac{b^2 \text{arctanh}(bx+a) \ln(bx+a-1)}{2(a-1)^2} + \frac{b^2 \text{arctanh}(bx+a) \ln(bx+a+1)}{2(1+a)^2} + \frac{b \text{arctanh}(bx+a)}{(a-1)(1+a)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(b*x+a)^2/x^3,x)
```

```
[Out] -1/2*arctanh(b*x+a)^2/x^2-1/2*b^2*arctanh(b*x+a)/(a-1)^2*ln(b*x+a-1)+1/2*b^2*arctanh(b*x+a)/(1+a)^2*ln(b*x+a+1)+b*arctanh(b*x+a)/(a-1)/(1+a)/x+2*b^2*a*arctanh(b*x+a)*a/(a-1)^2/(1+a)^2*ln(b*x)-1/8*b^2/(a-1)^2*ln(b*x+a-1)^2+1/4*b^2/(a-1)^2*dilog(1/2+1/2*b*x+1/2*a)+1/4*b^2/(a-1)^2*ln(b*x+a-1)*ln(1/2+1/2*b*x+1/2*a)-1/8*b^2/(1+a)^2*ln(b*x+a+1)^2+1/4*b^2/(1+a)^2*ln(-1/2*b*x-1/2*a+1/2)*ln(b*x+a+1)-1/4*b^2/(1+a)^2*ln(-1/2*b*x-1/2*a+1/2)*ln(1/2+1/2*b*x+1/2*a)-1/4*b^2/(1+a)^2*dilog(1/2+1/2*b*x+1/2*a)-b^2/(a-1)/(1+a)/(2*a-2)*ln(b*x
```

$$a-1)+b^2/(a-1)/(1+a)/(2+2*a)*\ln(b*x+a+1)+b^2/(a-1)^2/(1+a)^2*\ln(b*x)+b^2*a/(a-1)^2/(1+a)^2*dilog((b*x+a-1)/(a-1))+b^2*a/(a-1)^2/(1+a)^2*\ln(b*x)*\ln((b*x+a-1)/(a-1))-b^2*a/(a-1)^2/(1+a)^2*dilog((b*x+a+1)/(1+a))-b^2*a/(a-1)^2/(1+a)^2*\ln(b*x)*\ln((b*x+a+1)/(1+a))$$

maxima [A] time = 0.35, size = 360, normalized size = 0.97

$$\frac{1}{8} \left(\frac{8 \left(\log(bx + a - 1) \log\left(\frac{1}{2}bx + \frac{1}{2}a + \frac{1}{2}\right) + \text{Li}_2\left(-\frac{1}{2}bx - \frac{1}{2}a + \frac{1}{2}\right)\right)a}{a^4 - 2a^2 + 1} - \frac{8 \left(\log\left(\frac{bx}{a+1} + 1\right) \log(x) + \text{Li}_2\left(-\frac{bx}{a+1}\right)\right)a}{a^4 - 2a^2 + 1} \right) +$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctanh(b*x+a)^2/x^3,x, algorithm="maxima")

[Out] 1/8*(8*(log(b*x + a - 1)*log(1/2*b*x + 1/2*a + 1/2) + dilog(-1/2*b*x - 1/2*a + 1/2))*a/(a^4 - 2*a^2 + 1) - 8*(log(b*x/(a + 1) + 1)*log(x) + dilog(-b*x/(a + 1)))*a/(a^4 - 2*a^2 + 1) + 8*(log(b*x/(a - 1) + 1)*log(x) + dilog(-b*x/(a - 1)))*a/(a^4 - 2*a^2 + 1) - ((a^2 - 2*a + 1)*log(b*x + a + 1)^2 - 2*(a^2 - 2*a + 1)*log(b*x + a + 1)*log(b*x + a - 1) + (a^2 + 2*a + 1)*log(b*x + a - 1)^2)/(a^4 - 2*a^2 + 1) + 4*log(b*x + a + 1)/(a^3 + a^2 - a - 1) - 4*log(b*x + a - 1)/(a^3 - a^2 - a + 1) + 8*log(x)/(a^4 - 2*a^2 + 1))*b^2 + 1/2*(4*a*b*log(x)/(a^4 - 2*a^2 + 1) + b*log(b*x + a + 1)/(a^2 + 2*a + 1) - b*log(b*x + a - 1)/(a^2 - 2*a + 1) + 2/((a^2 - 1)*x))*b*arctanh(b*x + a) - 1/2*arctanh(b*x + a)^2/x^2

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{atanh}(a + bx)^2}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(atanh(a + b*x)^2/x^3,x)

[Out] int(atanh(a + b*x)^2/x^3, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}^2(a + bx)}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(atanh(b*x+a)**2/x**3,x)

[Out] Integral(atanh(a + b*x)**2/x**3, x)

$$3.8 \quad \int \frac{\tanh^{-1}(1+bx)^2}{x} dx$$

Optimal. Leaf size=56

$$\frac{1}{2}\text{Li}_3\left(1 + \frac{2}{bx}\right) - \text{Li}_2\left(1 + \frac{2}{bx}\right)\tanh^{-1}(bx+1) - \log\left(-\frac{2}{bx}\right)\tanh^{-1}(bx+1)^2$$

[Out] $-\text{arctanh}(b*x+1)^2*\ln(-2/b/x) - \text{arctanh}(b*x+1)*\text{polylog}(2, 1+2/b/x) + 1/2*\text{polylog}(3, 1+2/b/x)$

Rubi [A] time = 0.13, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {6111, 5918, 5948, 6058, 6610}

$$\frac{1}{2}\text{PolyLog}\left(3, \frac{2}{bx} + 1\right) - \tanh^{-1}(bx+1)\text{PolyLog}\left(2, \frac{2}{bx} + 1\right) - \log\left(-\frac{2}{bx}\right)\tanh^{-1}(bx+1)^2$$

Antiderivative was successfully verified.

[In] Int[ArcTanh[1 + b*x]^2/x, x]

[Out] $-(\text{ArcTanh}[1 + b*x]^2*\text{Log}[-2/(b*x)]) - \text{ArcTanh}[1 + b*x]*\text{PolyLog}[2, 1 + 2/(b*x)] + \text{PolyLog}[3, 1 + 2/(b*x)]/2$

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_.)]*(b_.))^(p_.)/((d_.) + (e_.)*(x_.)), x_Symbol]
:> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c^p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)]]/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_.)]*(b_.))^(p_.)/((d_.) + (e_.)*(x_.)^2), x_Symbol]
:> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6058

```
Int[(Log[u_] * ((a_.) + ArcTanh[(c_.)*(x_.)]*(b_.))^(p_.))/((d_.) + (e_.)*(x_.)^2), x_Symbol]
:> -Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d), x] + Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
```

e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]

Rule 6111

Int[((a_.) + ArcTanh[(c_.) + (d_.)*(x_.)]*(b_.))^ (p_.)*((e_.) + (f_.)*(x_.))^ (m_.), x_Symbol] := Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*ArcTanh[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]

Rule 6610

Int[(u_)*PolyLog[n_, v_], x_Symbol] := With[{w = DerivativeDivides[v, u*v, x]}, Simp[w*PolyLog[n + 1, v], x] /; !FalseQ[w]] /; FreeQ[n, x]

Rubi steps

$$\begin{aligned} \int \frac{\tanh^{-1}(1+bx)^2}{x} dx &= \frac{\text{Subst}\left(\int \frac{\tanh^{-1}(x)^2}{-\frac{1}{b} + \frac{x}{b}} dx, x, 1+bx\right)}{b} \\ &= -\tanh^{-1}(1+bx)^2 \log\left(-\frac{2}{bx}\right) + 2 \text{Subst}\left(\int \frac{\tanh^{-1}(x) \log\left(\frac{2}{1-x}\right)}{1-x^2} dx, x, 1+bx\right) \\ &= -\tanh^{-1}(1+bx)^2 \log\left(-\frac{2}{bx}\right) - \tanh^{-1}(1+bx) \text{Li}_2\left(1 + \frac{2}{bx}\right) + \text{Subst}\left(\int \frac{\text{Li}_2\left(1 - \frac{2}{1-x}\right)}{1-x^2} dx, x, 1+bx\right) \\ &= -\tanh^{-1}(1+bx)^2 \log\left(-\frac{2}{bx}\right) - \tanh^{-1}(1+bx) \text{Li}_2\left(1 + \frac{2}{bx}\right) + \frac{1}{2} \text{Li}_3\left(1 + \frac{2}{bx}\right) \end{aligned}$$

Mathematica [A] time = 0.09, size = 75, normalized size = 1.34

$$\tanh^{-1}(bx+1) \text{Li}_2\left(-e^{-2 \tanh^{-1}(bx+1)}\right) + \frac{1}{2} \text{Li}_3\left(-e^{-2 \tanh^{-1}(bx+1)}\right) - \frac{2}{3} \tanh^{-1}(bx+1)^3 - \tanh^{-1}(bx+1)^2 \log\left(e^{-2 \tanh^{-1}(bx+1)}\right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[ArcTanh[1 + b*x]^2/x, x]

[Out] (-2*ArcTanh[1 + b*x]^3)/3 - ArcTanh[1 + b*x]^2*Log[1 + E^(-2*ArcTanh[1 + b*x])] + ArcTanh[1 + b*x]*PolyLog[2, -E^(-2*ArcTanh[1 + b*x])] + PolyLog[3, -E^(-2*ArcTanh[1 + b*x])]/2

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\text{artanh}(bx+1)^2}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctanh(b*x+1)^2/x,x, algorithm="fricas")

[Out] integral(arctanh(b*x + 1)^2/x, x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{artanh}(bx+1)^2}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctanh(b*x+1)^2/x,x, algorithm="giac")

[Out] integrate(arctanh(b*x + 1)^2/x, x)

maple [C] time = 0.45, size = 160, normalized size = 2.86

$$\ln(bx) \text{arctanh}(bx+1)^2 - \text{arctanh}(bx+1) \text{polylog}\left(2, -\frac{(bx+2)^2}{-(bx+1)^2+1}\right) + \frac{\text{polylog}\left(3, -\frac{(bx+2)^2}{-(bx+1)^2+1}\right)}{2} - \left(i\pi \text{csgn}\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arctanh(b*x+1)^2/x,x)

[Out] ln(b*x)*arctanh(b*x+1)^2-arctanh(b*x+1)*polylog(2, -(b*x+2)^2/(-(b*x+1)^2+1))+1/2*polylog(3, -(b*x+2)^2/(-(b*x+1)^2+1))-(I*Pi*csgn(I/(1+(b*x+2)^2/(-(b*x+1)^2+1))))^3-I*Pi*csgn(I/(1+(b*x+2)^2/(-(b*x+1)^2+1)))^2+I*Pi+ln(2))*arctanh(b*x+1)^2

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{12} \log(-bx)^3 + \frac{1}{4} \log(bx+2)^2 \log(-x) - \frac{1}{4} \int \frac{2(bx \log(b) + 2(bx+1) \log(-x) + 2 \log(b)) \log(bx+2)}{bx^2+2x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctanh(b*x+1)^2/x,x, algorithm="maxima")

[Out] $1/12*\log(-b*x)^3 + 1/4*\log(b*x + 2)^2*\log(-x) - 1/4*\text{integrate}(2*(b*x*\log(b) + 2*(b*x + 1)*\log(-x) + 2*\log(b))*\log(b*x + 2)/(b*x^2 + 2*x), x)$

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\operatorname{atanh}(bx + 1)^2}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(atanh(b*x + 1)^2/x, x)`

[Out] `int(atanh(b*x + 1)^2/x, x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}^2(bx + 1)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(atanh(b*x+1)**2/x, x)`

[Out] `Integral(atanh(b*x + 1)**2/x, x)`

3.9 $\int (ce + dex)^3 (a + b \tanh^{-1}(c + dx)) dx$

Optimal. Leaf size=72

$$\frac{e^3(c + dx)^4 (a + b \tanh^{-1}(c + dx))}{4d} + \frac{be^3(c + dx)^3}{12d} - \frac{be^3 \tanh^{-1}(c + dx)}{4d} + \frac{1}{4}be^3x$$

[Out] $1/4*b*e^3*x+1/12*b*e^3*(d*x+c)^3/d-1/4*b*e^3*\operatorname{arctanh}(d*x+c)/d+1/4*e^3*(d*x+c)^4*(a+b*\operatorname{arctanh}(d*x+c))/d$

Rubi [A] time = 0.07, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {6107, 12, 5916, 302, 206}

$$\frac{e^3(c + dx)^4 (a + b \tanh^{-1}(c + dx))}{4d} + \frac{be^3(c + dx)^3}{12d} - \frac{be^3 \tanh^{-1}(c + dx)}{4d} + \frac{1}{4}be^3x$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(c*e + d*e*x)^3*(a + b*\operatorname{ArcTanh}[c + d*x]), x]$

[Out] $(b*e^3*x)/4 + (b*e^3*(c + d*x)^3)/(12*d) - (b*e^3*\operatorname{ArcTanh}[c + d*x])/(4*d) + (e^3*(c + d*x)^4*(a + b*\operatorname{ArcTanh}[c + d*x]))/(4*d)$

Rule 12

$\operatorname{Int}[(a_*)*(u_), x_Symbol] \rightarrow \operatorname{Dist}[a, \operatorname{Int}[u, x], x] /; \operatorname{FreeQ}[a, x] \&\& \operatorname{!Match} Q[u, (b_)*(v_)] /; \operatorname{FreeQ}[b, x]$

Rule 206

$\operatorname{Int}[((a_) + (b_)*(x_)^2)^{-1}, x_Symbol] \rightarrow \operatorname{Simp}[(1*\operatorname{ArcTanh}[\operatorname{Rt}[-b, 2]*x]/\operatorname{Rt}[a, 2])]/(\operatorname{Rt}[a, 2]*\operatorname{Rt}[-b, 2]), x] /; \operatorname{FreeQ}\{a, b\}, x \&\& \operatorname{NegQ}[a/b] \&\& (\operatorname{Gt} Q[a, 0] \operatorname{||} \operatorname{Lt} Q[b, 0])$

Rule 302

$\operatorname{Int}[(x_)^{(m)}/((a_) + (b_)*(x_)^{(n)}), x_Symbol] \rightarrow \operatorname{Int}[\operatorname{PolynomialDivide}[x^m, a + b*x^n, x], x] /; \operatorname{FreeQ}\{a, b\}, x \&\& \operatorname{IGt} Q[m, 0] \&\& \operatorname{IGt} Q[n, 0] \&\& \operatorname{Gt} Q[m, 2*n - 1]$

Rule 5916

$\operatorname{Int}[(a_*) + \operatorname{ArcTanh}[(c_)*(x_)]*(b_*)^{(p_)*((d_)*(x_))^{(m_)}}, x_Symbol] \rightarrow \operatorname{Simp}[(d*x)^{(m+1)}*(a + b*\operatorname{ArcTanh}[c*x])^p/(d*(m+1)), x] - \operatorname{Dist}[(b*c$

p)/(d(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || IntegerQ[m]) && NeQ[m, -1]

Rule 6107

Int[((a_.) + ArcTanh[(c_.) + (d_.)*(x_.)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_.))^(m_.), x_Symbol] :> Dist[1/d, Subst[Int[((f*x)/d)^m*(a + b*ArcTanh[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] && IGtQ[p, 0]

Rubi steps

$$\begin{aligned}
 \int (ce + dex)^3 (a + b \tanh^{-1}(c + dx)) dx &= \frac{\text{Subst}\left(\int e^3 x^3 (a + b \tanh^{-1}(x)) dx, x, c + dx\right)}{d} \\
 &= \frac{e^3 \text{Subst}\left(\int x^3 (a + b \tanh^{-1}(x)) dx, x, c + dx\right)}{d} \\
 &= \frac{e^3 (c + dx)^4 (a + b \tanh^{-1}(c + dx))}{4d} - \frac{(be^3) \text{Subst}\left(\int \frac{x^4}{1-x^2} dx, x, c + dx\right)}{4d} \\
 &= \frac{e^3 (c + dx)^4 (a + b \tanh^{-1}(c + dx))}{4d} - \frac{(be^3) \text{Subst}\left(\int \left(-1 - x^2 + \frac{1}{1-x^2}\right) dx, x, c + dx\right)}{4d} \\
 &= \frac{1}{4} be^3 x + \frac{be^3 (c + dx)^3}{12d} + \frac{e^3 (c + dx)^4 (a + b \tanh^{-1}(c + dx))}{4d} - \frac{(be^3) \text{Subst}\left(\int \frac{1}{1-x^2} dx, x, c + dx\right)}{4d} \\
 &= \frac{1}{4} be^3 x + \frac{be^3 (c + dx)^3}{12d} - \frac{be^3 \tanh^{-1}(c + dx)}{4d} + \frac{e^3 (c + dx)^4 (a + b \tanh^{-1}(c + dx))}{4d}
 \end{aligned}$$

Mathematica [A] time = 0.10, size = 78, normalized size = 1.08

$$\frac{e^3 (6a(c + dx)^4 + 2b(c + dx)^3 + 6b(c + dx) + 3b \log(-c - dx + 1) - 3b \log(c + dx + 1) + 6b(c + dx)^4 \tanh^{-1}(c + dx))}{24d}$$

Antiderivative was successfully verified.

[In] Integrate[(c*e + d*e*x)^3*(a + b*ArcTanh[c + d*x]), x]

[Out] (e^3*(6*b*(c + d*x) + 2*b*(c + d*x)^3 + 6*a*(c + d*x)^4 + 6*b*(c + d*x)^4*ArcTanh[c + d*x] + 3*b*Log[1 - c - d*x] - 3*b*Log[1 + c + d*x]))/(24*d)

fricas [B] time = 0.56, size = 159, normalized size = 2.21

$$\frac{6ad^4e^3x^4 + 2(12ac + b)d^3e^3x^3 + 6(6ac^2 + bc)d^2e^3x^2 + 6(4ac^3 + bc^2 + b)de^3x + 3(bd^4e^3x^4 + 4bcd^3e^3x^3 + 6bd^2e^3x^2 + 6bd^2e^3x^2 + 4b^2c^3d^2e^3x + (b^2c^4 - b^2)e^3) \log(-(dx + c + 1)/(dx + c - 1))}{24d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*e*x+c*e)^3*(a+b*arctanh(d*x+c)),x, algorithm="fricas")

[Out] 1/24*(6*a*d^4*e^3*x^4 + 2*(12*a*c + b)*d^3*e^3*x^3 + 6*(6*a*c^2 + b*c)*d^2*e^3*x^2 + 6*(4*a*c^3 + b*c^2 + b)*d*e^3*x + 3*(b*d^4*e^3*x^4 + 4*b*c*d^3*e^3*x^3 + 6*b*c^2*d^2*e^3*x^2 + 4*b*c^3*d^2*e^3*x + (b*c^4 - b)*e^3)*log(-(d*x + c + 1)/(d*x + c - 1))/d

giac [B] time = 3.08, size = 282, normalized size = 3.92

$$\frac{\left(\frac{3(dx+c+1)^3be^3 \log\left(\frac{dx+c+1}{dx+c-1}\right)}{(dx+c-1)^3} + \frac{3(dx+c+1)be^3 \log\left(\frac{dx+c+1}{dx+c-1}\right)}{dx+c-1} + \frac{6(dx+c+1)^3ae^3}{(dx+c-1)^3} + \frac{6(dx+c+1)ae^3}{dx+c-1} + \frac{3(dx+c+1)^3be^3}{(dx+c-1)^3} - \frac{6(dx+c+1)^2be^3}{(dx+c-1)^2} + \frac{5(dx+c+1)^2be^3}{(dx+c-1)^2} \right)}{6 \left(\frac{(dx+c+1)^4d^2}{(dx+c-1)^4} - \frac{4(dx+c+1)^3d^2}{(dx+c-1)^3} + \frac{6(dx+c+1)^2d^2}{(dx+c-1)^2} - \frac{4(dx+c+1)d^2}{dx+c-1} + d^2 \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*e*x+c*e)^3*(a+b*arctanh(d*x+c)),x, algorithm="giac")

[Out] 1/6*(3*(d*x + c + 1)^3*b*e^3*log(-(d*x + c + 1)/(d*x + c - 1))/(d*x + c - 1)^3 + 3*(d*x + c + 1)*b*e^3*log(-(d*x + c + 1)/(d*x + c - 1))/(d*x + c - 1) + 6*(d*x + c + 1)^3*a*e^3/(d*x + c - 1)^3 + 6*(d*x + c + 1)*a*e^3/(d*x + c - 1) + 3*(d*x + c + 1)^3*b*e^3/(d*x + c - 1)^3 - 6*(d*x + c + 1)^2*b*e^3/(d*x + c - 1)^2 + 5*(d*x + c + 1)*b*e^3/(d*x + c - 1) - 2*b*e^3)*((c + 1)*d - (c - 1)*d)/((d*x + c + 1)^4*d^2/(d*x + c - 1)^4 - 4*(d*x + c + 1)^3*d^2/(d*x + c - 1)^3 + 6*(d*x + c + 1)^2*d^2/(d*x + c - 1)^2 - 4*(d*x + c + 1)*d^2/(d*x + c - 1) + d^2)

maple [B] time = 0.03, size = 242, normalized size = 3.36

$$\frac{d^3x^4ae^3}{4} + d^2x^3ace^3 + \frac{3dx^2a^2c^2e^3}{2} + xac^3e^3 + \frac{ac^4e^3}{4d} + \frac{d^3 \operatorname{arctanh}(dx+c)x^4be^3}{4} + d^2 \operatorname{arctanh}(dx+c)x^3bce^3 + \frac{3d \operatorname{arctanh}(dx+c)x^2bce^3}{4} + \frac{3d \operatorname{arctanh}(dx+c)x^2bce^3}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*e*x+c*e)^3*(a+b*arctanh(d*x+c)),x)

[Out] 1/4*d^3*x^4*a*e^3+d^2*x^3*a*c^2*e^3+3/2*d*x^2*a*c^2*e^3+x*a*c^3*e^3+1/4/d*a*c^4*e^3+1/4*d^3*arctanh(d*x+c)*x^4*b*e^3+d^2*arctanh(d*x+c)*x^3*b*c*e^3+3/2*d

$d \cdot \operatorname{arctanh}(d \cdot x + c) \cdot x^2 \cdot b \cdot c^2 \cdot e^3 + \operatorname{arctanh}(d \cdot x + c) \cdot x \cdot b \cdot c^3 \cdot e^3 + 1/4 \cdot d \cdot \operatorname{arctanh}(d \cdot x + c) \cdot b \cdot c^4 \cdot e^3 + 1/12 \cdot d^2 \cdot x^3 \cdot b \cdot e^3 + 1/4 \cdot d \cdot x^2 \cdot b \cdot c \cdot e^3 + 1/4 \cdot x \cdot b \cdot c^2 \cdot e^3 + 1/12 \cdot d \cdot b \cdot c^3 \cdot e^3 + 1/4 \cdot b \cdot e^3 \cdot x + 1/4 \cdot d \cdot b \cdot c \cdot e^3 + 1/8 \cdot d \cdot e^3 \cdot b \cdot \ln(d \cdot x + c - 1) - 1/8 \cdot d \cdot e^3 \cdot b \cdot \ln(d \cdot x + c + 1)$

maxima [B] time = 0.33, size = 357, normalized size = 4.96

$$\frac{1}{4} a d^3 e^3 x^4 + a c d^2 e^3 x^3 + \frac{3}{2} a c^2 d e^3 x^2 + \frac{3}{4} \left(2 x^2 \operatorname{artanh}(d x + c) + d \left(\frac{2 x}{d^2} - \frac{(c^2 + 2 c + 1) \log(d x + c + 1)}{d^3} + \frac{(c^2 - 2 c + 1) \log(d x + c - 1)}{d^3} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*e*x+c*e)^3*(a+b*arctanh(d*x+c)),x, algorithm="maxima")

[Out] $\frac{1}{4} a d^3 e^3 x^4 + a c d^2 e^3 x^3 + \frac{3}{2} a c^2 d e^3 x^2 + \frac{3}{4} (2 x^2 \operatorname{arctanh}(d x + c) + d (2 x / d^2 - (c^2 + 2 c + 1) \log(d x + c + 1) / d^3 + (c^2 - 2 c + 1) \log(d x + c - 1) / d^3)) \cdot b \cdot c^2 \cdot d \cdot e^3 + \frac{1}{2} (2 x^3 \operatorname{arctanh}(d x + c) + d ((d x^2 - 4 c x) / d^3 + (c^3 + 3 c^2 + 3 c + 1) \log(d x + c + 1) / d^4 - (c^3 - 3 c^2 + 3 c - 1) \log(d x + c - 1) / d^4)) \cdot b \cdot c \cdot d^2 \cdot e^3 + \frac{1}{24} (6 x^4 \operatorname{arctanh}(d x + c) + d (2 (d^2 x^3 - 3 c d x^2 + 3 (3 c^2 + 1) x) / d^4 - 3 (c^4 + 4 c^3 + 6 c^2 + 4 c + 1) \log(d x + c + 1) / d^5 + 3 (c^4 - 4 c^3 + 6 c^2 - 4 c + 1) \log(d x + c - 1) / d^5)) \cdot b \cdot d^3 \cdot e^3 + a c^3 e^3 x + \frac{1}{2} (2 (d x + c) \operatorname{arctanh}(d x + c) + \log(-(d x + c)^2 + 1)) \cdot b \cdot c^3 \cdot e^3 / d$

mupad [B] time = 1.42, size = 414, normalized size = 5.75

$$x^3 \left(\frac{d^2 e^3 (b + 20 a c)}{12} - \frac{2 a c d^2 e^3}{3} \right) + \ln(c + d x + 1) \left(\frac{b c^3 e^3 x}{2} + \frac{3 b c^2 d e^3 x^2}{4} + \frac{b c d^2 e^3 x^3}{2} + \frac{b d^3 e^3 x^4}{8} \right) - \ln(1 - d x - c) \left(\frac{b c^3 e^3 x^4}{8} + \frac{b c^3 e^3 x^3}{2} + \frac{3 b c^2 d e^3 x^2}{4} + \frac{b c d^2 e^3 x^3}{2} \right) - x^2 \left(\frac{c ((d^2 e^3 (b + 20 a c)) / 4 - 2 a c d^2 e^3)}{d} - \frac{d e^3 (b c - a + 10 a c^2)}{2} + \frac{a d e^3 (4 c^2 - 4)}{8} \right) + x \left(\frac{c e^3 (3 b c - 6 a + 20 a c^2)}{2} - \frac{(4 c^2 - 4) ((d^2 e^3 (b + 20 a c)) / 4 - 2 a c d^2 e^3)}{(4 d^2)} + \frac{2 c ((2 c ((d^2 e^3 (b + 20 a c)) / 4 - 2 a c d^2 e^3)) / d - d e^3 (b c - a + 10 a c^2) + (a d e^3 (4 c^2 - 4)) / 4)}{d} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*e + d*e*x)^3*(a + b*atanh(c + d*x)),x)

[Out] $x^3 ((d^2 e^3 (b + 20 a c)) / 12 - (2 a c d^2 e^3) / 3) + \log(c + d x + 1) ((b d^3 e^3 x^4) / 8 + (b c^3 e^3 x^3) / 2 + (3 b c^2 d e^3 x^2) / 4 + (b c d^2 e^3 x^3) / 2) - \log(1 - d x - c) ((b d^3 e^3 x^4) / 8 + (b c^3 e^3 x^3) / 2 + (3 b c^2 d e^3 x^2) / 4 + (b c d^2 e^3 x^3) / 2) - x^2 ((c ((d^2 e^3 (b + 20 a c)) / 4 - 2 a c d^2 e^3)) / d - (d e^3 (b c - a + 10 a c^2)) / 2 + (a d e^3 (4 c^2 - 4)) / 8) + x ((c e^3 (3 b c - 6 a + 20 a c^2)) / 2 - ((4 c^2 - 4) ((d^2 e^3 (b + 20 a c)) / 4 - 2 a c d^2 e^3)) / (4 d^2) + (2 c ((2 c ((d^2 e^3 (b + 20 a c)) / 4 - 2 a c d^2 e^3)) / d - d e^3 (b c - a + 10 a c^2) + (a d e^3 (4 c^2 - 4)) / 4)) / d)$

+ (log(c + d*x - 1)*(b*e^3 - b*c^4*e^3))/(8*d) + (a*d^3*e^3*x^4)/4 + (b*e^3 * log(c + d*x + 1)*(c^2 + 1)*(c - 1)*(c + 1))/(8*d)

sympy [A] time = 4.27, size = 231, normalized size = 3.21

$$\left\{ \begin{array}{l} ac^3e^3x + \frac{3ac^2de^3x^2}{2} + acd^2e^3x^3 + \frac{ad^3e^3x^4}{4} + \frac{bc^4e^3 \operatorname{atanh}(c+dx)}{4d} + bc^3e^3x \operatorname{atanh}(c + dx) + \frac{3bc^2de^3x^2 \operatorname{atanh}(c+dx)}{2} + \frac{bc^2e^3x}{4} + \\ c^3e^3x(a + b \operatorname{atanh}(c)) \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*e*x+c*e)**3*(a+b*atanh(d*x+c)), x)

[Out] Piecewise((a*c**3*e**3*x + 3*a*c**2*d*e**3*x**2/2 + a*c*d**2*e**3*x**3 + a*d**3*e**3*x**4/4 + b*c**4*e**3*atanh(c + d*x)/(4*d) + b*c**3*e**3*x*atanh(c + d*x) + 3*b*c**2*d*e**3*x**2*atanh(c + d*x)/2 + b*c**2*e**3*x/4 + b*c*d**2*e**3*x**3*atanh(c + d*x) + b*c*d*e**3*x**2/4 + b*d**3*e**3*x**4*atanh(c + d*x)/4 + b*d**2*e**3*x**3/12 + b*e**3*x/4 - b*e**3*atanh(c + d*x)/(4*d), Ne(d, 0)), (c**3*e**3*x*(a + b*atanh(c)), True))

3.10 $\int (ce + dex)^2 (a + b \tanh^{-1}(c + dx)) dx$

Optimal. Leaf size=69

$$\frac{e^2(c + dx)^3 (a + b \tanh^{-1}(c + dx))}{3d} + \frac{be^2(c + dx)^2}{6d} + \frac{be^2 \log(1 - (c + dx)^2)}{6d}$$

[Out] $1/6*b*e^2*(d*x+c)^2/d+1/3*e^2*(d*x+c)^3*(a+b*\operatorname{arctanh}(d*x+c))/d+1/6*b*e^2*\ln(1-(d*x+c)^2)/d$

Rubi [A] time = 0.07, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {6107, 12, 5916, 266, 43}

$$\frac{e^2(c + dx)^3 (a + b \tanh^{-1}(c + dx))}{3d} + \frac{be^2(c + dx)^2}{6d} + \frac{be^2 \log(1 - (c + dx)^2)}{6d}$$

Antiderivative was successfully verified.

[In] `Int[(c*e + d*e*x)^2*(a + b*ArcTanh[c + d*x]),x]`

[Out] $(b*e^2*(c + d*x)^2)/(6*d) + (e^2*(c + d*x)^3*(a + b*ArcTanh[c + d*x]))/(3*d) + (b*e^2*Log[1 - (c + d*x)^2])/(6*d)$

Rule 12

`Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]`

Rule 43

`Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])`

Rule 266

`Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]`

Rule 5916

`Int[((a_.) + ArcTanh[(c_.)*(x_)])*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol] := Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c`

p)/(d(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || IntegerQ[m]) && NeQ[m, -1]

Rule 6107

Int[((a_.) + ArcTanh[(c_) + (d_.)*(x_.)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_.))^(m_.), x_Symbol] :> Dist[1/d, Subst[Int[((f*x)/d)^m*(a + b*ArcTanh[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] && IGtQ[p, 0]

Rubi steps

$$\begin{aligned}
 \int (ce + dex)^2 (a + b \tanh^{-1}(c + dx)) dx &= \frac{\text{Subst}\left(\int e^2 x^2 (a + b \tanh^{-1}(x)) dx, x, c + dx\right)}{d} \\
 &= \frac{e^2 \text{Subst}\left(\int x^2 (a + b \tanh^{-1}(x)) dx, x, c + dx\right)}{d} \\
 &= \frac{e^2 (c + dx)^3 (a + b \tanh^{-1}(c + dx))}{3d} - \frac{(be^2) \text{Subst}\left(\int \frac{x^3}{1-x^2} dx, x, c + dx\right)}{3d} \\
 &= \frac{e^2 (c + dx)^3 (a + b \tanh^{-1}(c + dx))}{3d} - \frac{(be^2) \text{Subst}\left(\int \frac{x}{1-x} dx, x, (c + dx)\right)}{6d} \\
 &= \frac{e^2 (c + dx)^3 (a + b \tanh^{-1}(c + dx))}{3d} - \frac{(be^2) \text{Subst}\left(\int \left(-1 + \frac{1}{1-x}\right) dx, x, (c + dx)\right)}{6d} \\
 &= \frac{be^2 (c + dx)^2}{6d} + \frac{e^2 (c + dx)^3 (a + b \tanh^{-1}(c + dx))}{3d} + \frac{be^2 \log(1 - (c + dx))}{6d}
 \end{aligned}$$

Mathematica [A] time = 0.08, size = 59, normalized size = 0.86

$$\frac{e^2 \left((c + dx)^2 (2a(c + dx) + b) + b \log(1 - (c + dx)^2) + 2b(c + dx)^3 \tanh^{-1}(c + dx) \right)}{6d}$$

Antiderivative was successfully verified.

[In] Integrate[(c*e + d*e*x)^2*(a + b*ArcTanh[c + d*x]),x]

[Out] (e^2*((c + d*x)^2*(b + 2*a*(c + d*x)) + 2*b*(c + d*x)^3*ArcTanh[c + d*x] + b*Log[1 - (c + d*x)^2]))/(6*d)

fricas [B] time = 0.50, size = 144, normalized size = 2.09

$$\frac{2ad^3e^2x^3 + (6ac + b)d^2e^2x^2 + 2(3ac^2 + bc)de^2x + (bc^3 + b)e^2 \log(dx + c + 1) - (bc^3 - b)e^2 \log(dx + c - 1) + (b^2d^3e^2x^3 + 3b^2c^2d^2e^2x^2 + 3b^2c^2d^2e^2x) \log(-\frac{dx+c+1}{dx+c-1})}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*e*x+c*e)^2*(a+b*arctanh(d*x+c)),x, algorithm="fricas")

[Out] 1/6*(2*a*d^3*e^2*x^3 + (6*a*c + b)*d^2*e^2*x^2 + 2*(3*a*c^2 + b*c)*d*e^2*x + (b*c^3 + b)*e^2*log(d*x + c + 1) - (b*c^3 - b)*e^2*log(d*x + c - 1) + (b*d^3*e^2*x^3 + 3*b*c*d^2*e^2*x^2 + 3*b*c^2*d*e^2*x)*log(-\frac{dx+c+1}{dx+c-1}))/d

giac [B] time = 0.34, size = 364, normalized size = 5.28

$$\frac{\left(\frac{(dx+c+1)^3 be^2 \log\left(-\frac{dx+c+1}{dx+c-1}+1\right)}{(dx+c-1)^3} - \frac{3(dx+c+1)^2 be^2 \log\left(-\frac{dx+c+1}{dx+c-1}+1\right)}{(dx+c-1)^2} + \frac{3(dx+c+1) be^2 \log\left(-\frac{dx+c+1}{dx+c-1}+1\right)}{dx+c-1} - be^2 \log\left(-\frac{dx+c+1}{dx+c-1}+1\right) - \frac{(dx+c+1)^3 d^2}{(dx+c-1)^3} - \frac{3(dx+c+1)^2 d^2}{(dx+c-1)^2} \right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*e*x+c*e)^2*(a+b*arctanh(d*x+c)),x, algorithm="giac")

[Out] -1/6*((d*x + c + 1)^3*b*e^2*log(-\frac{dx+c+1}{dx+c-1}+1)/(\frac{dx+c+1}{dx+c-1}+1) - 3*(d*x + c + 1)^2*b*e^2*log(-\frac{dx+c+1}{dx+c-1}+1)/(\frac{dx+c+1}{dx+c-1}+1)^2 + 3*(d*x + c + 1)*b*e^2*log(-\frac{dx+c+1}{dx+c-1}+1)/(\frac{dx+c+1}{dx+c-1}+1) - b*e^2*log(-\frac{dx+c+1}{dx+c-1}+1) - (d*x + c + 1)^3*b*e^2*log(-\frac{dx+c+1}{dx+c-1}+1)/(\frac{dx+c+1}{dx+c-1}+1)^3 - 3*(d*x + c + 1)^2*b*e^2*log(-\frac{dx+c+1}{dx+c-1}+1)/(\frac{dx+c+1}{dx+c-1}+1)^2 - 6*(d*x + c + 1)^2*a*e^2/(\frac{dx+c+1}{dx+c-1}+1)^2 - 2*a*e^2 - 2*(d*x + c + 1)^2*b*e^2/(\frac{dx+c+1}{dx+c-1}+1)^2 + 2*(d*x + c + 1)*b*e^2/(\frac{dx+c+1}{dx+c-1}+1))*((c + 1)*d - (c - 1)*d)/((d*x + c + 1)^3*d^2/(\frac{dx+c+1}{dx+c-1}+1)^3 - 3*(d*x + c + 1)^2*d^2/(\frac{dx+c+1}{dx+c-1}+1)^2 + 3*(d*x + c + 1)*d^2/(\frac{dx+c+1}{dx+c-1}+1) - d^2)

maple [B] time = 0.03, size = 174, normalized size = 2.52

$$\frac{d^2x^3ae^2}{3} + dx^2ace^2 + xac^2e^2 + \frac{ac^3e^2}{3d} + \frac{d^2 \operatorname{arctanh}(dx+c)x^3be^2}{3} + d \operatorname{arctanh}(dx+c)x^2bce^2 + \operatorname{arctanh}(dx+c)xbce^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*e*x+c*e)^2*(a+b*arctanh(d*x+c)),x)

[Out] $\frac{1}{3}d^2x^3 + acde^2x^2 + \frac{1}{2}\left(2x^2 \operatorname{arctanh}(dx+c) + d\left(\frac{2x}{d^2} - \frac{(c^2+2c+1)\log(dx+c+1)}{d^3} + \frac{(c^2-2c+1)\log(dx+c-1)}{d^3}\right)\right) + \frac{1}{6}d^2x^3 + \frac{1}{6}d^2x^2 + \frac{1}{6}d^2x + \frac{1}{6}d^2 + \frac{1}{6}d^2 \ln(dx+c-1) + \frac{1}{6}d^2 \ln(dx+c+1)$

maxima [B] time = 0.32, size = 225, normalized size = 3.26

$$\frac{1}{3}ad^2e^2x^3 + acde^2x^2 + \frac{1}{2}\left(2x^2 \operatorname{arctanh}(dx+c) + d\left(\frac{2x}{d^2} - \frac{(c^2+2c+1)\log(dx+c+1)}{d^3} + \frac{(c^2-2c+1)\log(dx+c-1)}{d^3}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*e*x+c*e)^2*(a+b*arctanh(d*x+c)),x, algorithm="maxima")`

[Out] $\frac{1}{3}a*d^2*e^2*x^3 + a*c*d*e^2*x^2 + \frac{1}{2}*(2*x^2*\operatorname{arctanh}(d*x+c) + d*(2*x/d^2 - (c^2+2*c+1)*\log(d*x+c+1)/d^3 + (c^2-2*c+1)*\log(d*x+c-1)/d^3))*b*c*d*e^2 + \frac{1}{6}*(2*x^3*\operatorname{arctanh}(d*x+c) + d*((d*x^2-4*c*x)/d^3 + (c^3+3*c^2+3*c+1)*\log(d*x+c+1)/d^4 - (c^3-3*c^2+3*c-1)*\log(d*x+c-1)/d^4))*b*d^2*e^2 + a*c^2*e^2*x + \frac{1}{2}*(2*(d*x+c)*\operatorname{arctanh}(d*x+c) + \log(-(d*x+c)^2+1))*b*c^2*e^2/d$

mupad [B] time = 0.62, size = 237, normalized size = 3.43

$$\frac{ad^2e^2x^3}{3} + \frac{bce^2x}{3} + \frac{be^2 \ln(c+dx-1)}{6d} + \frac{be^2 \ln(c+dx+1)}{6d} + ac^2e^2x + \frac{bd^2e^2x^2}{6} + acde^2x^2 + \frac{bc^2e^2x \ln(c+dx)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c*e+d*e*x)^2*(a+b*atanh(c+d*x)),x)`

[Out] $(a*d^2*e^2*x^3)/3 + (b*c*e^2*x)/3 + (b*e^2*\log(c+d*x-1))/(6*d) + (b*e^2*\log(c+d*x+1))/(6*d) + a*c^2*e^2*x + (b*d*e^2*x^2)/6 + a*c*d*e^2*x^2 + (b*c^2*e^2*x*\log(c+d*x+1))/2 - (b*c^3*e^2*\log(c+d*x-1))/(6*d) + (b*c^3*e^2*\log(c+d*x+1))/(6*d) - (b*c^2*e^2*x*\log(1-d*x-c))/2 + (b*d^2*e^2*x^3*\log(c+d*x+1))/6 - (b*d^2*e^2*x^3*\log(1-d*x-c))/6 + (b*c*d*e^2*x^2*\log(c+d*x+1))/2 - (b*c*d*e^2*x^2*\log(1-d*x-c))/2$

sympy [A] time = 3.03, size = 180, normalized size = 2.61

$$\begin{cases} ac^2e^2x + acde^2x^2 + \frac{ad^2e^2x^3}{3} + \frac{bc^3e^2 \operatorname{atanh}(c+dx)}{3d} + bc^2e^2x \operatorname{atanh}(c+dx) + bcde^2x^2 \operatorname{atanh}(c+dx) + \frac{bce^2x}{3} + \frac{bd^2e^2x^3}{6} \\ c^2e^2x(a+b \operatorname{atanh}(c)) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**2*(a+b*atanh(d*x+c)),x)
```

```
[Out] Piecewise((a*c**2*e**2*x + a*c*d*e**2*x**2 + a*d**2*e**2*x**3/3 + b*c**3*e*  
*2*atanh(c + d*x)/(3*d) + b*c**2*e**2*x*atanh(c + d*x) + b*c*d*e**2*x**2*at  
anh(c + d*x) + b*c*e**2*x/3 + b*d**2*e**2*x**3*atanh(c + d*x)/3 + b*d*e**2*  
x**2/6 + b*e**2*log(c/d + x + 1/d)/(3*d) - b*e**2*atanh(c + d*x)/(3*d), Ne(  
d, 0)), (c**2*e**2*x*(a + b*atanh(c)), True))
```

3.11 $\int (ce + dex) (a + b \tanh^{-1}(c + dx)) dx$

Optimal. Leaf size=48

$$\frac{e(c + dx)^2 (a + b \tanh^{-1}(c + dx))}{2d} - \frac{be \tanh^{-1}(c + dx)}{2d} + \frac{bex}{2}$$

[Out] $1/2*b*e*x-1/2*b*e*arctanh(d*x+c)/d+1/2*e*(d*x+c)^2*(a+b*arctanh(d*x+c))/d$

Rubi [A] time = 0.03, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {6107, 12, 5916, 321, 206}

$$\frac{e(c + dx)^2 (a + b \tanh^{-1}(c + dx))}{2d} - \frac{be \tanh^{-1}(c + dx)}{2d} + \frac{bex}{2}$$

Antiderivative was successfully verified.

[In] Int[(c*e + d*e*x)*(a + b*ArcTanh[c + d*x]),x]

[Out] (b*e*x)/2 - (b*e*ArcTanh[c + d*x])/(2*d) + (e*(c + d*x)^2*(a + b*ArcTanh[c + d*x]))/(2*d)

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 321

Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 5916

Int[((a_.) + ArcTanh[(c_.)*(x_)])*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol] := Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c

$\ast p)/(d\ast(m + 1)), \text{Int}[(d\ast x)^{(m + 1)}\ast(a + b\ast \text{ArcTanh}[c\ast x])^{(p - 1)}/(1 - c^2\ast x^2), x], x] /; \text{FreeQ}\{a, b, c, d, m\}, x\} \&\& \text{IGtQ}[p, 0] \&\& (\text{EqQ}[p, 1] \parallel \text{IntegerQ}[m]) \&\& \text{NeQ}[m, -1]$

Rule 6107

$\text{Int}[(a_.) + \text{ArcTanh}[c_.] + (d_.)\ast(x_.)\ast(b_.)^{(p_.)}\ast((e_.) + (f_.)\ast(x_.)^{(m_.)}), x_Symbol] \text{:>} \text{Dist}[1/d, \text{Subst}[\text{Int}[(f\ast x)/d]^{m\ast}(a + b\ast \text{ArcTanh}[x])^p, x], x, c + d\ast x], x] /; \text{FreeQ}\{a, b, c, d, e, f, m\}, x\} \&\& \text{EqQ}[d\ast e - c\ast f, 0] \&\& \text{IGtQ}[p, 0]$

Rubi steps

$$\begin{aligned} \int (ce + dex) (a + b \tanh^{-1}(c + dx)) dx &= \frac{\text{Subst}\left(\int ex (a + b \tanh^{-1}(x)) dx, x, c + dx\right)}{d} \\ &= \frac{e \text{Subst}\left(\int x (a + b \tanh^{-1}(x)) dx, x, c + dx\right)}{d} \\ &= \frac{e(c + dx)^2 (a + b \tanh^{-1}(c + dx))}{2d} - \frac{(be) \text{Subst}\left(\int \frac{x^2}{1-x^2} dx, x, c + dx\right)}{2d} \\ &= \frac{bex}{2} + \frac{e(c + dx)^2 (a + b \tanh^{-1}(c + dx))}{2d} - \frac{(be) \text{Subst}\left(\int \frac{1}{1-x^2} dx, x, c + dx\right)}{2d} \\ &= \frac{bex}{2} - \frac{be \tanh^{-1}(c + dx)}{2d} + \frac{e(c + dx)^2 (a + b \tanh^{-1}(c + dx))}{2d} \end{aligned}$$

Mathematica [A] time = 0.05, size = 77, normalized size = 1.60

$$\frac{e(2ac^2 + 4acdx + 2ad^2x^2 + b \log(-c - dx + 1) - b \log(c + dx + 1) + 2b(c + dx)^2 \tanh^{-1}(c + dx) + 2bc + 2bdx)}{4d}$$

Antiderivative was successfully verified.

[In] Integrate[(c*e + d*e*x)*(a + b*ArcTanh[c + d*x]), x]

[Out] (e*(2*b*c + 2*a*c^2 + 2*b*d*x + 4*a*c*d*x + 2*a*d^2*x^2 + 2*b*(c + d*x)^2*ArcTanh[c + d*x] + b*Log[1 - c - d*x] - b*Log[1 + c + d*x]))/(4*d)

fricas [A] time = 0.56, size = 73, normalized size = 1.52

$$\frac{2ad^2ex^2 + 2(2ac + b)dex + (bd^2ex^2 + 2bcdex + (bc^2 - b)e) \log\left(-\frac{dx+c+1}{dx+c-1}\right)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*e*x+c*e)*(a+b*arctanh(d*x+c)),x, algorithm="fricas")

[Out] $\frac{1}{4}*(2*a*d^2*e*x^2 + 2*(2*a*c + b)*d*e*x + (b*d^2*e*x^2 + 2*b*c*d*e*x + (b*c^2 - b)*e)*\log(-(d*x + c + 1)/(d*x + c - 1)))/d$

giac [B] time = 0.39, size = 137, normalized size = 2.85

$$\frac{\left(\frac{(dx+c+1)be \log\left(-\frac{dx+c+1}{dx+c-1}\right)}{dx+c-1} + \frac{2(dx+c+1)ae}{dx+c-1} + \frac{(dx+c+1)be}{dx+c-1} - be\right)\left((c+1)d - (c-1)d\right)}{2\left(\frac{(dx+c+1)^2 d^2}{(dx+c-1)^2} - \frac{2(dx+c+1)d^2}{dx+c-1} + d^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*e*x+c*e)*(a+b*arctanh(d*x+c)),x, algorithm="giac")

[Out] $\frac{1}{2}*((d*x + c + 1)*b*e*\log(-(d*x + c + 1)/(d*x + c - 1))/(d*x + c - 1) + 2*(d*x + c + 1)*a*e/(d*x + c - 1) + (d*x + c + 1)*b*e/(d*x + c - 1) - b*e)*((c + 1)*d - (c - 1)*d)/((d*x + c + 1)^2*d^2/(d*x + c - 1)^2 - 2*(d*x + c + 1)*d^2/(d*x + c - 1) + d^2)$

maple [B] time = 0.03, size = 107, normalized size = 2.23

$$\frac{ade x^2}{2} + xace + \frac{a c^2 e}{2d} + \frac{d \operatorname{arctanh}(dx + c) x^2 be}{2} + \operatorname{arctanh}(dx + c) x bce + \frac{\operatorname{arctanh}(dx + c) b c^2 e}{2d} + \frac{bex}{2} + \frac{bec}{2d} + \frac{be \ln(a)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*e*x+c*e)*(a+b*arctanh(d*x+c)),x)

[Out] $\frac{1}{2}*a*d*e*x^2 + x*a*c*e + \frac{1}{2}*d*a*c^2*e + \frac{1}{2}*d*\operatorname{arctanh}(d*x+c)*x^2*b*e + \operatorname{arctanh}(d*x+c)*x*b*c*e + \frac{1}{2}*d*\operatorname{arctanh}(d*x+c)*b*c^2*e + \frac{1}{2}*b*e*x + \frac{1}{2}*d*b*e*c + \frac{1}{4}*d*b*e*\ln(d*x+c-1) - \frac{1}{4}*d*b*e*\ln(d*x+c+1)$

maxima [B] time = 0.31, size = 113, normalized size = 2.35

$$\frac{1}{2} adex^2 + \frac{1}{4} \left(2x^2 \operatorname{artanh}(dx + c) + d \left(\frac{2x}{d^2} - \frac{(c^2 + 2c + 1) \log(dx + c + 1)}{d^3} + \frac{(c^2 - 2c + 1) \log(dx + c - 1)}{d^3} \right) \right) bde$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*e*x+c*e)*(a+b*arctanh(d*x+c)),x, algorithm="maxima")

[Out] $\frac{1}{2}*a*d*e*x^2 + \frac{1}{4}*(2*x^2*\operatorname{arctanh}(d*x + c) + d*(2*x/d^2 - (c^2 + 2*c + 1)*\log(d*x + c + 1)/d^3 + (c^2 - 2*c + 1)*\log(d*x + c - 1)/d^3))*b*d*e + a*c*e*x + \frac{1}{2}*(2*(d*x + c)*\operatorname{arctanh}(d*x + c) + \log(-(d*x + c)^2 + 1))*b*c*e/d$

mupad [B] time = 1.84, size = 73, normalized size = 1.52

$$\frac{bex}{2} + acex - \frac{be \operatorname{atanh}(c + dx)}{2d} + \frac{adex^2}{2} + \frac{bc^2e \operatorname{atanh}(c + dx)}{2d} + bcex \operatorname{atanh}(c + dx) + \frac{bdex^2 \operatorname{atanh}(c + dx)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c*e + d*e*x)*(a + b*atanh(c + d*x)),x)`

[Out] `(b*e*x)/2 + a*c*e*x - (b*e*atanh(c + d*x))/(2*d) + (a*d*e*x^2)/2 + (b*c^2*e*atanh(c + d*x))/(2*d) + b*c*e*x*atanh(c + d*x) + (b*d*e*x^2*atanh(c + d*x))/2`

sympy [A] time = 1.66, size = 95, normalized size = 1.98

$$\begin{cases} acex + \frac{adex^2}{2} + \frac{bc^2e \operatorname{atanh}(c+dx)}{2d} + bcex \operatorname{atanh}(c + dx) + \frac{bdex^2 \operatorname{atanh}(c+dx)}{2} + \frac{bex}{2} - \frac{be \operatorname{atanh}(c+dx)}{2d} & \text{for } d \neq 0 \\ cex(a + b \operatorname{atanh}(c)) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*e*x+c*e)*(a+b*atanh(d*x+c)),x)`

[Out] `Piecewise((a*c*e*x + a*d*e*x**2/2 + b*c**2*e*atanh(c + d*x)/(2*d) + b*c*e*x*atanh(c + d*x) + b*d*e*x**2*atanh(c + d*x)/2 + b*e*x/2 - b*e*atanh(c + d*x))/(2*d), Ne(d, 0)), (c*e*x*(a + b*atanh(c)), True))`

$$3.12 \quad \int \frac{a+b \tanh^{-1}(c+dx)}{ce+dex} dx$$

Optimal. Leaf size=54

$$\frac{a \log(c+dx)}{de} - \frac{b \operatorname{Li}_2(-c-dx)}{2de} + \frac{b \operatorname{Li}_2(c+dx)}{2de}$$

[Out] a*ln(d*x+c)/d/e-1/2*b*polylog(2,-d*x-c)/d/e+1/2*b*polylog(2,d*x+c)/d/e

Rubi [A] time = 0.04, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {6107, 12, 5912}

$$-\frac{b \operatorname{PolyLog}(2, -c-dx)}{2de} + \frac{b \operatorname{PolyLog}(2, c+dx)}{2de} + \frac{a \log(c+dx)}{de}$$

Antiderivative was successfully verified.

[In] Int[(a + b*ArcTanh[c + d*x])/(c*e + d*e*x), x]

[Out] (a*Log[c + d*x])/(d*e) - (b*PolyLog[2, -c - d*x])/(2*d*e) + (b*PolyLog[2, c + d*x])/(2*d*e)

Rule 12

Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 5912

Int[((a_) + ArcTanh[(c_)*(x_)])*(b_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] + (-Simp[(b*PolyLog[2, -(c*x)])]/2, x] + Simp[(b*PolyLog[2, c*x])/2, x] /; FreeQ[{a, b, c}, x]

Rule 6107

Int[((a_) + ArcTanh[(c_) + (d_.)*(x_)])*(b_.))^p_)*((e_.) + (f_.)*(x_))^(m_.), x_Symbol] :> Dist[1/d, Subst[Int[((f*x)/d)^m*(a + b*ArcTanh[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] & IGtQ[p, 0]

Rubi steps

$$\int \frac{a + b \tanh^{-1}(c + dx)}{ce + dex} dx = \frac{\text{Subst}\left(\int \frac{a+b \tanh^{-1}(x)}{ex} dx, x, c + dx\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{a+b \tanh^{-1}(x)}{x} dx, x, c + dx\right)}{de}$$

$$= \frac{a \log(c + dx)}{de} - \frac{b \text{Li}_2(-c - dx)}{2de} + \frac{b \text{Li}_2(c + dx)}{2de}$$

Mathematica [A] time = 0.03, size = 54, normalized size = 1.00

$$\frac{a \log(c + dx)}{de} - \frac{b \text{Li}_2(-c - dx)}{2de} + \frac{b \text{Li}_2(c + dx)}{2de}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*ArcTanh[c + d*x])/(c*e + d*e*x), x]

[Out] (a*Log[c + d*x])/(d*e) - (b*PolyLog[2, -c - d*x])/(2*d*e) + (b*PolyLog[2, c + d*x])/(2*d*e)

fricas [F] time = 0.52, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b \text{artanh}(dx + c) + a}{dex + ce}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arctanh(d*x+c))/(d*e*x+c*e), x, algorithm="fricas")

[Out] integral((b*arctanh(d*x + c) + a)/(d*e*x + c*e), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b \text{artanh}(dx + c) + a}{dex + ce} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arctanh(d*x+c))/(d*e*x+c*e), x, algorithm="giac")

[Out] integrate((b*arctanh(d*x + c) + a)/(d*e*x + c*e), x)

maple [A] time = 0.05, size = 89, normalized size = 1.65

$$\frac{a \ln(dx + c)}{de} + \frac{b \ln(dx + c) \text{arctanh}(dx + c)}{de} - \frac{b \text{dilog}(dx + c)}{2de} - \frac{b \text{dilog}(dx + c + 1)}{2de} - \frac{b \ln(dx + c) \ln(dx + c + 1)}{2de}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b*arctanh(d*x+c))/(d*e*x+c*e),x)`

[Out] $1/d*a/e*\ln(d*x+c)+1/d*b/e*\ln(d*x+c)*\operatorname{arctanh}(d*x+c)-1/2/d*b/e*\operatorname{dilog}(d*x+c)-1/2/d*b/e*\operatorname{dilog}(d*x+c+1)-1/2/d*b/e*\ln(d*x+c)*\ln(d*x+c+1)$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2} b \int \frac{\log(dx + c + 1) - \log(-dx - c + 1)}{dex + ce} dx + \frac{a \log(dex + ce)}{de}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arctanh(d*x+c))/(d*e*x+c*e),x, algorithm="maxima")`

[Out] $1/2*b*\operatorname{integrate}((\log(d*x + c + 1) - \log(-d*x - c + 1))/(d*e*x + c*e), x) + a*\log(d*e*x + c*e)/(d*e)$

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{a + b \operatorname{atanh}(c + dx)}{ce + dex} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b*atanh(c + d*x))/(c*e + d*e*x),x)`

[Out] `int((a + b*atanh(c + d*x))/(c*e + d*e*x), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a}{c+dx} dx + \int \frac{b \operatorname{atanh}(c+dx)}{c+dx} dx}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*atanh(d*x+c))/(d*e*x+c*e),x)`

[Out] $(\operatorname{Integral}(a/(c + d*x), x) + \operatorname{Integral}(b*\operatorname{atanh}(c + d*x)/(c + d*x), x))/e$

$$3.13 \quad \int \frac{a+b \tanh^{-1}(c+dx)}{(ce+dex)^2} dx$$

Optimal. Leaf size=63

$$-\frac{a+b \tanh^{-1}(c+dx)}{de^2(c+dx)} + \frac{b \log(c+dx)}{de^2} - \frac{b \log(1-(c+dx)^2)}{2de^2}$$

[Out] $(-a-b*\operatorname{arctanh}(d*x+c))/d/e^2/(d*x+c)+b*\ln(d*x+c)/d/e^2-1/2*b*\ln(1-(d*x+c)^2)/d/e^2$

Rubi [A] time = 0.06, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6107, 12, 5916, 266, 36, 31, 29}

$$-\frac{a+b \tanh^{-1}(c+dx)}{de^2(c+dx)} + \frac{b \log(c+dx)}{de^2} - \frac{b \log(1-(c+dx)^2)}{2de^2}$$

Antiderivative was successfully verified.

[In] `Int[(a + b*ArcTanh[c + d*x])/(c*e + d*e*x)^2, x]`

[Out] $-\frac{(a + b*\operatorname{ArcTanh}[c + d*x])}{(d*e^2*(c + d*x))} + \frac{(b*\operatorname{Log}[c + d*x])}{(d*e^2)} - \frac{(b*\operatorname{Log}[1 - (c + d*x)^2])}{(2*d*e^2)}$

Rule 12

`Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]`

Rule 29

`Int[(x_)^(-1), x_Symbol] := Simp[Log[x], x]`

Rule 31

`Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]`

Rule 36

`Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] := Dist[b/(b*c - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]`

Rule 266

```
Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5916

```
Int[((a_) + ArcTanh[(c_)*(x_)]*(b_))^(p_)*((d_)*(x_))^(m_), x_Symbol]
:= Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 6107

```
Int[((a_) + ArcTanh[(c_) + (d_)*(x_)]*(b_))^(p_)*((e_) + (f_)*(x_))^(
m_), x_Symbol] := Dist[1/d, Subst[Int[((f*x)/d)^m*(a + b*ArcTanh[x])^p, x]
, x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] &
& IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{a + b \tanh^{-1}(c + dx)}{(ce + dex)^2} dx &= \frac{\text{Subst}\left(\int \frac{a + b \tanh^{-1}(x)}{e^2 x^2} dx, x, c + dx\right)}{d} \\
&= \frac{\text{Subst}\left(\int \frac{a + b \tanh^{-1}(x)}{x^2} dx, x, c + dx\right)}{de^2} \\
&= -\frac{a + b \tanh^{-1}(c + dx)}{de^2(c + dx)} + \frac{b \text{Subst}\left(\int \frac{1}{x(1-x^2)} dx, x, c + dx\right)}{de^2} \\
&= -\frac{a + b \tanh^{-1}(c + dx)}{de^2(c + dx)} + \frac{b \text{Subst}\left(\int \frac{1}{(1-x)x} dx, x, (c + dx)^2\right)}{2de^2} \\
&= -\frac{a + b \tanh^{-1}(c + dx)}{de^2(c + dx)} + \frac{b \text{Subst}\left(\int \frac{1}{1-x} dx, x, (c + dx)^2\right)}{2de^2} + \frac{b \text{Subst}\left(\int \frac{1}{x} dx, x, (c + dx)^2\right)}{2de^2} \\
&= -\frac{a + b \tanh^{-1}(c + dx)}{de^2(c + dx)} + \frac{b \log(c + dx)}{de^2} - \frac{b \log(1 - (c + dx)^2)}{2de^2}
\end{aligned}$$

Mathematica [A] time = 0.07, size = 69, normalized size = 1.10

$$\frac{\frac{2a}{c+dx} + b \log(-c^2 - 2cdx - d^2x^2 + 1) - 2b \log(c + dx) + \frac{2b \tanh^{-1}(c+dx)}{c+dx}}{2de^2}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*ArcTanh[c + d*x])/(c*e + d*e*x)^2,x]

[Out] -1/2*((2*a)/(c + d*x) + (2*b*ArcTanh[c + d*x])/(c + d*x) - 2*b*Log[c + d*x] + b*Log[1 - c^2 - 2*c*d*x - d^2*x^2])/(d*e^2)

fricas [A] time = 0.82, size = 85, normalized size = 1.35

$$\frac{(bdx + bc) \log(d^2x^2 + 2cdx + c^2 - 1) - 2(bdx + bc) \log(dx + c) + b \log\left(-\frac{dx+c+1}{dx+c-1}\right) + 2a}{2(d^2e^2x + cde^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arctanh(d*x+c))/(d*e*x+c*e)^2,x, algorithm="fricas")

[Out] -1/2*((b*d*x + b*c)*log(d^2*x^2 + 2*c*d*x + c^2 - 1) - 2*(b*d*x + b*c)*log(d*x + c) + b*log(-(d*x + c + 1)/(d*x + c - 1)) + 2*a)/(d^2*e^2*x + c*d*e^2)

giac [B] time = 0.18, size = 137, normalized size = 2.17

$$\frac{((c + 1)d - (c - 1)d) \left(\frac{(dx+c+1)b \log\left(-\frac{dx+c+1}{dx+c-1} - 1\right)}{dx+c-1} + b \log\left(-\frac{dx+c+1}{dx+c-1} - 1\right) - \frac{(dx+c+1)b \log\left(-\frac{dx+c+1}{dx+c-1}\right)}{dx+c-1} + 2a \right)}{2 \left(\frac{(dx+c+1)d^2e^2}{dx+c-1} + d^2e^2 \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arctanh(d*x+c))/(d*e*x+c*e)^2,x, algorithm="giac")

[Out] 1/2*((c + 1)*d - (c - 1)*d)*((d*x + c + 1)*b*log(-(d*x + c + 1)/(d*x + c - 1) - 1)/(d*x + c - 1) + b*log(-(d*x + c + 1)/(d*x + c - 1) - 1) - (d*x + c + 1)*b*log(-(d*x + c + 1)/(d*x + c - 1))/(d*x + c - 1) + 2*a)/((d*x + c + 1)*d^2*e^2/(d*x + c - 1) + d^2*e^2)

maple [A] time = 0.04, size = 86, normalized size = 1.37

$$-\frac{a}{d^2(dx+c)} - \frac{b \operatorname{arctanh}(dx+c)}{d^2(dx+c)} + \frac{b \ln(dx+c)}{d^2} - \frac{b \ln(dx+c-1)}{2d^2} - \frac{b \ln(dx+c+1)}{2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b*arctanh(d*x+c))/(d*e*x+c*e)^2,x)`

[Out] $-1/d*a/e^2/(d*x+c)-1/d*b/e^2/(d*x+c)*\operatorname{arctanh}(d*x+c)+b*\ln(d*x+c)/d/e^2-1/2/d*b/e^2*\ln(d*x+c-1)-1/2/d*b/e^2*\ln(d*x+c+1)$

maxima [A] time = 0.32, size = 95, normalized size = 1.51

$$-\frac{1}{2} \left(d \left(\frac{\log(dx+c+1)}{d^2 e^2} - \frac{2 \log(dx+c)}{d^2 e^2} + \frac{\log(dx+c-1)}{d^2 e^2} \right) + \frac{2 \operatorname{artanh}(dx+c)}{d^2 e^2 x + c d e^2} \right) b - \frac{a}{d^2 e^2 x + c d e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arctanh(d*x+c))/(d*e*x+c*e)^2,x, algorithm="maxima")`

[Out] $-1/2*(d*(\log(d*x+c+1)/(d^2*e^2) - 2*\log(d*x+c)/(d^2*e^2) + \log(d*x+c-1)/(d^2*e^2)) + 2*\operatorname{arctanh}(d*x+c)/(d^2*e^2*x+c*d*e^2))*b - a/(d^2*e^2*x+c*d*e^2)$

mupad [B] time = 1.37, size = 122, normalized size = 1.94

$$\frac{b \ln(1-dx-c)}{2x d^2 e^2 + 2c d e^2} - \frac{b \ln(c+dx+1)}{2(x d^2 e^2 + c d e^2)} - \frac{a}{x d^2 e^2 + c d e^2} - \frac{b \ln(c^2 + 2c dx + d^2 x^2 - 1)}{2 d e^2} + \frac{b \ln(c+dx)}{d e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b*atanh(c+d*x))/(c*e+d*e*x)^2,x)`

[Out] $(b*\log(1-d*x-c))/(2*d^2*e^2*x+2*c*d*e^2) - (b*\log(c+d*x+1))/(2*(d^2*e^2*x+c*d*e^2)) - a/(d^2*e^2*x+c*d*e^2) - (b*\log(c^2+d^2*x^2+2*c*d*x-1))/(2*d*e^2) + (b*\log(c+d*x))/(d*e^2)$

sympy [A] time = 2.83, size = 270, normalized size = 4.29

$$\left\{ \begin{array}{l} \frac{\infty a}{e^2 x} \\ \frac{-\frac{a}{x} + b d \log(x) - b d \log\left(x - \frac{1}{d}\right) - b d \operatorname{atanh}(dx) - \frac{b \operatorname{atanh}(dx)}{x}}{d^2 e^2} \\ \frac{x(a+b \operatorname{atanh}(c))}{c^2 e^2} \\ -\frac{a}{c d e^2 + d^2 e^2 x} + \frac{b c \log\left(\frac{c}{d} + x\right)}{c d e^2 + d^2 e^2 x} - \frac{b c \log\left(\frac{c}{d} + x + \frac{1}{d}\right)}{c d e^2 + d^2 e^2 x} + \frac{b c \operatorname{atanh}(c+dx)}{c d e^2 + d^2 e^2 x} + \frac{b d x \log\left(\frac{c}{d} + x\right)}{c d e^2 + d^2 e^2 x} - \frac{b d x \log\left(\frac{c}{d} + x + \frac{1}{d}\right)}{c d e^2 + d^2 e^2 x} + \frac{b d x \operatorname{atanh}(c+dx)}{c d e^2 + d^2 e^2 x} - \frac{b \operatorname{atanh}(c)}{c d e^2 + d^2 e^2 x} \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*atanh(d*x+c))/(d*e*x+c*e)**2,x)`

```
[Out] Piecewise((zoo*a/(e**2*x), Eq(c, 0) & Eq(d, 0)), ((-a/x + b*d*log(x) - b*d*
log(x - 1/d) - b*d*atanh(d*x) - b*atanh(d*x)/x)/(d**2*e**2), Eq(c, 0)), (x*
(a + b*atanh(c))/(c**2*e**2), Eq(d, 0)), (-a/(c*d*e**2 + d**2*e**2*x) + b*c
*log(c/d + x)/(c*d*e**2 + d**2*e**2*x) - b*c*log(c/d + x + 1/d)/(c*d*e**2 +
d**2*e**2*x) + b*c*atanh(c + d*x)/(c*d*e**2 + d**2*e**2*x) + b*d*x*log(c/d
+ x)/(c*d*e**2 + d**2*e**2*x) - b*d*x*log(c/d + x + 1/d)/(c*d*e**2 + d**2*
e**2*x) + b*d*x*atanh(c + d*x)/(c*d*e**2 + d**2*e**2*x) - b*atanh(c + d*x)/
(c*d*e**2 + d**2*e**2*x), True))
```

$$3.14 \quad \int \frac{a+b \tanh^{-1}(c+dx)}{(ce+dex)^3} dx$$

Optimal. Leaf size=63

$$-\frac{a+b \tanh^{-1}(c+dx)}{2de^3(c+dx)^2} - \frac{b}{2de^3(c+dx)} + \frac{b \tanh^{-1}(c+dx)}{2de^3}$$

[Out] $-1/2*b/d/e^3/(d*x+c)+1/2*b*\operatorname{arctanh}(d*x+c)/d/e^3+1/2*(-a-b*\operatorname{arctanh}(d*x+c))/d/e^3/(d*x+c)^2$

Rubi [A] time = 0.05, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {6107, 12, 5916, 325, 206}

$$-\frac{a+b \tanh^{-1}(c+dx)}{2de^3(c+dx)^2} - \frac{b}{2de^3(c+dx)} + \frac{b \tanh^{-1}(c+dx)}{2de^3}$$

Antiderivative was successfully verified.

[In] Int[(a + b*ArcTanh[c + d*x])/(c*e + d*e*x)^3,x]

[Out] $-b/(2*d*e^3*(c+d*x)) + (b*\operatorname{ArcTanh}[c+d*x])/(2*d*e^3) - (a+b*\operatorname{ArcTanh}[c+d*x])/(2*d*e^3*(c+d*x)^2)$

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 325

Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[((c*x)^(m+1)*(a+b*x^n)^(p+1))/(a*c*(m+1)), x] - Dist[(b*(m+n*(p+1)+1))/(a*c^n*(m+1)), Int[(c*x)^(m+n)*(a+b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_.)]*(b_.))^(p_.)*((d_.)*(x_.))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 6107

```
Int[((a_.) + ArcTanh[(c_.) + (d_.)*(x_.)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_.))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((f*x)/d)^m*(a + b*ArcTanh[x])^p, x]
, x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] &
& IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{a + b \tanh^{-1}(c + dx)}{(ce + dex)^3} dx &= \frac{\text{Subst}\left(\int \frac{a + b \tanh^{-1}(x)}{e^3 x^3} dx, x, c + dx\right)}{d} \\
&= \frac{\text{Subst}\left(\int \frac{a + b \tanh^{-1}(x)}{x^3} dx, x, c + dx\right)}{de^3} \\
&= -\frac{a + b \tanh^{-1}(c + dx)}{2de^3(c + dx)^2} + \frac{b \text{Subst}\left(\int \frac{1}{x^2(1-x^2)} dx, x, c + dx\right)}{2de^3} \\
&= -\frac{b}{2de^3(c + dx)} - \frac{a + b \tanh^{-1}(c + dx)}{2de^3(c + dx)^2} + \frac{b \text{Subst}\left(\int \frac{1}{1-x^2} dx, x, c + dx\right)}{2de^3} \\
&= -\frac{b}{2de^3(c + dx)} + \frac{b \tanh^{-1}(c + dx)}{2de^3} - \frac{a + b \tanh^{-1}(c + dx)}{2de^3(c + dx)^2}
\end{aligned}$$

Mathematica [A] time = 0.06, size = 100, normalized size = 1.59

$$-\frac{a}{2de^3(c + dx)^2} - \frac{b}{2de^3(c + dx)} - \frac{b \log(-c - dx + 1)}{4de^3} + \frac{b \log(c + dx + 1)}{4de^3} - \frac{b \tanh^{-1}(c + dx)}{2de^3(c + dx)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c + d*x])/(c*e + d*e*x)^3, x]
```

```
[Out] -1/2*a/(d*e^3*(c + d*x)^2) - b/(2*d*e^3*(c + d*x)) - (b*ArcTanh[c + d*x])/(
2*d*e^3*(c + d*x)^2) - (b*Log[1 - c - d*x])/(4*d*e^3) + (b*Log[1 + c + d*x]
)/(4*d*e^3)
```


fricas [A] time = 0.57, size = 88, normalized size = 1.40

$$\frac{2bdx + 2bc - (bd^2x^2 + 2bcdx + bc^2 - b) \log\left(-\frac{dx+c+1}{dx+c-1}\right) + 2a}{4(d^3e^3x^2 + 2cd^2e^3x + c^2de^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arctanh(d*x+c))/(d*e*x+c*e)^3,x, algorithm="fricas")

[Out] -1/4*(2*b*d*x + 2*b*c - (b*d^2*x^2 + 2*b*c*d*x + b*c^2 - b)*log(-(d*x + c + 1)/(d*x + c - 1)) + 2*a)/(d^3*e^3*x^2 + 2*c*d^2*e^3*x + c^2*d*e^3)

giac [B] time = 0.33, size = 134, normalized size = 2.13

$$\frac{((c+1)d - (c-1)d) \left(\frac{(dx+c+1)b \log\left(-\frac{dx+c+1}{dx+c-1}\right)}{dx+c-1} + \frac{2(dx+c+1)a}{dx+c-1} + \frac{(dx+c+1)b}{dx+c-1} + b \right)}{2 \left(\frac{(dx+c+1)^2 d^2 e^3}{(dx+c-1)^2} + \frac{2(dx+c+1)d^2 e^3}{dx+c-1} + d^2 e^3 \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arctanh(d*x+c))/(d*e*x+c*e)^3,x, algorithm="giac")

[Out] 1/2*((c+1)*d - (c-1)*d)*((d*x + c + 1)*b*log(-(d*x + c + 1)/(d*x + c - 1))/(d*x + c - 1) + 2*(d*x + c + 1)*a/(d*x + c - 1) + (d*x + c + 1)*b/(d*x + c - 1) + b)/((d*x + c + 1)^2*d^2*e^3/(d*x + c - 1)^2 + 2*(d*x + c + 1)*d^2*e^3/(d*x + c - 1) + d^2*e^3)

maple [A] time = 0.04, size = 88, normalized size = 1.40

$$-\frac{a}{2de^3(dx+c)^2} - \frac{b \operatorname{arctanh}(dx+c)}{2de^3(dx+c)^2} - \frac{b}{2de^3(dx+c)} - \frac{b \ln(dx+c-1)}{4de^3} + \frac{b \ln(dx+c+1)}{4de^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*arctanh(d*x+c))/(d*e*x+c*e)^3,x)

[Out] -1/2/d*a/e^3/(d*x+c)^2-1/2/d*b/e^3/(d*x+c)^2*arctanh(d*x+c)-1/2*b/d/e^3/(d*x+c)-1/4/d*b/e^3*ln(d*x+c-1)+1/4/d*b/e^3*ln(d*x+c+1)

maxima [B] time = 0.32, size = 131, normalized size = 2.08

$$-\frac{1}{4} \left(d \left(\frac{2}{d^3e^3x + cd^2e^3} - \frac{\log(dx+c+1)}{d^2e^3} + \frac{\log(dx+c-1)}{d^2e^3} \right) + \frac{2 \operatorname{artanh}(dx+c)}{d^3e^3x^2 + 2cd^2e^3x + c^2de^3} \right) b - \frac{a}{2(d^3e^3x^2 + 2cd^2e^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arctanh(d*x+c))/(d*e*x+c*e)^3,x, algorithm="maxima")

[Out] $-1/4*(d*(2/(d^3*e^3*x + c*d^2*e^3) - \log(d*x + c + 1)/(d^2*e^3) + \log(d*x + c - 1)/(d^2*e^3)) + 2*\arctanh(d*x + c)/(d^3*e^3*x^2 + 2*c*d^2*e^3*x + c^2*d*e^3))*b - 1/2*a/(d^3*e^3*x^2 + 2*c*d^2*e^3*x + c^2*d*e^3)$

mupad [B] time = 1.73, size = 67, normalized size = 1.06

$$\frac{b \operatorname{atanh}(c + dx)}{2 d e^3} - \frac{\frac{a}{2} + \frac{bc}{2} + \frac{b \ln(c+dx+1)}{4} - \frac{b \ln(1-dx-c)}{4} + \frac{bdx}{2}}{d e^3 (c + dx)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*atanh(c + d*x))/(c*e + d*e*x)^3,x)

[Out] $(b*\operatorname{atanh}(c + d*x))/(2*d*e^3) - (a/2 + (b*c)/2 + (b*\log(c + d*x + 1))/4 - (b*\log(1 - d*x - c))/4 + (b*d*x)/2)/(d*e^3*(c + d*x)^2)$

sympy [A] time = 3.55, size = 313, normalized size = 4.97

$$\left\{ \begin{array}{l} -\frac{a}{2c^2de^3+4cd^2e^3x+2d^3e^3x^2} + \frac{bc^2 \operatorname{atanh}(c+dx)}{2c^2de^3+4cd^2e^3x+2d^3e^3x^2} + \frac{2bcdx \operatorname{atanh}(c+dx)}{2c^2de^3+4cd^2e^3x+2d^3e^3x^2} - \frac{bc}{2c^2de^3+4cd^2e^3x+2d^3e^3x^2} + \frac{bd^2x^2 \operatorname{atanh}(c+dx)}{2c^2de^3+4cd^2e^3x+2d^3e^3x^2} \\ \frac{x(a+b \operatorname{atanh}(c))}{c^3e^3} \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*atanh(d*x+c))/(d*e*x+c*e)**3,x)

[Out] Piecewise((-a/(2*c**2*d*e**3 + 4*c*d**2*e**3*x + 2*d**3*e**3*x**2) + b*c**2*atanh(c + d*x)/(2*c**2*d*e**3 + 4*c*d**2*e**3*x + 2*d**3*e**3*x**2) + 2*b*c*d*x*atanh(c + d*x)/(2*c**2*d*e**3 + 4*c*d**2*e**3*x + 2*d**3*e**3*x**2) - b*c/(2*c**2*d*e**3 + 4*c*d**2*e**3*x + 2*d**3*e**3*x**2) + b*d**2*x**2*atanh(c + d*x)/(2*c**2*d*e**3 + 4*c*d**2*e**3*x + 2*d**3*e**3*x**2) - b*d*x/(2*c**2*d*e**3 + 4*c*d**2*e**3*x + 2*d**3*e**3*x**2) - b*atanh(c + d*x)/(2*c**2*d*e**3 + 4*c*d**2*e**3*x + 2*d**3*e**3*x**2), Ne(d, 0)), (x*(a + b*atanh(c))/(c**3*e**3), True))

3.15 $\int (ce + dex)^3 \left(a + b \tanh^{-1}(c + dx)\right)^2 dx$

Optimal. Leaf size=159

$$\frac{e^3(c+dx)^4 \left(a + b \tanh^{-1}(c+dx)\right)^2}{4d} + \frac{be^3(c+dx)^3 \left(a + b \tanh^{-1}(c+dx)\right)}{6d} - \frac{e^3 \left(a + b \tanh^{-1}(c+dx)\right)^2}{4d} + \frac{1}{2}abe^3x +$$

[Out] $\frac{1}{2}ab^2e^3x + \frac{1}{12}b^2e^3(d^2x+c)^2/d + \frac{1}{2}b^2e^3(d^2x+c)\operatorname{arctanh}(d^2x+c)/d + \frac{1}{6}b^2e^3(d^2x+c)^3(a+b\operatorname{arctanh}(d^2x+c))/d - \frac{1}{4}e^3(a+b\operatorname{arctanh}(d^2x+c))^2/d + \frac{1}{4}e^3(d^2x+c)^4(a+b\operatorname{arctanh}(d^2x+c))^2/d + \frac{1}{3}b^2e^3\ln(1-(d^2x+c)^2)/d$

Rubi [A] time = 0.25, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {6107, 12, 5916, 5980, 266, 43, 5910, 260, 5948}

$$\frac{e^3(c+dx)^4 \left(a + b \tanh^{-1}(c+dx)\right)^2}{4d} + \frac{be^3(c+dx)^3 \left(a + b \tanh^{-1}(c+dx)\right)}{6d} - \frac{e^3 \left(a + b \tanh^{-1}(c+dx)\right)^2}{4d} + \frac{1}{2}abe^3x +$$

Antiderivative was successfully verified.

[In] $\text{Int}[(c*e + d*e*x)^3*(a + b*\text{ArcTanh}[c + d*x])^2, x]$

[Out] $(a*b*e^3*x)/2 + (b^2*e^3*(c + d*x)^2)/(12*d) + (b^2*e^3*(c + d*x)*\text{ArcTanh}[c + d*x])/(2*d) + (b*e^3*(c + d*x)^3*(a + b*\text{ArcTanh}[c + d*x]))/(6*d) - (e^3*(a + b*\text{ArcTanh}[c + d*x])^2)/(4*d) + (e^3*(c + d*x)^4*(a + b*\text{ArcTanh}[c + d*x])^2)/(4*d) + (b^2*e^3*\text{Log}[1 - (c + d*x)^2])/(3*d)$

Rule 12

$\text{Int}[(a_*)(u_), x_Symbol] \rightarrow \text{Dist}[a, \text{Int}[u, x], x] /; \text{FreeQ}[a, x] \ \&\& \ !\text{MatchQ}[u, (b_*)(v_)] /; \text{FreeQ}[b, x]$

Rule 43

$\text{Int}[(a_*) + (b_*)(x_)]^{(m_*)} * ((c_*) + (d_*)(x_))^{(n_*)}, x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*x)^m*(c + d*x)^n, x], x] /; \text{FreeQ}[\{a, b, c, d, n\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{IGtQ}[m, 0] \ \&\& \ (\ !\text{IntegerQ}[n] \ || \ (\text{EqQ}[c, 0] \ \&\& \ \text{LeQ}[7*m + 4*n + 4, 0]) \ || \ \text{LtQ}[9*m + 5*(n + 1), 0] \ || \ \text{GtQ}[m + n + 2, 0])$

Rule 260

$\text{Int}[(x_)]^{(m_*)} / ((a_*) + (b_*)(x_)]^{(n_*)}, x_Symbol] \rightarrow \text{Simp}[\text{Log}[\text{RemoveContent}[a + b*x^n, x]]/(b*n), x] /; \text{FreeQ}[\{a, b, m, n\}, x] \ \&\& \ \text{EqQ}[m, n - 1]$

Rule 266

```
Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5910

```
Int[((a_) + ArcTanh[(c_)*(x_)]*(b_))^(p_), x_Symbol] := Simp[x*(a + b*ArcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 5916

```
Int[((a_) + ArcTanh[(c_)*(x_)]*(b_))^(p_)*((d_)*(x_))^(m_), x_Symbol]
:= Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || IntegerQ[m]) && NeQ[m, -1]
```

Rule 5948

```
Int[((a_) + ArcTanh[(c_)*(x_)]*(b_))^(p_)/((d_) + (e_)*(x_)^2), x_Symbol] := Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 5980

```
Int((((a_) + ArcTanh[(c_)*(x_)]*(b_))^(p_)*((f_)*(x_))^(m_))/((d_) + (e_)*(x_)^2), x_Symbol] := Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x])^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTanh[c*x])^p)/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 6107

```
Int[((a_) + ArcTanh[(c_) + (d_)*(x_)]*(b_))^(p_)*((e_) + (f_)*(x_))^(m_), x_Symbol] := Dist[1/d, Subst[Int[((f*x)/d)^m*(a + b*ArcTanh[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int (ce + dex)^3 (a + b \tanh^{-1}(c + dx))^2 dx &= \frac{\text{Subst}\left(\int e^3 x^3 (a + b \tanh^{-1}(x))^2 dx, x, c + dx\right)}{d} \\
&= \frac{e^3 \text{Subst}\left(\int x^3 (a + b \tanh^{-1}(x))^2 dx, x, c + dx\right)}{d} \\
&= \frac{e^3 (c + dx)^4 (a + b \tanh^{-1}(c + dx))^2}{4d} - \frac{(be^3) \text{Subst}\left(\int \frac{x^4 (a + b \tanh^{-1}(x))}{1-x^2}\right)}{2d} \\
&= \frac{e^3 (c + dx)^4 (a + b \tanh^{-1}(c + dx))^2}{4d} + \frac{(be^3) \text{Subst}\left(\int x^2 (a + b \tanh^{-1}(x))\right)}{2d} \\
&= \frac{be^3 (c + dx)^3 (a + b \tanh^{-1}(c + dx))}{6d} + \frac{e^3 (c + dx)^4 (a + b \tanh^{-1}(c + dx))}{4d} \\
&= \frac{1}{2}abe^3x + \frac{be^3 (c + dx)^3 (a + b \tanh^{-1}(c + dx))}{6d} - \frac{e^3 (a + b \tanh^{-1}(c + dx))}{4d} \\
&= \frac{1}{2}abe^3x + \frac{b^2e^3 (c + dx) \tanh^{-1}(c + dx)}{2d} + \frac{be^3 (c + dx)^3 (a + b \tanh^{-1}(c + dx))}{6d} \\
&= \frac{1}{2}abe^3x + \frac{b^2e^3 (c + dx)^2}{12d} + \frac{b^2e^3 (c + dx) \tanh^{-1}(c + dx)}{2d} + \frac{be^3 (c + dx)^3 (a + b \tanh^{-1}(c + dx))}{6d}
\end{aligned}$$

Mathematica [A] time = 0.19, size = 148, normalized size = 0.93

$$\frac{e^3 (3a^2(c + dx)^4 + 2ab(c + dx)^3 + 6ab(c + dx) + b(3a + 4b) \log(-c - dx + 1) + b(4b - 3a) \log(c + dx + 1) + 2b(c + dx))}{12d}$$

Antiderivative was successfully verified.

[In] Integrate[(c*e + d*e*x)^3*(a + b*ArcTanh[c + d*x])^2,x]

[Out] (e^3*(6*a*b*(c + d*x) + b^2*(c + d*x)^2 + 2*a*b*(c + d*x)^3 + 3*a^2*(c + d*x)^4 + 2*b*(c + d*x)*(3*b + b*(c + d*x)^2 + 3*a*(c + d*x)^3)*ArcTanh[c + d*x] + 3*b^2*(-1 + (c + d*x)^4)*ArcTanh[c + d*x]^2 + b*(3*a + 4*b)*Log[1 - c - d*x] + b*(-3*a + 4*b)*Log[1 + c + d*x]))/(12*d)

fricas [B] time = 0.74, size = 383, normalized size = 2.41

$$12 a^2 d^4 e^3 x^4 + 8 (6 a^2 c + ab) d^3 e^3 x^3 + 4 (18 a^2 c^2 + 6 abc + b^2) d^2 e^3 x^2 + 8 (6 a^2 c^3 + 3 abc^2 + b^2 c + 3 ab) d e^3 x + 4 ($$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*e*x+c*e)^3*(a+b*arctanh(d*x+c))^2,x, algorithm="fricas")

[Out] $\frac{1}{48}*(12*a^2*d^4*e^3*x^4 + 8*(6*a^2*c + a*b)*d^3*e^3*x^3 + 4*(18*a^2*c^2 + 6*a*b*c + b^2)*d^2*e^3*x^2 + 8*(6*a^2*c^3 + 3*a*b*c^2 + b^2*c + 3*a*b)*d*e^3*x + 4*(3*a*b*c^4 + b^2*c^3 + 3*b^2*c - 3*a*b + 4*b^2)*e^3*\log(d*x + c + 1) - 4*(3*a*b*c^4 + b^2*c^3 + 3*b^2*c - 3*a*b - 4*b^2)*e^3*\log(d*x + c - 1) + 3*(b^2*d^4*e^3*x^4 + 4*b^2*c*d^3*e^3*x^3 + 6*b^2*c^2*d^2*e^3*x^2 + 4*b^2*c^3*d*e^3*x + (b^2*c^4 - b^2)*e^3)*\log(-(d*x + c + 1)/(d*x + c - 1))^2 + 4*(3*a*b*d^4*e^3*x^4 + (12*a*b*c + b^2)*d^3*e^3*x^3 + 3*(6*a*b*c^2 + b^2*c)*d^2*e^3*x^2 + 3*(4*a*b*c^3 + b^2*c^2 + b^2)*d*e^3*x)*\log(-(d*x + c + 1)/(d*x + c - 1)))/d$

giac [B] time = 0.25, size = 791, normalized size = 4.97

$$\frac{3(dx+c+1)^3 b^2 e^3 \log\left(-\frac{dx+c+1}{dx+c-1}\right)^2}{(dx+c-1)^3} + \frac{3(dx+c+1) b^2 e^3 \log\left(-\frac{dx+c+1}{dx+c-1}\right)^2}{dx+c-1} - \frac{4(dx+c+1)^4 b^2 e^3 \log\left(-\frac{dx+c+1}{dx+c-1}+1\right)}{(dx+c-1)^4} + \frac{16(dx+c+1)^3 b^2 e^3 \log\left(-\frac{dx+c+1}{dx+c-1}+1\right)}{(dx+c-1)^3} -$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*e*x+c*e)^3*(a+b*arctanh(d*x+c))^2,x, algorithm="giac")

[Out] $\frac{1}{12}*(3*(d*x + c + 1)^3*b^2*e^3*\log(-(d*x + c + 1)/(d*x + c - 1))^2/(d*x + c - 1)^3 + 3*(d*x + c + 1)*b^2*e^3*\log(-(d*x + c + 1)/(d*x + c - 1))^2/(d*x + c - 1) - 4*(d*x + c + 1)^4*b^2*e^3*\log(-(d*x + c + 1)/(d*x + c - 1) + 1)/(d*x + c - 1)^4 + 16*(d*x + c + 1)^3*b^2*e^3*\log(-(d*x + c + 1)/(d*x + c - 1) + 1)/(d*x + c - 1)^3 - 24*(d*x + c + 1)^2*b^2*e^3*\log(-(d*x + c + 1)/(d*x + c - 1) + 1)/(d*x + c - 1)^2 + 16*(d*x + c + 1)*b^2*e^3*\log(-(d*x + c + 1)/(d*x + c - 1) + 1)/(d*x + c - 1) - 4*b^2*e^3*\log(-(d*x + c + 1)/(d*x + c - 1) + 1)/(d*x + c - 1) + 12*(d*x + c + 1)^3*a*b*e^3*\log(-(d*x + c + 1)/(d*x + c - 1))/(d*x + c - 1)^3 + 12*(d*x + c + 1)*a*b*e^3*\log(-(d*x + c + 1)/(d*x + c - 1))/(d*x + c - 1) + 4*(d*x + c + 1)^4*b^2*e^3*\log(-(d*x + c + 1)/(d*x + c - 1))/(d*x + c - 1)^4 - 10*(d*x + c + 1)^3*b^2*e^3*\log(-(d*x + c + 1)/(d*x + c - 1))/(d*x + c - 1)^3 + 12*(d*x + c + 1)^2*b^2*e^3*\log(-(d*x + c + 1)/(d*x + c - 1))/(d*x + c - 1)^2 - 6*(d*x + c + 1)*b^2*e^3*\log(-(d*x + c + 1)/(d*x + c - 1))/(d*x + c - 1) + 12*(d*x + c + 1)^3*a^2*e^3/(d*x + c - 1)^3 + 12*(d*x + c + 1)*a^2*e^3/(d*x + c - 1) + 12*(d*x + c + 1)^3*a*b*e^3/(d*x + c - 1)^3 - 24*(d*x + c + 1)^2*a*b*e^3/(d*x + c - 1)^2 + 20*(d*x + c + 1)*a*b*e^3/(d*x + c - 1) - 8*a*b*e^3 + 2*(d*x + c + 1)^3*b^2*e^3/(d*x + c - 1)^3 - 4*(d*x + c + 1)^2*b^2*e^3/(d*x + c - 1)^2 + 2*(d*x + c + 1)*b^2*e^3/(d*x + c - 1))*((c + 1)*d - (c - 1)*d)/((d*x + c + 1)^4*d^2/(d*x + c - 1)^4 - 4*($

$$d^3x + c + 1)^3 d^2 / (d^3x + c - 1)^3 + 6(d^3x + c + 1)^2 d^2 / (d^3x + c - 1)^2 - 4(d^3x + c + 1) d^2 / (d^3x + c - 1) + d^2$$

maple [B] time = 0.07, size = 732, normalized size = 4.60

$$\frac{ab e^3 x}{2} + \frac{d^3 x^4 a^2 e^3}{4} + \frac{\operatorname{arctanh}(dx + c) x b^2 e^3}{2} + 2d^2 \operatorname{arctanh}(dx + c) x^3 abc e^3 + 3d \operatorname{arctanh}(dx + c) x^2 ab c^2 e^3 + \frac{e^3 b^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d*e*x+c*e)^3*(a+b*arctanh(d*x+c))^2,x)`

[Out] $\frac{1}{2} a b e^3 x + \frac{1}{4} d^3 x^4 a^2 e^3 + 3 d \operatorname{arctanh}(d x + c) x^2 a b c^2 e^3 + 2 d^2 \operatorname{arctanh}(d x + c) x^3 a b c e^3 + \frac{1}{16} d e^3 b^2 \ln(d x + c - 1)^2 + \frac{1}{3} d e^3 b^2 \ln(d x + c + 1) + \frac{1}{3} d e^3 b^2 \ln(d x + c - 1) + \frac{1}{16} d e^3 b^2 \ln(d x + c + 1)^2 + \frac{1}{2} \operatorname{arctanh}(d x + c) x b^2 e^3 + \frac{1}{12} d e^3 b^2 c^2 + \frac{1}{4} d a^2 c^4 e^3 + 2 \operatorname{arctanh}(d x + c) x a b c^3 e^3 + \frac{1}{2} d \operatorname{arctanh}(d x + c) x^2 b^2 c e^3 + \frac{1}{2} d^3 \operatorname{arctanh}(d x + c) x^4 a b e^3 + \frac{3}{2} d \operatorname{arctanh}(d x + c)^2 x^2 b^2 c^2 e^3 + \frac{1}{2} d \operatorname{arctanh}(d x + c) a b c^4 e^3 + d^2 \operatorname{arctanh}(d x + c)^2 x^3 b^2 c e^3 + \frac{1}{2} d x^2 a b c e^3 + \frac{1}{6} e^3 b^2 x c + \frac{1}{2} \operatorname{arctanh}(d x + c) x b^2 c^2 e^3 + \operatorname{arctanh}(d x + c)^2 x b^2 c^3 e^3 + \frac{1}{4} d e^3 b^2 \operatorname{arctanh}(d x + c) \ln(d x + c - 1) + x a^2 c^3 e^3 + \frac{1}{12} d e^3 b^2 x^2 + \frac{1}{2} x a b c^2 e^3 + \frac{3}{2} d x^2 a^2 c^2 e^3 + d^2 x^3 a^2 c e^3 + \frac{1}{4} d e^3 a b \ln(d x + c - 1) - \frac{1}{4} d e^3 a b \ln(d x + c + 1) + \frac{1}{6} d \operatorname{arctanh}(d x + c) b^2 c^3 e^3 + \frac{1}{2} d \operatorname{arctanh}(d x + c) b^2 c e^3 + \frac{1}{6} d^2 \operatorname{arctanh}(d x + c) x^3 b^2 e^3 + \frac{1}{4} d^3 \operatorname{arctanh}(d x + c)^2 x^4 b^2 e^3 - \frac{1}{4} d e^3 b^2 \operatorname{arctanh}(d x + c) \ln(d x + c + 1) + \frac{1}{4} d \operatorname{arctanh}(d x + c)^2 b^2 c^4 e^3 - \frac{1}{8} d e^3 b^2 \ln(d x + c - 1) \ln\left(\frac{1}{2} + \frac{1}{2} d x + \frac{1}{2} c\right) + \frac{1}{8} d e^3 b^2 \ln\left(-\frac{1}{2} d x - \frac{1}{2} c + \frac{1}{2}\right) \ln\left(\frac{1}{2} + \frac{1}{2} d x + \frac{1}{2} c\right) - \frac{1}{8} d e^3 b^2 \ln\left(-\frac{1}{2} d x - \frac{1}{2} c + \frac{1}{2}\right) \ln(d x + c + 1) + \frac{1}{6} d^2 x^3 a b e^3 + \frac{1}{6} d a b c^3 e^3 + \frac{1}{2} d a b c e^3$

maxima [B] time = 0.62, size = 827, normalized size = 5.20

$$\frac{1}{4} a^2 d^3 e^3 x^4 + a^2 c d^2 e^3 x^3 + \frac{3}{2} a^2 c^2 d e^3 x^2 + \frac{3}{2} \left(2 x^2 \operatorname{arctanh}(dx + c) + d \left(\frac{2x}{d^2} - \frac{(c^2 + 2c + 1) \log(dx + c + 1)}{d^3} + \frac{(c^2 - 2c + 1) \log(dx + c - 1)}{d^3} \right) \right) a b c^2 d e^3 + (2 x^3 \operatorname{arctanh}(dx + c) + d \left(\frac{d x^2 - 4 c x}{d^3} + \frac{c^3 + 3 c^2 + 3 c + 1}{d^4} \log(dx + c + 1) - \frac{c^3 - 3 c^2 + 3 c - 1}{d^4} \log(dx + c - 1) \right)) a b c d^2 e^3 + \frac{1}{12} (6 x^4 \operatorname{arctanh}(dx + c) + d (2 (d^2 x^3 - 3 c d x^2 + 3 (3 c^2 + 1) x) / d^4 - 3 (c^4 + 4 c^3 + 6 c^2 + 4 c + 1) \log(dx + c + 1) / d^5 + 3 (c^4 - 4 c^3 + 6 c^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*e*x+c*e)^3*(a+b*arctanh(d*x+c))^2,x, algorithm="maxima")`

[Out] $\frac{1}{4} a^2 d^3 e^3 x^4 + a^2 c d^2 e^3 x^3 + \frac{3}{2} a^2 c^2 d e^3 x^2 + \frac{3}{2} (2 x^2 \operatorname{arctanh}(d x + c) + d (2 x / d^2 - (c^2 + 2 c + 1) \log(d x + c + 1) / d^3 + (c^2 - 2 c + 1) \log(d x + c - 1) / d^3)) a b c^2 d e^3 + (2 x^3 \operatorname{arctanh}(d x + c) + d ((d x^2 - 4 c x) / d^3 + (c^3 + 3 c^2 + 3 c + 1) \log(d x + c + 1) / d^4 - (c^3 - 3 c^2 + 3 c - 1) \log(d x + c - 1) / d^4)) a b c d^2 e^3 + \frac{1}{12} (6 x^4 \operatorname{arctanh}(d x + c) + d (2 (d^2 x^3 - 3 c d x^2 + 3 (3 c^2 + 1) x) / d^4 - 3 (c^4 + 4 c^3 + 6 c^2 + 4 c + 1) \log(d x + c + 1) / d^5 + 3 (c^4 - 4 c^3 + 6 c^2$

$$\begin{aligned}
& 1) + d*(c + 1)) + 8*b^2*d^4*e^3*(c - 1))/d^2 - 48*b^2*c^2*d^3*e^3 + 8*b^2 \\
& *d^3*e^3*(c - 1)*(c + 1) - 32*b^2*c*d^3*e^3*(c - 1))/d^2 + 32*b^2*c^3*d^2* \\
& e^3 - ((c - 1)*(c + 1)*(32*b^2*c*d^4*e^3 - 8*b^2*d^3*e^3*(d*(c - 1) + d*(c \\
& + 1)) + 8*b^2*d^4*e^3*(c - 1))/d^2 + 48*b^2*c^2*d^2*e^3*(c - 1))/(64*d^2) \\
& - (b^2*d^3*e^3*x^4)/32 + (b*d^3*e^3*x^4*(8*a + b))/32 + x^2*((c*(2*a^2*c* \\
& d^2*e^3 - (a*d^2*e^3*(b + 10*a*c))/2))/d + (d*e^3*(b^2 - 6*a^2 + 60*a^2*c^2 \\
& + 12*a*b*c))/12 - (a^2*d*e^3*(6*c^2 - 6))/12) - x^3*((2*a^2*c*d^2*e^3)/3 - \\
& (a*d^2*e^3*(b + 10*a*c))/6) + \log(c + d*x + 1)^2*((b^2*c^3*e^3*x)/4 - (b^2 \\
& *e^3 - b^2*c^4*e^3)/(16*d) + (b^2*d^3*e^3*x^4)/16 + (3*b^2*c^2*d*e^3*x^2)/8 \\
& + (b^2*c*d^2*e^3*x^3)/4) - (\log(c + d*x - 1)*(3*b^2*c*e^3 - 4*b^2*e^3 + b^ \\
& 2*c^3*e^3 - 3*a*b*e^3 + 3*a*b*c^4*e^3))/(12*d) + (\log(c + d*x + 1)*(4*b^2*e \\
& ^3 + 3*b^2*c*e^3 + b^2*c^3*e^3 - 3*a*b*e^3 + 3*a*b*c^4*e^3))/(12*d) + d*\log \\
& (c + d*x + 1)*(x^2*((b^2*c*e^3)/4 + (3*a*b*c^2*e^3)/2) + x^3*((b^2*d*e^3)/1 \\
& 2 + a*b*c*d*e^3) + (x*(b^2*e^3 + b^2*c^2*e^3 + 4*a*b*c^3*e^3))/(4*d) + (a*b \\
& *d^2*e^3*x^4)/4) + (a^2*d^3*e^3*x^4)/4
\end{aligned}$$

sympy [A] time = 8.07, size = 581, normalized size = 3.65

$$\left\{ \begin{array}{l} a^2c^3e^3x + \frac{3a^2c^2de^3x^2}{2} + a^2cd^2e^3x^3 + \frac{a^2d^3e^3x^4}{4} + \frac{abc^4e^3 \operatorname{atanh}(c+dx)}{2d} + 2abc^3e^3x \operatorname{atanh}(c+dx) + 3abc^2de^3x^2 \operatorname{atanh}(c \\ c^3e^3x(a+b \operatorname{atanh}(c))^2 \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*e*x+c*e)**3*(a+b*atanh(d*x+c))**2,x)

[Out] Piecewise((a**2*c**3*e**3*x + 3*a**2*c**2*d*e**3*x**2/2 + a**2*c*d**2*e**3*x**3 + a**2*d**3*e**3*x**4/4 + a*b*c**4*e**3*atanh(c + d*x)/(2*d) + 2*a*b*c**3*e**3*x*atanh(c + d*x) + 3*a*b*c**2*d*e**3*x**2*atanh(c + d*x) + a*b*c**2*e**3*x/2 + 2*a*b*c*d**2*e**3*x**3*atanh(c + d*x) + a*b*c*d*e**3*x**2/2 + a*b*d**3*e**3*x**4*atanh(c + d*x)/2 + a*b*d**2*e**3*x**3/6 + a*b*e**3*x/2 - a*b*e**3*atanh(c + d*x)/(2*d) + b**2*c**4*e**3*atanh(c + d*x)**2/(4*d) + b**2*c**3*e**3*x*atanh(c + d*x)**2 + b**2*c**3*e**3*atanh(c + d*x)/(6*d) + 3*b**2*c**2*d*e**3*x**2*atanh(c + d*x)**2/2 + b**2*c**2*e**3*x*atanh(c + d*x)/2 + b**2*c*d**2*e**3*x**3*atanh(c + d*x)**2 + b**2*c*d*e**3*x**2*atanh(c + d*x)/2 + b**2*c*e**3*x/6 + b**2*c*e**3*atanh(c + d*x)/(2*d) + b**2*d**3*e**3*x**4*atanh(c + d*x)**2/4 + b**2*d**2*e**3*x**3*atanh(c + d*x)/6 + b**2*d*e**3*x**2/12 + b**2*e**3*x*atanh(c + d*x)/2 + 2*b**2*e**3*log(c/d + x + 1/d)/(3*d) - b**2*e**3*atanh(c + d*x)**2/(4*d) - 2*b**2*e**3*atanh(c + d*x)/(3*d), Ne(d, 0)), (c**3*e**3*x*(a + b*atanh(c))**2, True))

3.16 $\int (ce + dex)^2 (a + b \tanh^{-1}(c + dx))^2 dx$

Optimal. Leaf size=179

$$\frac{e^2(c + dx)^3 (a + b \tanh^{-1}(c + dx))^2}{3d} + \frac{be^2(c + dx)^2 (a + b \tanh^{-1}(c + dx))}{3d} + \frac{e^2 (a + b \tanh^{-1}(c + dx))^2}{3d} - \frac{2be^2 \log\left(\frac{2(-dx-c-1)}{-dx-c+1}\right)}{d}$$

[Out] $\frac{1}{3}b^2e^{2x} - \frac{1}{3}b^2e^{2\operatorname{arctanh}(dx+c)}/d + \frac{1}{3}b^2e^{2(dx+c)^2}(a+b\operatorname{arctanh}(dx+c))/d + \frac{1}{3}e^{2(a+b\operatorname{arctanh}(dx+c))^2}/d + \frac{1}{3}e^{2(dx+c)^3}(a+b\operatorname{arctanh}(dx+c))^2/d - \frac{2}{3}b^2e^{2(a+b\operatorname{arctanh}(dx+c))}\ln(2/(-dx-c+1))/d - \frac{1}{3}b^2e^{2\operatorname{polylog}(2,(-dx-c-1)/(-dx-c+1))}/d$

Rubi [A] time = 0.24, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {6107, 12, 5916, 5980, 321, 206, 5984, 5918, 2402, 2315}

$$-\frac{b^2e^2\operatorname{PolyLog}\left(2, -\frac{c+dx+1}{-c-dx+1}\right)}{3d} + \frac{e^2(c + dx)^3 (a + b \tanh^{-1}(c + dx))^2}{3d} + \frac{be^2(c + dx)^2 (a + b \tanh^{-1}(c + dx))}{3d} + \frac{e^2 (a + b \tanh^{-1}(c + dx))^2}{3d}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(c*e + d*e*x)^2*(a + b*\operatorname{ArcTanh}[c + d*x])^2, x]$

[Out] $\frac{(b^2e^{2x})}{3} - \frac{(b^2e^{2\operatorname{ArcTanh}[c + d*x]})}{(3*d)} + \frac{(b^2e^{2(c + d*x)^2}(a + b*\operatorname{ArcTanh}[c + d*x]))}{(3*d)} + \frac{(e^{2(a + b*\operatorname{ArcTanh}[c + d*x])^2})}{(3*d)} + \frac{(e^{2(c + d*x)^3}(a + b*\operatorname{ArcTanh}[c + d*x])^2)}{(3*d)} - \frac{(2*b^2e^{2(a + b*\operatorname{ArcTanh}[c + d*x])}\operatorname{Log}[2/(1 - c - d*x)])}{(3*d)} - \frac{(b^2e^{2\operatorname{PolyLog}[2, -((1 + c + d*x)/(1 - c - d*x)])})}{(3*d)}$

Rule 12

$\operatorname{Int}[(a_*)(u_), x_Symbol] \rightarrow \operatorname{Dist}[a, \operatorname{Int}[u, x], x] /; \operatorname{FreeQ}[a, x] \ \&\& \ !\operatorname{Match}Q[u, (b_*)(v_)] /; \operatorname{FreeQ}[b, x]$

Rule 206

$\operatorname{Int}[(a_*) + (b_*)(x_)^2)^{-1}, x_Symbol] \rightarrow \operatorname{Simp}[(1*\operatorname{ArcTanh}[\operatorname{Rt}[-b, 2]*x]/\operatorname{Rt}[a, 2])]/(\operatorname{Rt}[a, 2]*\operatorname{Rt}[-b, 2]), x] /; \operatorname{FreeQ}[\{a, b\}, x] \ \&\& \ \operatorname{NegQ}[a/b] \ \&\& \ (\operatorname{Gt}Q[a, 0] \ || \ \operatorname{Lt}Q[b, 0])$

Rule 321

$\operatorname{Int}[(c_*)(x_)^m * ((a_*) + (b_*)(x_)^n)^p, x_Symbol] \rightarrow \operatorname{Simp}[(c^{n-1} * (c*x)^{m-n+1} * (a + b*x^n)^{p+1}) / (b*(m + n*p + 1)), x] - \operatorname{Dist}[\dots]$

$(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), \text{Int}[(c*x)^{(m - n)}*(a + b*x^n)^p, x],$
 $x] /; \text{FreeQ}\{a, b, c, p\}, x\} \&\& \text{IGtQ}[n, 0] \&\& \text{GtQ}[m, n - 1] \&\& \text{NeQ}[m + n*p$
 $+ 1, 0] \&\& \text{IntBinomialQ}[a, b, c, n, m, p, x]$

Rule 2315

$\text{Int}[\text{Log}[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] \text{ :> } -\text{Simp}[\text{PolyLog}[2, 1 -$
 $c*x]/e, x] /; \text{FreeQ}\{c, d, e\}, x\} \&\& \text{EqQ}[e + c*d, 0]$

Rule 2402

$\text{Int}[\text{Log}[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] \text{ :> } -\text{Dis}$
 $t[e/g, \text{Subst}[\text{Int}[\text{Log}[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; \text{FreeQ}\{$
 $c, d, e, f, g\}, x\} \&\& \text{EqQ}[c, 2*d] \&\& \text{EqQ}[e^2*f + d^2*g, 0]$

Rule 5916

$\text{Int}[((a_.) + \text{ArcTanh}[(c_.)*(x_)]*(b_.))^{(p_.)}*((d_.)*(x_))^{(m_.)}, x_Symbol]$
 $\text{ :> } \text{Simp}[(d*x)^{(m + 1)}*(a + b*\text{ArcTanh}[c*x])^p/(d*(m + 1)), x] - \text{Dist}[(b*c$
 $*p)/(d*(m + 1)), \text{Int}[(d*x)^{(m + 1)}*(a + b*\text{ArcTanh}[c*x])^{(p - 1)}]/(1 - c^2*$
 $x^2), x], x] /; \text{FreeQ}\{a, b, c, d, m\}, x\} \&\& \text{IGtQ}[p, 0] \&\& (\text{EqQ}[p, 1] \parallel \text{In}$
 $\text{tegerQ}[m]) \&\& \text{NeQ}[m, -1]$

Rule 5918

$\text{Int}[((a_.) + \text{ArcTanh}[(c_.)*(x_)]*(b_.))^{(p_.)}/((d_) + (e_.)*(x_)), x_Symbol]$
 $\text{ :> } -\text{Simp}[(a + b*\text{ArcTanh}[c*x])^p*\text{Log}[2/(1 + (e*x)/d)]/e, x] + \text{Dist}[(b*c*$
 $p)/e, \text{Int}[(a + b*\text{ArcTanh}[c*x])^{(p - 1)}*\text{Log}[2/(1 + (e*x)/d)]/(1 - c^2*x^2)$
 $, x], x] /; \text{FreeQ}\{a, b, c, d, e\}, x\} \&\& \text{IGtQ}[p, 0] \&\& \text{EqQ}[c^2*d^2 - e^2, 0]$
 $]$

Rule 5980

$\text{Int}[(((a_.) + \text{ArcTanh}[(c_.)*(x_)]*(b_.))^{(p_.)}*((f_.)*(x_))^{(m_.)})/((d_) + ($
 $e_.)*(x_)^2), x_Symbol] \text{ :> } \text{Dist}[f^2/e, \text{Int}[(f*x)^{(m - 2)}*(a + b*\text{ArcTanh}[c*x$
 $])^p, x], x] - \text{Dist}[(d*f^2)/e, \text{Int}[(f*x)^{(m - 2)}*(a + b*\text{ArcTanh}[c*x])^p]/($
 $d + e*x^2), x], x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x\} \&\& \text{GtQ}[p, 0] \&\& \text{GtQ}[m, 1]$
 $]$

Rule 5984

$\text{Int}[(((a_.) + \text{ArcTanh}[(c_.)*(x_)]*(b_.))^{(p_.)}*(x_))/((d_) + (e_.)*(x_)^2),$
 $x_Symbol] \text{ :> } \text{Simp}[(a + b*\text{ArcTanh}[c*x])^{(p + 1)}/(b*e*(p + 1)), x] + \text{Dist}[1/$
 $(c*d), \text{Int}[(a + b*\text{ArcTanh}[c*x])^p/(1 - c*x), x], x] /; \text{FreeQ}\{a, b, c, d, e$

}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]

Rule 6107

Int[((a_.) + ArcTanh[(c_.) + (d_.)*(x_.)]*(b_.))^p_.)*((e_.) + (f_.)*(x_.))^m_.), x_Symbol] :> Dist[1/d, Subst[Int[((f*x)/d)^m*(a + b*ArcTanh[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] && IGtQ[p, 0]

Rubi steps

$$\begin{aligned}
 \int (ce + dex)^2 (a + b \tanh^{-1}(c + dx))^2 dx &= \frac{\text{Subst}\left(\int e^2 x^2 (a + b \tanh^{-1}(x))^2 dx, x, c + dx\right)}{d} \\
 &= \frac{e^2 \text{Subst}\left(\int x^2 (a + b \tanh^{-1}(x))^2 dx, x, c + dx\right)}{d} \\
 &= \frac{e^2 (c + dx)^3 (a + b \tanh^{-1}(c + dx))^2}{3d} - \frac{(2be^2) \text{Subst}\left(\int \frac{x^3 (a + b \tanh^{-1}(x))}{1-x^2} dx, x, c + dx\right)}{3d} \\
 &= \frac{e^2 (c + dx)^3 (a + b \tanh^{-1}(c + dx))^2}{3d} + \frac{(2be^2) \text{Subst}\left(\int x (a + b \tanh^{-1}(x)) dx, x, c + dx\right)}{3d} \\
 &= \frac{be^2 (c + dx)^2 (a + b \tanh^{-1}(c + dx))}{3d} + \frac{e^2 (a + b \tanh^{-1}(c + dx))^2}{3d} + \frac{e^2 (c + dx)^2}{3d} \\
 &= \frac{1}{3} b^2 e^2 x + \frac{be^2 (c + dx)^2 (a + b \tanh^{-1}(c + dx))}{3d} + \frac{e^2 (a + b \tanh^{-1}(c + dx))^2}{3d} \\
 &= \frac{1}{3} b^2 e^2 x - \frac{b^2 e^2 \tanh^{-1}(c + dx)}{3d} + \frac{be^2 (c + dx)^2 (a + b \tanh^{-1}(c + dx))}{3d} \\
 &= \frac{1}{3} b^2 e^2 x - \frac{b^2 e^2 \tanh^{-1}(c + dx)}{3d} + \frac{be^2 (c + dx)^2 (a + b \tanh^{-1}(c + dx))}{3d}
 \end{aligned}$$

Mathematica [A] time = 0.47, size = 150, normalized size = 0.84

$$\frac{e^2 \left(a^2 (c + dx)^3 + ab \left((c + dx)^2 + \log \left((c + dx)^2 - 1 \right) + 2(c + dx)^3 \tanh^{-1}(c + dx) \right) + b^2 \left(\text{Li}_2 \left(-e^{-2 \tanh^{-1}(c + dx)} \right) + (c + dx)^2 \right) \right)}{3d}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(c*e + d*e*x)^2*(a + b*ArcTanh[c + d*x])^2,x]

[Out] (e^2*(a^2*(c + d*x)^3 + a*b*((c + d*x)^2 + 2*(c + d*x)^3*ArcTanh[c + d*x] + Log[-1 + (c + d*x)^2]) + b^2*(c + d*x - ArcTanh[c + d*x] + (c + d*x)^2*ArcTanh[c + d*x] - ArcTanh[c + d*x]^2 + (c + d*x)^3*ArcTanh[c + d*x]^2 - 2*ArcTanh[c + d*x]*Log[1 + E^(-2*ArcTanh[c + d*x])]) + PolyLog[2, -E^(-2*ArcTanh[c + d*x])])))/(3*d)

fricas [F] time = 0.66, size = 0, normalized size = 0.00

integral (a^2*d^2*e^2*x^2 + 2*a^2*c*d*e^2*x + a^2*c^2*e^2 + (b^2*d^2*e^2*x^2 + 2*b^2*c*d*e^2*x + b^2*c^2*e^2) artanh(dx + c)^2 + 2*(abd^2*e^2*x^2 + 2*a

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*e*x+c*e)^2*(a+b*arctanh(d*x+c))^2,x, algorithm="fricas")

[Out] integral(a^2*d^2*e^2*x^2 + 2*a^2*c*d*e^2*x + a^2*c^2*e^2 + (b^2*d^2*e^2*x^2 + 2*b^2*c*d*e^2*x + b^2*c^2*e^2)*arctanh(d*x + c)^2 + 2*(a*b*d^2*e^2*x^2 + 2*a*b*c*d*e^2*x + a*b*c^2*e^2)*arctanh(d*x + c), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dex + ce)^2 (b \operatorname{artanh}(dx + c) + a)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*e*x+c*e)^2*(a+b*arctanh(d*x+c))^2,x, algorithm="giac")

[Out] integrate((d*e*x + c*e)^2*(b*arctanh(d*x + c) + a)^2, x)

maple [B] time = 0.06, size = 583, normalized size = 3.26

$$-\frac{e^2 b^2 \ln(dx + c + 1)^2}{12d} + \frac{d^2 x^3 a^2 e^2}{3} + x a^2 c^2 e^2 - \frac{e^2 b^2 \operatorname{dilog}\left(\frac{1}{2} + \frac{dx}{2} + \frac{c}{2}\right)}{3d} - \frac{e^2 b^2 \ln(dx + c + 1)}{6d} + \frac{e^2 b^2 \ln(dx + c - 1)}{6d} +$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*e*x+c*e)^2*(a+b*arctanh(d*x+c))^2,x)

[Out] -1/3/d*e^2*b^2*dilog(1/2+1/2*d*x+1/2*c)-1/12/d*e^2*b^2*ln(d*x+c+1)^2+1/3*d^2*x^3*a^2*e^2+x*a^2*c^2*e^2-1/6/d*e^2*b^2*ln(d*x+c+1)+1/6/d*e^2*b^2*ln(d*x+c-1)+1/12/d*e^2*b^2*ln(d*x+c-1)^2+1/3/d*b^2*c*e^2+1/3/d*a^2*c^3*e^2+2*d*arctanh(d*x+c)*x^2*a*b*c*e^2+d*arctanh(d*x+c)^2*x^2*b^2*c*e^2+2*arctanh(d*x+c)*x*a*b*c^2*e^2+2/3/d*arctanh(d*x+c)*a*b*c^3*e^2+2/3*d^2*arctanh(d*x+c)*x^3*

$a*b*e^{2+1/3/d*a*b*c^2*e^{2+2/3*x*a*b*c*e^{2+1/3*b^2*e^2*x+1/3*d^2*arctanh(d*x+c)}^2*x^3*b^2*e^{2+1/3/d*e^2*b^2*arctanh(d*x+c)*ln(d*x+c-1)+d*x^2*a^2*c*e^{2+1/3*d*x^2*a*b*e^{2+2/3*arctanh(d*x+c)*x*b^2*c*e^{2+arctanh(d*x+c)}^2*x*b^2*c^2*e^{2+1/3*d*arctanh(d*x+c)*x^2*b^2*e^{2+1/3/d*arctanh(d*x+c)}^2*b^2*c^3*e^{2+1/3/d*arctanh(d*x+c)*b^2*c^2*e^{2+1/3/d*e^2*b^2*arctanh(d*x+c)*ln(d*x+c+1)-1/6/d*e^2*b^2*ln(d*x+c-1)*ln(1/2+1/2*d*x+1/2*c)+1/6/d*e^2*b^2*ln(-1/2*d*x-1/2*c+1/2)*ln(d*x+c+1)-1/6/d*e^2*b^2*ln(-1/2*d*x-1/2*c+1/2)*ln(1/2+1/2*d*x+1/2*c)+1/3/d*e^2*a*b*ln(d*x+c+1)+1/3/d*e^2*a*b*ln(d*x+c-1)}$

maxima [B] time = 0.63, size = 619, normalized size = 3.46

$$\frac{1}{3} a^2 d^2 e^2 x^3 + a^2 c d e^2 x^2 + \left(2 x^2 \operatorname{artanh}(d x + c) + d \left(\frac{2 x}{d^2} - \frac{(c^2 + 2 c + 1) \log(d x + c + 1)}{d^3} + \frac{(c^2 - 2 c + 1) \log(d x + c - 1)}{d^3} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*e*x+c*e)^2*(a+b*arctanh(d*x+c))^2,x, algorithm="maxima")

[Out] $\frac{1}{3} a^2 d^2 e^2 x^3 + a^2 c d e^2 x^2 + (2 x^2 \operatorname{arctanh}(d x + c) + d (2 x / d^2 - (c^2 + 2 c + 1) \log(d x + c + 1) / d^3 + (c^2 - 2 c + 1) \log(d x + c - 1) / d^3)) a b c d e^2 + 1 / 3 (2 x^3 \operatorname{arctanh}(d x + c) + d ((d x^2 - 4 c x) / d^3 + (c^3 + 3 c^2 + 3 c + 1) \log(d x + c + 1) / d^4 - (c^3 - 3 c^2 + 3 c - 1) \log(d x + c - 1) / d^4)) a b d^2 e^2 + a^2 c^2 e^2 x + (2 (d x + c) \operatorname{arctanh}(d x + c) + \log(-(d x + c)^2 + 1)) a b c^2 e^2 / d + 1 / 3 (\log(d x + c + 1) \log(-1 / 2 d x - 1 / 2 c + 1 / 2) + \operatorname{dilog}(1 / 2 d x + 1 / 2 c + 1 / 2)) b^2 e^2 / d + 1 / 6 (c^2 e^2 - e^2) b^2 \log(d x + c + 1) / d - 1 / 6 (c^2 e^2 - e^2) b^2 \log(d x + c - 1) / d + 1 / 12 (4 b^2 d e^2 x + (b^2 d^3 e^2 x^3 + 3 b^2 c d^2 e^2 x^2 + 3 b^2 c^2 d e^2 x + (c^3 e^2 + e^2) b^2) \log(d x + c + 1)^2 + (b^2 d^3 e^2 x^3 + 3 b^2 c d^2 e^2 x^2 + 3 b^2 c^2 d e^2 x + (c^3 e^2 - e^2) b^2) \log(-d x - c + 1)^2 + 2 (b^2 d^2 e^2 x^2 + 2 b^2 c d e^2 x) \log(d x + c + 1) - 2 (b^2 d^2 e^2 x^2 + 2 b^2 c d e^2 x + (b^2 d^3 e^2 x^3 + 3 b^2 c d^2 e^2 x^2 + 3 b^2 c^2 d e^2 x + (c^3 e^2 + e^2) b^2) \log(d x + c + 1)) \log(-d x - c + 1)) / d$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (c e + d e x)^2 (a + b \operatorname{atanh}(c + d x))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*e + d*e*x)^2*(a + b*atanh(c + d*x))^2,x)

[Out] int((c*e + d*e*x)^2*(a + b*atanh(c + d*x))^2, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$e^2 \left(\int a^2 c^2 dx + \int a^2 d^2 x^2 dx + \int b^2 c^2 \operatorname{atanh}^2(c + d x) dx + \int 2 a b c^2 \operatorname{atanh}(c + d x) dx + \int 2 a^2 c d x dx + \int b^2 d^2 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**2*(a+b*atanh(d*x+c))**2,x)
```

```
[Out] e**2*(Integral(a**2*c**2, x) + Integral(a**2*d**2*x**2, x) + Integral(b**2*c**2*atanh(c + d*x)**2, x) + Integral(2*a*b*c**2*atanh(c + d*x), x) + Integral(2*a**2*c*d*x, x) + Integral(b**2*d**2*x**2*atanh(c + d*x)**2, x) + Integral(2*a*b*d**2*x**2*atanh(c + d*x), x) + Integral(2*b**2*c*d*x*atanh(c + d*x)**2, x) + Integral(4*a*b*c*d*x*atanh(c + d*x), x))
```

3.17 $\int (ce + dex) \left(a + b \tanh^{-1}(c + dx) \right)^2 dx$

Optimal. Leaf size=95

$$\frac{e(c+dx)^2 (a+b \tanh^{-1}(c+dx))^2}{2d} - \frac{e(a+b \tanh^{-1}(c+dx))^2}{2d} + abex + \frac{b^2 e \log(1-(c+dx)^2)}{2d} + \frac{b^2 e(c+dx) \tanh^{-1}(c+dx)}{d}$$

[Out] a*b*e*x+b^2*e*(d*x+c)*arctanh(d*x+c)/d-1/2*e*(a+b*arctanh(d*x+c))^2/d+1/2*e*(d*x+c)^2*(a+b*arctanh(d*x+c))^2/d+1/2*b^2*e*ln(1-(d*x+c)^2)/d

Rubi [A] time = 0.14, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6107, 12, 5916, 5980, 5910, 260, 5948}

$$\frac{e(c+dx)^2 (a+b \tanh^{-1}(c+dx))^2}{2d} - \frac{e(a+b \tanh^{-1}(c+dx))^2}{2d} + abex + \frac{b^2 e \log(1-(c+dx)^2)}{2d} + \frac{b^2 e(c+dx) \tanh^{-1}(c+dx)}{d}$$

Antiderivative was successfully verified.

[In] Int[(c*e + d*e*x)*(a + b*ArcTanh[c + d*x])^2,x]

[Out] a*b*e*x + (b^2*e*(c + d*x)*ArcTanh[c + d*x])/d - (e*(a + b*ArcTanh[c + d*x])^2)/(2*d) + (e*(c + d*x)^2*(a + b*ArcTanh[c + d*x])^2)/(2*d) + (b^2*e*Log[1 - (c + d*x)^2])/(2*d)

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 260

Int[(x_)^(m_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 5910

Int[((a_.) + ArcTanh[(c_.)*(x_)])*(b_.))^(p_.), x_Symbol] := Simp[x*(a + b*ArcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]

Rule 5916

Int[((a_.) + ArcTanh[(c_.)*(x_)])*(b_.))^(p_.)*((d_.)*(x_)^(m_.)), x_Symbol] := Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c

p)/(d(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || IntegerQ[m]) && NeQ[m, -1]

Rule 5948

Int[((a_.) + ArcTanh[(c_.)*(x_.)]*(b_.))^(p_.)/((d_.) + (e_.)*(x_.)^2), x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]

Rule 5980

Int[(((a_.) + ArcTanh[(c_.)*(x_.)]*(b_.))^(p_.)*((f_.)*(x_.))^(m_.))/((d_.) + (e_.)*(x_.)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x])^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTanh[c*x])^p)/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]

Rule 6107

Int[((a_.) + ArcTanh[(c_.) + (d_.)*(x_.)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_.))^(m_.), x_Symbol] :> Dist[1/d, Subst[Int[((f*x)/d)^m*(a + b*ArcTanh[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] && IGtQ[p, 0]

Rubi steps

$$\begin{aligned}
\int (ce + dex) (a + b \tanh^{-1}(c + dx))^2 dx &= \frac{\text{Subst}\left(\int ex (a + b \tanh^{-1}(x))^2 dx, x, c + dx\right)}{d} \\
&= \frac{e \text{Subst}\left(\int x (a + b \tanh^{-1}(x))^2 dx, x, c + dx\right)}{d} \\
&= \frac{e(c + dx)^2 (a + b \tanh^{-1}(c + dx))^2}{2d} - \frac{(be) \text{Subst}\left(\int \frac{x^2 (a + b \tanh^{-1}(x))}{1-x^2} dx, x, c + dx\right)}{d} \\
&= \frac{e(c + dx)^2 (a + b \tanh^{-1}(c + dx))^2}{2d} + \frac{(be) \text{Subst}\left(\int (a + b \tanh^{-1}(x)) dx, x, c + dx\right)}{d} \\
&= abex - \frac{e (a + b \tanh^{-1}(c + dx))^2}{2d} + \frac{e(c + dx)^2 (a + b \tanh^{-1}(c + dx))^2}{2d} \\
&= abex + \frac{b^2 e(c + dx) \tanh^{-1}(c + dx)}{d} - \frac{e (a + b \tanh^{-1}(c + dx))^2}{2d} + \frac{e(c + dx)^2 (a + b \tanh^{-1}(c + dx))^2}{2d} \\
&= abex + \frac{b^2 e(c + dx) \tanh^{-1}(c + dx)}{d} - \frac{e (a + b \tanh^{-1}(c + dx))^2}{2d} + \frac{e(c + dx)^2 (a + b \tanh^{-1}(c + dx))^2}{2d}
\end{aligned}$$

Mathematica [A] time = 0.10, size = 134, normalized size = 1.41

$$e \left(\frac{a^2(c + dx)^2}{2d} + \frac{(ab + b^2) \log(-c - dx + 1)}{2d} + \frac{(b^2 - ab) \log(c + dx + 1)}{2d} + \frac{ab(c + dx)}{d} + \frac{b(c + dx) \tanh^{-1}(c + dx)}{d} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(c*e + d*e*x)*(a + b*ArcTanh[c + d*x])^2,x]

[Out] e*((a*b*(c + d*x))/d + (a^2*(c + d*x)^2)/(2*d) + (b*(c + d*x)*(b + a*(c + d*x))*ArcTanh[c + d*x])/d + ((-b^2 + b^2*(c + d*x)^2)*ArcTanh[c + d*x]^2)/(2*d) + ((a*b + b^2)*Log[1 - c - d*x])/(2*d) + ((-(a*b) + b^2)*Log[1 + c + d*x])/(2*d))

fricas [B] time = 0.47, size = 191, normalized size = 2.01

$$\frac{4 a^2 d^2 e x^2 + 8 (a^2 c + a b) d e x + 4 (a b c^2 + b^2 c - a b + b^2) e \log (d x + c + 1) - 4 (a b c^2 + b^2 c - a b - b^2) e \log (d x + c - 1)}{8 d}$$

Verification of antiderivative is not currently implemented for this CAS.

$+c+1)+1/4/d*e*b^2*\ln(-1/2*d*x-1/2*c+1/2)*\ln(1/2+1/2*d*x+1/2*c)+d*\operatorname{arctanh}(d*x+c)*x^2*a*b*e+2*\operatorname{arctanh}(d*x+c)*x*a*b*c*e+1/d*\operatorname{arctanh}(d*x+c)*a*b*c^2*e+a*b*e*x+1/d*a*b*c*e+1/2/d*e*a*b*\ln(d*x+c-1)-1/2/d*e*a*b*\ln(d*x+c+1)$

maxima [B] time = 0.60, size = 316, normalized size = 3.33

$$\frac{1}{2} a^2 d e x^2 + \frac{1}{2} \left(2 x^2 \operatorname{artanh}(d x + c) + d \left(\frac{2 x}{d^2} - \frac{(c^2 + 2 c + 1) \log(d x + c + 1)}{d^3} + \frac{(c^2 - 2 c + 1) \log(d x + c - 1)}{d^3} \right) \right) a b d e$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*e*x+c*e)*(a+b*arctanh(d*x+c))^2,x, algorithm="maxima")

[Out] $1/2*a^2*d*e*x^2 + 1/2*(2*x^2*\operatorname{arctanh}(d*x + c) + d*(2*x/d^2 - (c^2 + 2*c + 1)*\log(d*x + c + 1)/d^3 + (c^2 - 2*c + 1)*\log(d*x + c - 1)/d^3))*a*b*d*e + a^2*c*e*x + (2*(d*x + c)*\operatorname{arctanh}(d*x + c) + \log(-(d*x + c)^2 + 1))*a*b*c*e/d + 1/8*((b^2*d^2*e*x^2 + 2*b^2*c*d*e*x + (c^2*e - e)*b^2)*\log(d*x + c + 1)^2 + (b^2*d^2*e*x^2 + 2*b^2*c*d*e*x + (c^2*e - e)*b^2)*\log(-d*x - c + 1)^2 + 4*(b^2*d*e*x + (c*e + e)*b^2)*\log(d*x + c + 1) - 2*(2*b^2*d*e*x + 2*(c*e - e)*b^2 + (b^2*d^2*e*x^2 + 2*b^2*c*d*e*x + (c^2*e - e)*b^2)*\log(d*x + c + 1))*\log(-d*x - c + 1))/d$

mupad [B] time = 1.56, size = 432, normalized size = 4.55

$$x \left(a e (b + 3 a c) - 2 a^2 c e \right) + \ln(1 - d x - c)^2 \left(\frac{b^2 c e x}{4} - \frac{b^2 e - b^2 c^2 e}{8 d} + \frac{b^2 d e x^2}{8} \right) - \ln(1 - d x - c) \left(\ln(c + d x + 1) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*e + d*e*x)*(a + b*atanh(c + d*x))^2,x)

[Out] $x*(a*e*(b + 3*a*c) - 2*a^2*c*e) + \log(1 - d*x - c)^2*((b^2*c*e*x)/4 - (b^2*e - b^2*c^2*e)/(8*d) + (b^2*d*e*x^2)/8) - \log(1 - d*x - c)*(\log(c + d*x + 1))*((b^2*c*e*x)/2 - ((b^2*e)/2 - (b^2*c^2*e)/2)/(2*d) + (b^2*d*e*x^2)/4) - (x*(4*b^2*d^2*e*(c - 1) - 4*b^2*d*e*(d*(c - 1) + d*(c + 1)) + 8*b^2*c*d^2*e))/(16*d^2) + (x*(8*b*d^2*e*(4*a*c - 2*a + b*c) + 4*b*d^2*e*(4*a + b)*(c + 1) - 4*b*d*e*(d*(c - 1) + d*(c + 1))*(4*a + b)))/(16*d^2) - (b^2*d*e*x^2)/8 + (b*d*e*x^2*(4*a + b))/8 + \log(c + d*x + 1)^2*((b^2*c*e*x)/4 - (b^2*e - b^2*c^2*e)/(8*d) + (b^2*d*e*x^2)/8) + (\log(c + d*x + 1)*(b^2*e - a*b*e + b^2*c*e + a*b*c^2*e))/(2*d) + (\log(c + d*x - 1)*(b^2*e + a*b*e - b^2*c*e - a*b*c^2*e))/(2*d) + d*\log(c + d*x + 1)*((x*(b^2*e + 2*a*b*c*e))/(2*d) + (a*b*e*x^2)/2) + (a^2*d*e*x^2)/2$

sympy [A] time = 3.23, size = 238, normalized size = 2.51

$$\begin{cases} a^2 c e x + \frac{a^2 d e x^2}{2} + \frac{a b c^2 e \operatorname{atanh}(c+d x)}{d} + 2 a b c e x \operatorname{atanh}(c+d x) + a b d e x^2 \operatorname{atanh}(c+d x) + a b e x - \frac{a b e \operatorname{atanh}(c+d x)}{d} + \frac{b^2 c^2}{d} \\ c e x (a + b \operatorname{atanh}(c))^2 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*e*x+c*e)*(a+b*atanh(d*x+c))**2,x)

[Out] Piecewise((a**2*c*e*x + a**2*d*e*x**2/2 + a*b*c**2*e*atanh(c + d*x)/d + 2*a*b*c*e*x*atanh(c + d*x) + a*b*d*e*x**2*atanh(c + d*x) + a*b*e*x - a*b*e*atanh(c + d*x)/d + b**2*c**2*e*atanh(c + d*x)**2/(2*d) + b**2*c*e*x*atanh(c + d*x)**2 + b**2*c*e*atanh(c + d*x)/d + b**2*d*e*x**2*atanh(c + d*x)**2/2 + b**2*e*x*atanh(c + d*x) + b**2*e*log(c/d + x + 1/d)/d - b**2*e*atanh(c + d*x)**2/(2*d) - b**2*e*atanh(c + d*x)/d, Ne(d, 0)), (c*e*x*(a + b*atanh(c))**2, True))

$$3.18 \quad \int \frac{(a+b \tanh^{-1}(c+dx))^2}{ce+dex} dx$$

Optimal. Leaf size=168

$$\frac{b \operatorname{Li}_2\left(1 - \frac{2}{-c-dx+1}\right)(a+b \tanh^{-1}(c+dx))}{de} + \frac{b \operatorname{Li}_2\left(\frac{2}{-c-dx+1} - 1\right)(a+b \tanh^{-1}(c+dx))}{de} + \frac{2 \tanh^{-1}\left(1 - \frac{2}{-c-dx+1}\right)(a+b \tanh^{-1}(c+dx))}{de}$$

[Out] $-2*(a+b*\operatorname{arctanh}(d*x+c))^2*\operatorname{arctanh}(-1+2/(-d*x-c+1))/d/e-b*(a+b*\operatorname{arctanh}(d*x+c))*\operatorname{polylog}(2,1-2/(-d*x-c+1))/d/e+b*(a+b*\operatorname{arctanh}(d*x+c))*\operatorname{polylog}(2,-1+2/(-d*x-c+1))/d/e+1/2*b^2*\operatorname{polylog}(3,1-2/(-d*x-c+1))/d/e-1/2*b^2*\operatorname{polylog}(3,-1+2/(-d*x-c+1))/d/e$

Rubi [A] time = 0.30, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {6107, 12, 5914, 6052, 5948, 6058, 6610}

$$\frac{b \operatorname{PolyLog}\left(2, 1 - \frac{2}{-c-dx+1}\right)(a+b \tanh^{-1}(c+dx))}{de} + \frac{b \operatorname{PolyLog}\left(2, \frac{2}{-c-dx+1} - 1\right)(a+b \tanh^{-1}(c+dx))}{de} + \frac{b^2 \operatorname{PolyLog}\left(3, 1 - \frac{2}{-c-dx+1}\right)(a+b \tanh^{-1}(c+dx))}{de}$$

Antiderivative was successfully verified.

[In] `Int[(a + b*ArcTanh[c + d*x])^2/(c*e + d*e*x), x]`

[Out] $(2*(a + b*\operatorname{ArcTanh}[c + d*x])^2*\operatorname{ArcTanh}[1 - 2/(1 - c - d*x)])/(d*e) - (b*(a + b*\operatorname{ArcTanh}[c + d*x])*\operatorname{PolyLog}[2, 1 - 2/(1 - c - d*x)])/(d*e) + (b*(a + b*\operatorname{ArcTanh}[c + d*x])*\operatorname{PolyLog}[2, -1 + 2/(1 - c - d*x)])/(d*e) + (b^2*\operatorname{PolyLog}[3, 1 - 2/(1 - c - d*x)])/(2*d*e) - (b^2*\operatorname{PolyLog}[3, -1 + 2/(1 - c - d*x)])/(2*d*e)$

Rule 12

`Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]`

Rule 5914

`Int[((a_.) + ArcTanh[(c_.)*(x_)])*(b_.)^p/(x_), x_Symbol] := Simp[2*(a + b*ArcTanh[c*x])^p*ArcTanh[1 - 2/(1 - c*x)], x] - Dist[2*b*c*p, Int[((a + b*ArcTanh[c*x])^(p - 1)*ArcTanh[1 - 2/(1 - c*x)]/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 1]`

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbol]
:> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b, c, d, e, p}, x]
&& EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6052

```
Int[(ArcTanh[u_]*)((a_.) + ArcTanh[(c_.)*(x_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbol]
:> Dist[1/2, Int[(Log[1 + u]*(a + b*ArcTanh[c*x])^p)/(d + e*x^2), x], x] - Dist[1/2, Int[(Log[1 - u]*(a + b*ArcTanh[c*x])^p)/(d + e*x^2), x], x] /;
FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e, 0] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6058

```
Int[(Log[u_]*)((a_.) + ArcTanh[(c_.)*(x_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbol]
:> -Simp[(a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u]/(2*c*d), x] + Dist[(b*p)/2, Int[(a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u]/(d + e*x^2), x], x] /;
FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6107

```
Int[((a_.) + ArcTanh[(c_) + (d_.)*(x_.)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_.))^(m_.), x_Symbol]
:> Dist[1/d, Subst[Int[((f*x)/d)^m*(a + b*ArcTanh[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
&& EqQ[d*e - c*f, 0] && IGtQ[p, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v, x]}, Simp[w*PolyLog[n + 1, v], x] /; !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a + b \tanh^{-1}(c + dx))^2}{ce + dex} dx &= \frac{\text{Subst} \left(\int \frac{(a+b \tanh^{-1}(x))^2}{ex} dx, x, c + dx \right)}{d} \\
&= \frac{\text{Subst} \left(\int \frac{(a+b \tanh^{-1}(x))^2}{x} dx, x, c + dx \right)}{de} \\
&= \frac{2(a + b \tanh^{-1}(c + dx))^2 \tanh^{-1} \left(1 - \frac{2}{1-c-dx} \right)}{de} - \frac{(4b) \text{Subst} \left(\int \frac{\tanh^{-1} \left(1 - \frac{2}{1-x} \right) (a+b \tanh^{-1}(x))}{1-x^2} dx, x, c + dx \right)}{de} \\
&= \frac{2(a + b \tanh^{-1}(c + dx))^2 \tanh^{-1} \left(1 - \frac{2}{1-c-dx} \right)}{de} - \frac{(2b) \text{Subst} \left(\int \frac{(a+b \tanh^{-1}(x)) \log(2)}{1-x^2} dx, x, c + dx \right)}{de} \\
&= \frac{2(a + b \tanh^{-1}(c + dx))^2 \tanh^{-1} \left(1 - \frac{2}{1-c-dx} \right)}{de} - \frac{b(a + b \tanh^{-1}(c + dx)) \text{Li}_2 \left(1 - \frac{2}{1-c-dx} \right)}{de} \\
&= \frac{2(a + b \tanh^{-1}(c + dx))^2 \tanh^{-1} \left(1 - \frac{2}{1-c-dx} \right)}{de} - \frac{b(a + b \tanh^{-1}(c + dx)) \text{Li}_2 \left(1 - \frac{2}{1-c-dx} \right)}{de}
\end{aligned}$$

Mathematica [C] time = 0.39, size = 424, normalized size = 2.52

$$a^2 \log(c + dx) - \frac{1}{4} ab \left(4\text{Li}_2 \left(e^{-2 \tanh^{-1}(c+dx)} \right) + 4\text{Li}_2 \left(-e^{2 \tanh^{-1}(c+dx)} \right) - 4i\pi \log \left(\frac{2}{\sqrt{1-(c+dx)^2}} \right) - 8 \tanh^{-1}(c + dx)^2 - \dots \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b*ArcTanh[c + d*x])^2/(c*e + d*e*x), x]

[Out] (a^2*Log[c + d*x] + 2*a*b*ArcTanh[c + d*x]*(-Log[1/Sqrt[1 - (c + d*x)^2]] + Log[(I*(c + d*x))/Sqrt[1 - (c + d*x)^2]]) - (a*b*(Pi^2 - (4*I)*Pi*ArcTanh[c + d*x] - 8*ArcTanh[c + d*x]^2 - 8*ArcTanh[c + d*x]*Log[1 - E^(-2*ArcTanh[c + d*x])]) + (4*I)*Pi*Log[1 + E^(2*ArcTanh[c + d*x])] + 8*ArcTanh[c + d*x]*Log[1 + E^(2*ArcTanh[c + d*x])] - (4*I)*Pi*Log[2/Sqrt[1 - (c + d*x)^2]] - 8*ArcTanh[c + d*x]*Log[2/Sqrt[1 - (c + d*x)^2]] + 8*ArcTanh[c + d*x]*Log[((2*I)*(c + d*x))/Sqrt[1 - (c + d*x)^2]] + 4*PolyLog[2, E^(-2*ArcTanh[c + d*x])] + 4*PolyLog[2, -E^(2*ArcTanh[c + d*x])])/4 + b^2*((I/24)*Pi^3 - (2*ArcTanh[c + d*x]^3)/3 - ArcTanh[c + d*x]^2*Log[1 + E^(-2*ArcTanh[c + d*x])] + ArcTanh[c + d*x]^2*Log[1 - E^(2*ArcTanh[c + d*x])] + ArcTanh[c + d*x]*PolyLo

$g[2, -E^{(-2*\text{ArcTanh}[c + d*x])}] + \text{ArcTanh}[c + d*x]*\text{PolyLog}[2, E^{(2*\text{ArcTanh}[c + d*x])}] + \text{PolyLog}[3, -E^{(-2*\text{ArcTanh}[c + d*x])}]/2 - \text{PolyLog}[3, E^{(2*\text{ArcTanh}[c + d*x])}]/2)/(d*e)$

fricas [F] time = 0.51, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^2 \operatorname{artanh}(dx + c)^2 + 2ab \operatorname{artanh}(dx + c) + a^2}{dex + ce}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arctanh(d*x+c))^2/(d*e*x+c*e),x, algorithm="fricas")

[Out] integral((b^2*arctanh(d*x + c)^2 + 2*a*b*arctanh(d*x + c) + a^2)/(d*e*x + c*e), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{artanh}(dx + c) + a)^2}{dex + ce} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arctanh(d*x+c))^2/(d*e*x+c*e),x, algorithm="giac")

[Out] integrate((b*arctanh(d*x + c) + a)^2/(d*e*x + c*e), x)

maple [C] time = 0.22, size = 893, normalized size = 5.32

$$\frac{a^2 \ln(dx + c)}{de} + \frac{b^2 \ln(dx + c) \operatorname{arctanh}(dx + c)^2}{de} - \frac{b^2 \operatorname{arctanh}(dx + c) \operatorname{polylog}\left(2, -\frac{(dx+c+1)^2}{1-(dx+c)^2}\right)}{de} + \frac{b^2 \operatorname{polylog}\left(3, -\frac{(dx+c+1)^2}{1-(dx+c)^2}\right)}{2de}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*arctanh(d*x+c))^2/(d*e*x+c*e),x)

[Out] $1/d*a^2/e*\ln(d*x+c)+1/d*b^2/e*\ln(d*x+c)*\operatorname{arctanh}(d*x+c)^2-1/d*b^2/e*\operatorname{arctanh}(d*x+c)*\operatorname{polylog}(2, -(d*x+c+1)^2/(1-(d*x+c)^2))+1/2/d*b^2/e*\operatorname{polylog}(3, -(d*x+c+1)^2/(1-(d*x+c)^2))-1/d*b^2/e*\operatorname{arctanh}(d*x+c)^2*\ln((d*x+c+1)^2/(1-(d*x+c)^2)-1)+1/d*b^2/e*\operatorname{arctanh}(d*x+c)^2*\ln(1-(d*x+c+1)/(1-(d*x+c)^2)^{(1/2}))+2/d*b^2/e*\operatorname{arctanh}(d*x+c)*\operatorname{polylog}(2, (d*x+c+1)/(1-(d*x+c)^2)^{(1/2}))-2/d*b^2/e*\operatorname{polylog}(3, (d*x+c+1)/(1-(d*x+c)^2)^{(1/2}))+1/d*b^2/e*\operatorname{arctanh}(d*x+c)^2*\ln(1+(d*x+c+1)/(1-(d*x+c)^2)^{(1/2}))+2/d*b^2/e*\operatorname{arctanh}(d*x+c)*\operatorname{polylog}(2, -(d*x+c+1)/(1-(d*x+c)^2)^{(1/2}))-2/d*b^2/e*\operatorname{polylog}(3, -(d*x+c+1)/(1-(d*x+c)^2)^{(1/2}))+1/2*I/d*b$

$$\begin{aligned} &^2/e*\text{Pi}*c\text{sgn}(I*((d*x+c+1)^2/(1-(d*x+c)^2)-1))*c\text{sgn}(I/(1+(d*x+c+1)^2/(1-(d*x+c)^2))) \\ &)*c\text{sgn}(I*((d*x+c+1)^2/(1-(d*x+c)^2)-1)/(1+(d*x+c+1)^2/(1-(d*x+c)^2))) \\ &)*\text{arctanh}(d*x+c)^2-1/2*I/d*b^2/e*\text{Pi}*c\text{sgn}(I*((d*x+c+1)^2/(1-(d*x+c)^2)-1))*c \\ &\text{sgn}(I*((d*x+c+1)^2/(1-(d*x+c)^2)-1)/(1+(d*x+c+1)^2/(1-(d*x+c)^2)))^2*\text{arctan} \\ &\text{h}(d*x+c)^2+1/2*I/d*b^2/e*\text{Pi}*c\text{sgn}(I*((d*x+c+1)^2/(1-(d*x+c)^2)-1)/(1+(d*x+c+1) \\ &)^2/(1-(d*x+c)^2)))^3*\text{arctanh}(d*x+c)^2-1/2*I/d*b^2/e*\text{Pi}*c\text{sgn}(I/(1+(d*x+c+1) \\ &)^2/(1-(d*x+c)^2))) *c\text{sgn}(I*((d*x+c+1)^2/(1-(d*x+c)^2)-1)/(1+(d*x+c+1)^2/(1- \\ &(d*x+c)^2)))^2*\text{arctanh}(d*x+c)^2+2/d*a*b/e*\ln(d*x+c)*\text{arctanh}(d*x+c)-1/d*a*b/ \\ &e*\text{dilog}(d*x+c)-1/d*a*b/e*\text{dilog}(d*x+c+1)-1/d*a*b/e*\ln(d*x+c)*\ln(d*x+c+1) \end{aligned}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{a^2 \log(dx + ce)}{de} + \int \frac{b^2 (\log(dx + c + 1) - \log(-dx - c + 1))^2}{4(dx + ce)} + \frac{ab (\log(dx + c + 1) - \log(-dx - c + 1))}{dex + ce} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arctanh(d*x+c))^2/(d*e*x+c*e),x, algorithm="maxima")

[Out] a^2*log(d*e*x + c*e)/(d*e) + integrate(1/4*b^2*(log(d*x + c + 1) - log(-d*x - c + 1))^2/(d*e*x + c*e) + a*b*(log(d*x + c + 1) - log(-d*x - c + 1))/(d*e*x + c*e), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + b \operatorname{atanh}(c + dx))^2}{ce + dex} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*atanh(c + d*x))^2/(c*e + d*e*x),x)

[Out] int((a + b*atanh(c + d*x))^2/(c*e + d*e*x), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a^2}{c+dx} dx + \int \frac{b^2 \operatorname{atanh}^2(c+dx)}{c+dx} dx + \int \frac{2ab \operatorname{atanh}(c+dx)}{c+dx} dx}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*atanh(d*x+c))**2/(d*e*x+c*e),x)

[Out] (Integral(a**2/(c + d*x), x) + Integral(b**2*atanh(c + d*x)**2/(c + d*x), x) + Integral(2*a*b*atanh(c + d*x)/(c + d*x), x))/e

$$3.19 \quad \int \frac{(a+b \tanh^{-1}(c+dx))^2}{(ce+dex)^2} dx$$

Optimal. Leaf size=104

$$\frac{(a+b \tanh^{-1}(c+dx))^2}{de^2(c+dx)} + \frac{(a+b \tanh^{-1}(c+dx))^2}{de^2} + \frac{2b \log\left(2 - \frac{2}{c+dx+1}\right)(a+b \tanh^{-1}(c+dx))}{de^2} - \frac{b^2 \text{Li}_2\left(\frac{2}{c+dx+1}\right)}{de^2}$$

[Out] (a+b*arctanh(d*x+c))^2/d/e^2-(a+b*arctanh(d*x+c))^2/d/e^2/(d*x+c)+2*b*(a+b*arctanh(d*x+c))*ln(2-2/(d*x+c+1))/d/e^2-b^2*polylog(2,-1+2/(d*x+c+1))/d/e^2

Rubi [A] time = 0.18, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {6107, 12, 5916, 5988, 5932, 2447}

$$\frac{b^2 \text{PolyLog}\left(2, \frac{2}{c+dx+1} - 1\right)}{de^2} - \frac{(a+b \tanh^{-1}(c+dx))^2}{de^2(c+dx)} + \frac{(a+b \tanh^{-1}(c+dx))^2}{de^2} + \frac{2b \log\left(2 - \frac{2}{c+dx+1}\right)(a+b \tanh^{-1}(c+dx))}{de^2}$$

Antiderivative was successfully verified.

[In] Int[(a + b*ArcTanh[c + d*x])^2/(c*e + d*e*x)^2,x]

[Out] (a + b*ArcTanh[c + d*x])^2/(d*e^2) - (a + b*ArcTanh[c + d*x])^2/(d*e^2*(c + d*x)) + (2*b*(a + b*ArcTanh[c + d*x])*Log[2 - 2/(1 + c + d*x)])/(d*e^2) - (b^2*PolyLog[2, -1 + 2/(1 + c + d*x)])/(d*e^2)

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 2447

Int[Log[u]*(Pq_)^(m_), x_Symbol] := With[{C = FullSimplify[(Pq^m*(1-u))/D[u, x]]}, Simp[C*PolyLog[2, 1-u], x] /; FreeQ[C, x] /; IntegerQ[m] && PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u, x][[2]], Expon[Pq, x]]

Rule 5916

Int[((a_.) + ArcTanh[(c_.)*(x_)])*(b_.)^(p_.)*((d_.)*(x_))^(m_.), x_Symbol] := Simp[((d*x)^(m+1)*(a + b*ArcTanh[c*x])^p)/(d*(m+1)), x] - Dist[(b*c*p)/(d*(m+1)), Int[((d*x)^(m+1)*(a + b*ArcTanh[c*x])^(p-1))/(1-c^2*x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In

tegerQ[m]) && NeQ[m, -1]

Rule 5932

Int[((a_.) + ArcTanh[(c_.)*(x_.)]*(b_.))^(p_.)/((x_.)*((d_.) + (e_.)*(x_.))), x_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] - Dist[(b*c*p)/d, Int[(a + b*ArcTanh[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)]]/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0]

Rule 5988

Int[((a_.) + ArcTanh[(c_.)*(x_.)]*(b_.))^(p_.)/((x_.)*((d_.) + (e_.)*(x_.)^2)), x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]

Rule 6107

Int[((a_.) + ArcTanh[(c_.) + (d_.)*(x_.)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_.))^(m_.), x_Symbol] :> Dist[1/d, Subst[Int[((f*x)/d)^m*(a + b*ArcTanh[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] && IGtQ[p, 0]

Rubi steps

$$\begin{aligned}
\int \frac{(a + b \tanh^{-1}(c + dx))^2}{(ce + dex)^2} dx &= \frac{\text{Subst} \left(\int \frac{(a+b \tanh^{-1}(x))^2}{e^2 x^2} dx, x, c + dx \right)}{d} \\
&= \frac{\text{Subst} \left(\int \frac{(a+b \tanh^{-1}(x))^2}{x^2} dx, x, c + dx \right)}{de^2} \\
&= -\frac{(a + b \tanh^{-1}(c + dx))^2}{de^2(c + dx)} + \frac{(2b) \text{Subst} \left(\int \frac{a+b \tanh^{-1}(x)}{x(1-x^2)} dx, x, c + dx \right)}{de^2} \\
&= \frac{(a + b \tanh^{-1}(c + dx))^2}{de^2} - \frac{(a + b \tanh^{-1}(c + dx))^2}{de^2(c + dx)} + \frac{(2b) \text{Subst} \left(\int \frac{a+b \tanh^{-1}(x)}{x(1+x)} dx, x, c + dx \right)}{de^2} \\
&= \frac{(a + b \tanh^{-1}(c + dx))^2}{de^2} - \frac{(a + b \tanh^{-1}(c + dx))^2}{de^2(c + dx)} + \frac{2b(a + b \tanh^{-1}(c + dx))}{de^2} \\
&= \frac{(a + b \tanh^{-1}(c + dx))^2}{de^2} - \frac{(a + b \tanh^{-1}(c + dx))^2}{de^2(c + dx)} + \frac{2b(a + b \tanh^{-1}(c + dx))}{de^2}
\end{aligned}$$

Mathematica [A] time = 0.25, size = 126, normalized size = 1.21

$$\frac{a \left(2b(c + dx) \log \left(\frac{c+dx}{\sqrt{1-(c+dx)^2}} \right) - a \right) + 2b \tanh^{-1}(c + dx) \left(b(c + dx) \log \left(1 - e^{-2 \tanh^{-1}(c+dx)} \right) - a \right) - b^2(c + dx) \text{Li}_2}{de^2(c + dx)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b*ArcTanh[c + d*x])^2/(c*e + d*e*x)^2,x]

[Out] (b^2*(-1 + c + d*x)*ArcTanh[c + d*x]^2 + 2*b*ArcTanh[c + d*x]*(-a + b*(c + d*x)*Log[1 - E^(-2*ArcTanh[c + d*x])]) + a*(-a + 2*b*(c + d*x)*Log[(c + d*x)/Sqrt[1 - (c + d*x)^2]]) - b^2*(c + d*x)*PolyLog[2, E^(-2*ArcTanh[c + d*x])])/(d*e^2*(c + d*x))

fricas [F] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral} \left(\frac{b^2 \operatorname{artanh}(dx + c)^2 + 2ab \operatorname{artanh}(dx + c) + a^2}{d^2 e^2 x^2 + 2cde^2 x + c^2 e^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arctanh(d*x+c))^2/(d*e*x+c*e)^2,x, algorithm="fricas")

[Out] integral((b^2*arctanh(d*x + c)^2 + 2*a*b*arctanh(d*x + c) + a^2)/(d^2*e^2*x^2 + 2*c*d*e^2*x + c^2*e^2), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{artanh}(dx + c) + a)^2}{(dex + ce)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arctanh(d*x+c))^2/(d*e*x+c*e)^2,x, algorithm="giac")

[Out] integrate((b*arctanh(d*x + c) + a)^2/(d*e*x + c*e)^2, x)

maple [B] time = 0.07, size = 396, normalized size = 3.81

$$-\frac{a^2}{d e^2 (dx + c)} - \frac{b^2 \operatorname{arctanh}(dx + c)^2}{d e^2 (dx + c)} + \frac{2b^2 \ln(dx + c) \operatorname{arctanh}(dx + c)}{d e^2} - \frac{b^2 \operatorname{arctanh}(dx + c) \ln(dx + c - 1)}{d e^2} - \frac{b^2 \operatorname{arctanh}(dx + c) \ln(dx + c + 1)}{d e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*arctanh(d*x+c))^2/(d*e*x+c*e)^2,x)

[Out] -1/d*a^2/e^2/(d*x+c)-1/d*b^2/e^2/(d*x+c)*arctanh(d*x+c)^2+2/d*b^2/e^2*ln(d*x+c)*arctanh(d*x+c)-1/d*b^2/e^2*arctanh(d*x+c)*ln(d*x+c-1)-1/d*b^2/e^2*arctanh(d*x+c)*ln(d*x+c+1)-1/4/d*b^2/e^2*ln(d*x+c-1)^2+1/d*b^2/e^2*dilog(1/2+1/2*d*x+1/2*c)+1/2/d*b^2/e^2*ln(d*x+c-1)*ln(1/2+1/2*d*x+1/2*c)+1/4/d*b^2/e^2*ln(d*x+c+1)^2-1/2/d*b^2/e^2*ln(-1/2*d*x-1/2*c+1/2)*ln(d*x+c+1)+1/2/d*b^2/e^2*ln(-1/2*d*x-1/2*c+1/2)*ln(1/2+1/2*d*x+1/2*c)-1/d*b^2/e^2*dilog(d*x+c)-1/d*b^2/e^2*dilog(d*x+c+1)-1/d*b^2/e^2*ln(d*x+c)*ln(d*x+c+1)-2/d*a*b/e^2/(d*x+c)*arctanh(d*x+c)+2/d*a*b/e^2*ln(d*x+c)-1/d*a*b/e^2*ln(d*x+c-1)-1/d*a*b/e^2*ln(d*x+c+1)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\left(d \left(\frac{\log(dx + c + 1)}{d^2 e^2} - \frac{2 \log(dx + c)}{d^2 e^2} + \frac{\log(dx + c - 1)}{d^2 e^2} \right) + \frac{2 \operatorname{artanh}(dx + c)}{d^2 e^2 x + c d e^2} \right) a b - \frac{1}{4} b^2 \left(\frac{\log(-dx - c + 1)^2}{d^2 e^2 x + c d e^2} + \int \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arctanh(d*x+c))^2/(d*e*x+c*e)^2,x, algorithm="maxima")

[Out] -(d*(log(d*x + c + 1)/(d^2*e^2) - 2*log(d*x + c)/(d^2*e^2) + log(d*x + c - 1)/(d^2*e^2)) + 2*arctanh(d*x + c)/(d^2*e^2*x + c*d*e^2))*a*b - 1/4*b^2*(lo

$$g(-d*x - c + 1)^2/(d^2*e^2*x + c*d*e^2) + \text{integrate}(-((d*x + c - 1)*\log(d*x + c + 1)^2 + 2*(d*x - (d*x + c - 1)*\log(d*x + c + 1) + c)*\log(-d*x - c + 1)))/(d^3*e^2*x^3 + c^3*e^2 - c^2*e^2 + (3*c*d^2*e^2 - d^2*e^2)*x^2 + (3*c^2*d*e^2 - 2*c*d*e^2)*x), x) - a^2/(d^2*e^2*x + c*d*e^2)$$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + b \operatorname{atanh}(c + dx))^2}{(ce + dex)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b*atanh(c + d*x))^2/(c*e + d*e*x)^2,x)`

[Out] `int((a + b*atanh(c + d*x))^2/(c*e + d*e*x)^2, x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a^2}{c^2+2cdx+d^2x^2} dx + \int \frac{b^2 \operatorname{atanh}^2(c+dx)}{c^2+2cdx+d^2x^2} dx + \int \frac{2ab \operatorname{atanh}(c+dx)}{c^2+2cdx+d^2x^2} dx}{e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*atanh(d*x+c))**2/(d*e*x+c*e)**2,x)`

[Out] `(Integral(a**2/(c**2 + 2*c*d*x + d**2*x**2), x) + Integral(b**2*atanh(c + d*x)**2/(c**2 + 2*c*d*x + d**2*x**2), x) + Integral(2*a*b*atanh(c + d*x)/(c**2 + 2*c*d*x + d**2*x**2), x))/e**2`

$$3.20 \quad \int \frac{(a+b \tanh^{-1}(c+dx))^2}{(ce+dex)^3} dx$$

Optimal. Leaf size=119

$$\frac{b(a+b \tanh^{-1}(c+dx))}{de^3(c+dx)} - \frac{(a+b \tanh^{-1}(c+dx))^2}{2de^3(c+dx)^2} + \frac{(a+b \tanh^{-1}(c+dx))^2}{2de^3} + \frac{b^2 \log(c+dx)}{de^3} - \frac{b^2 \log(1-(c+dx))}{2de^3}$$

[Out] $-b*(a+b*\operatorname{arctanh}(d*x+c))/d/e^3/(d*x+c)+1/2*(a+b*\operatorname{arctanh}(d*x+c))^2/d/e^3-1/2*(a+b*\operatorname{arctanh}(d*x+c))^2/d/e^3/(d*x+c)^2+b^2*\ln(d*x+c)/d/e^3-1/2*b^2*\ln(1-(d*x+c)^2)/d/e^3$

Rubi [A] time = 0.17, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {6107, 12, 5916, 5982, 266, 36, 31, 29, 5948}

$$\frac{b(a+b \tanh^{-1}(c+dx))}{de^3(c+dx)} - \frac{(a+b \tanh^{-1}(c+dx))^2}{2de^3(c+dx)^2} + \frac{(a+b \tanh^{-1}(c+dx))^2}{2de^3} + \frac{b^2 \log(c+dx)}{de^3} - \frac{b^2 \log(1-(c+dx))}{2de^3}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(a + b*\operatorname{ArcTanh}[c + d*x])^2/(c*e + d*e*x)^3, x]$

[Out] $-((b*(a + b*\operatorname{ArcTanh}[c + d*x]))/(d*e^3*(c + d*x))) + (a + b*\operatorname{ArcTanh}[c + d*x])^2/(2*d*e^3) - (a + b*\operatorname{ArcTanh}[c + d*x])^2/(2*d*e^3*(c + d*x)^2) + (b^2*\operatorname{Log}[c + d*x])/(d*e^3) - (b^2*\operatorname{Log}[1 - (c + d*x)^2])/(2*d*e^3)$

Rule 12

$\operatorname{Int}[(a_*)(u_), x_Symbol] \rightarrow \operatorname{Dist}[a, \operatorname{Int}[u, x], x] /; \operatorname{FreeQ}[a, x] \ \&\& \ !\operatorname{Match} Q[u, (b_*)(v_)] /; \operatorname{FreeQ}[b, x]$

Rule 29

$\operatorname{Int}[(x_)^{-1}, x_Symbol] \rightarrow \operatorname{Simp}[\operatorname{Log}[x], x]$

Rule 31

$\operatorname{Int}[((a_*) + (b_*)(x_))^{-1}, x_Symbol] \rightarrow \operatorname{Simp}[\operatorname{Log}[\operatorname{RemoveContent}[a + b*x, x]]/b, x] /; \operatorname{FreeQ}[\{a, b\}, x]$

Rule 36

$\operatorname{Int}[1/(((a_*) + (b_*)(x_))*((c_*) + (d_*)(x_))), x_Symbol] \rightarrow \operatorname{Dist}[b/(b*c - a*d), \operatorname{Int}[1/(a + b*x), x], x] - \operatorname{Dist}[d/(b*c - a*d), \operatorname{Int}[1/(c + d*x), x],$

$x] /; \text{FreeQ}\{a, b, c, d\}, x\} \&\& \text{NeQ}[b*c - a*d, 0]$

Rule 266

$\text{Int}[(x_)^{(m_.)}*((a_) + (b_.)*(x_)^{(n_.)})^{(p_.)}, x_Symbol] \rightarrow \text{Dist}[1/n, \text{Subst}[\text{Int}[x^{(\text{Simplify}[(m + 1)/n) - 1}*(a + b*x)^p, x], x, x^n], x] /; \text{FreeQ}\{a, b, m, n, p\}, x\} \&\& \text{IntegerQ}[\text{Simplify}[(m + 1)/n]]$

Rule 5916

$\text{Int}[((a_.) + \text{ArcTanh}[(c_.)*(x_)]*(b_.))^{(p_.)}*((d_.)*(x_))^{(m_.)}, x_Symbol] \rightarrow \text{Simp}[(d*x)^{(m + 1)}*(a + b*\text{ArcTanh}[c*x])^p/(d*(m + 1)), x] - \text{Dist}[(b*c*p)/(d*(m + 1)), \text{Int}[(d*x)^{(m + 1)}*(a + b*\text{ArcTanh}[c*x])^{(p - 1)}]/(1 - c^2*x^2), x], x] /; \text{FreeQ}\{a, b, c, d, m\}, x\} \&\& \text{IGtQ}[p, 0] \&\& (\text{EqQ}[p, 1] \parallel \text{IntegerQ}[m]) \&\& \text{NeQ}[m, -1]$

Rule 5948

$\text{Int}[((a_.) + \text{ArcTanh}[(c_.)*(x_)]*(b_.))^{(p_.)}/((d_) + (e_.)*(x_)^2), x_Symbol] \rightarrow \text{Simp}[(a + b*\text{ArcTanh}[c*x])^{(p + 1)}/(b*c*d*(p + 1)), x] /; \text{FreeQ}\{a, b, c, d, e, p\}, x\} \&\& \text{EqQ}[c^2*d + e, 0] \&\& \text{NeQ}[p, -1]$

Rule 5982

$\text{Int}[(((a_.) + \text{ArcTanh}[(c_.)*(x_)]*(b_.))^{(p_.)}*((f_.)*(x_))^{(m_.)})/((d_) + (e_.)*(x_)^2), x_Symbol] \rightarrow \text{Dist}[1/d, \text{Int}[(f*x)^m*(a + b*\text{ArcTanh}[c*x])^p, x], x] - \text{Dist}[e/(d*f^2), \text{Int}[(f*x)^{(m + 2)}*(a + b*\text{ArcTanh}[c*x])^p]/(d + e*x^2), x], x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x\} \&\& \text{GtQ}[p, 0] \&\& \text{LtQ}[m, -1]$

Rule 6107

$\text{Int}[((a_.) + \text{ArcTanh}[(c_) + (d_.)*(x_)]*(b_.))^{(p_.)}*((e_.) + (f_.)*(x_))^{(m_.)}, x_Symbol] \rightarrow \text{Dist}[1/d, \text{Subst}[\text{Int}[(f*x)/d]^m*(a + b*\text{ArcTanh}[x])^p, x], x, c + d*x], x] /; \text{FreeQ}\{a, b, c, d, e, f, m\}, x\} \&\& \text{EqQ}[d*e - c*f, 0] \&\& \text{IGtQ}[p, 0]$

Rubi steps

$$\begin{aligned}
\int \frac{(a + b \tanh^{-1}(c + dx))^2}{(ce + dex)^3} dx &= \frac{\text{Subst} \left(\int \frac{(a+b \tanh^{-1}(x))^2}{e^3 x^3} dx, x, c + dx \right)}{d} \\
&= \frac{\text{Subst} \left(\int \frac{(a+b \tanh^{-1}(x))^2}{x^3} dx, x, c + dx \right)}{de^3} \\
&= -\frac{(a + b \tanh^{-1}(c + dx))^2}{2de^3(c + dx)^2} + \frac{b \text{Subst} \left(\int \frac{a+b \tanh^{-1}(x)}{x^2(1-x^2)} dx, x, c + dx \right)}{de^3} \\
&= -\frac{(a + b \tanh^{-1}(c + dx))^2}{2de^3(c + dx)^2} + \frac{b \text{Subst} \left(\int \frac{a+b \tanh^{-1}(x)}{x^2} dx, x, c + dx \right)}{de^3} + \frac{b \text{Subst} \left(\int \frac{1}{1-x^2} dx, x, c + dx \right)}{de^3} \\
&= -\frac{b(a + b \tanh^{-1}(c + dx))}{de^3(c + dx)} + \frac{(a + b \tanh^{-1}(c + dx))^2}{2de^3} - \frac{(a + b \tanh^{-1}(c + dx))^2}{2de^3(c + dx)^2} \\
&= -\frac{b(a + b \tanh^{-1}(c + dx))}{de^3(c + dx)} + \frac{(a + b \tanh^{-1}(c + dx))^2}{2de^3} - \frac{(a + b \tanh^{-1}(c + dx))^2}{2de^3(c + dx)^2} \\
&= -\frac{b(a + b \tanh^{-1}(c + dx))}{de^3(c + dx)} + \frac{(a + b \tanh^{-1}(c + dx))^2}{2de^3} - \frac{(a + b \tanh^{-1}(c + dx))^2}{2de^3(c + dx)^2} \\
&= -\frac{b(a + b \tanh^{-1}(c + dx))}{de^3(c + dx)} + \frac{(a + b \tanh^{-1}(c + dx))^2}{2de^3} - \frac{(a + b \tanh^{-1}(c + dx))^2}{2de^3(c + dx)^2}
\end{aligned}$$

Mathematica [A] time = 0.17, size = 136, normalized size = 1.14

$$\frac{-\frac{a^2}{(c+dx)^2} - \frac{2ab}{c+dx} - b(a+b) \log(-c-dx+1) + b(a-b) \log(c+dx+1) - \frac{2b \tanh^{-1}(c+dx)(a+b(c+dx))}{(c+dx)^2} + \frac{b^2(c^2+2cdx+d^2x^2-1)}{(c+dx)^2}}{2de^3}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*ArcTanh[c + d*x])^2/(c*e + d*e*x)^3, x]

[Out]
$$\begin{aligned}
&-(a^2/(c + d*x)^2) - (2*a*b)/(c + d*x) - (2*b*(a + b*(c + d*x))*\text{ArcTanh}[c \\
&+ d*x])/(c + d*x)^2 + (b^2*(-1 + c^2 + 2*c*d*x + d^2*x^2)*\text{ArcTanh}[c + d*x]^2)/ \\
&(c + d*x)^2 - b*(a + b)*\text{Log}[1 - c - d*x] + 2*b^2*\text{Log}[c + d*x] + (a - b)* \\
&b*\text{Log}[1 + c + d*x])/(2*d*e^3)
\end{aligned}$$

fricas [B] time = 0.49, size = 270, normalized size = 2.27

$$8 abdx + 8 abc - (b^2 d^2 x^2 + 2 b^2 cdx + b^2 c^2 - b^2) \log\left(-\frac{dx+c+1}{dx+c-1}\right)^2 + 4 a^2 - 4((ab - b^2)d^2 x^2 + 2(ab - b^2)cdx + ($$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arctanh(d*x+c))^2/(d*e*x+c*e)^3,x, algorithm="fricas")

[Out]
$$-1/8*(8*a*b*d*x + 8*a*b*c - (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - b^2)*\log(-\frac{d*x + c + 1}{d*x + c - 1})^2 + 4*a^2 - 4*((a*b - b^2)*d^2*x^2 + 2*(a*b - b^2)*c*d*x + (a*b - b^2)*c^2)*\log(d*x + c + 1) - 8*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*\log(d*x + c) + 4*((a*b + b^2)*d^2*x^2 + 2*(a*b + b^2)*c*d*x + (a*b + b^2)*c^2)*\log(d*x + c - 1) + 4*(b^2*d*x + b^2*c + a*b)*\log(-\frac{d*x + c + 1}{d*x + c - 1}))/(\frac{d^3*e^3*x^2 + 2*c*d^2*e^3*x + c^2*d*e^3}{d^3})$$

giac [B] time = 0.18, size = 356, normalized size = 2.99

$$\left(\frac{(dx+c+1)b^2 \log\left(-\frac{dx+c+1}{dx+c-1}\right)^2}{dx+c-1} + \frac{2(dx+c+1)^2 b^2 \log\left(-\frac{dx+c+1}{dx+c-1}-1\right)}{(dx+c-1)^2} + \frac{4(dx+c+1)b^2 \log\left(-\frac{dx+c+1}{dx+c-1}-1\right)}{dx+c-1} + 2b^2 \log\left(-\frac{dx+c+1}{dx+c-1}-1\right) + \frac{4(dx+c+1)}{d}\right) \frac{4}{\left(\frac{(dx+c+1)^2 d^2 e^3}{(dx+c-1)^2} + \dots\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arctanh(d*x+c))^2/(d*e*x+c*e)^3,x, algorithm="giac")

[Out]
$$1/4*((d*x + c + 1)*b^2*\log(-\frac{d*x + c + 1}{d*x + c - 1})^2/(d*x + c - 1) + 2*(d*x + c + 1)^2*b^2*\log(-\frac{d*x + c + 1}{d*x + c - 1} - 1)/(d*x + c - 1)^2 + 4*(d*x + c + 1)*b^2*\log(-\frac{d*x + c + 1}{d*x + c - 1} - 1)/(d*x + c - 1) + 2*b^2*\log(-\frac{d*x + c + 1}{d*x + c - 1} - 1) + 4*(d*x + c + 1)*a*b*\log(-\frac{d*x + c + 1}{d*x + c - 1})/(d*x + c - 1) - 2*(d*x + c + 1)^2*b^2*\log(-\frac{d*x + c + 1}{d*x + c - 1})/(d*x + c - 1)^2 - 2*(d*x + c + 1)*b^2*\log(-\frac{d*x + c + 1}{d*x + c - 1})/(d*x + c - 1) + 4*(d*x + c + 1)*a^2/(d*x + c - 1) + 4*(d*x + c + 1)*a*b/(d*x + c - 1) + 4*a*b)*((c + 1)*d - (c - 1)*d)/((d*x + c + 1)^2*d^2*e^3/(d*x + c - 1)^2 + 2*(d*x + c + 1)*d^2*e^3/(d*x + c - 1) + d^2*e^3)$$

maple [B] time = 0.08, size = 371, normalized size = 3.12

$$\frac{a^2}{2d e^3 (dx + c)^2} - \frac{b^2 \operatorname{arctanh}(dx + c)^2}{2d e^3 (dx + c)^2} - \frac{b^2 \operatorname{arctanh}(dx + c)}{d e^3 (dx + c)} - \frac{b^2 \operatorname{arctanh}(dx + c) \ln(dx + c - 1)}{2d e^3} + \frac{b^2 \operatorname{arctanh}(dx + c)}{d e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*arctanh(d*x+c))^2/(d*e*x+c*e)^3,x)

[Out]
$$-1/2/d*a^2/e^3/(d*x+c)^2-1/2/d*b^2/e^3/(d*x+c)^2*arctanh(d*x+c)^2-1/d*b^2/e^3*arctanh(d*x+c)/(d*x+c)-1/2/d*b^2/e^3*arctanh(d*x+c)*ln(d*x+c-1)+1/2/d*b^2/e^3*arctanh(d*x+c)*ln(d*x+c+1)-1/8/d*b^2/e^3*ln(d*x+c-1)^2+1/4/d*b^2/e^3*ln(d*x+c-1)*ln(1/2+1/2*d*x+1/2*c)+b^2*ln(d*x+c)/d/e^3-1/2/d*b^2/e^3*ln(d*x+c-1)-1/2/d*b^2/e^3*ln(d*x+c+1)-1/8/d*b^2/e^3*ln(d*x+c+1)^2+1/4/d*b^2/e^3*ln(-1/2*d*x-1/2*c+1/2)*ln(d*x+c+1)-1/4/d*b^2/e^3*ln(-1/2*d*x-1/2*c+1/2)*ln(1/2+1/2*d*x+1/2*c)-1/d*a*b/e^3/(d*x+c)^2*arctanh(d*x+c)-1/d*a*b/e^3/(d*x+c)-1/2/d*a*b/e^3*ln(d*x+c-1)+1/2/d*a*b/e^3*ln(d*x+c+1)$$

maxima [B] time = 0.36, size = 329, normalized size = 2.76

$$-\frac{1}{2} \left(d \left(\frac{2}{d^3 e^3 x + c d^2 e^3} - \frac{\log(dx + c + 1)}{d^2 e^3} + \frac{\log(dx + c - 1)}{d^2 e^3} \right) + \frac{2 \operatorname{artanh}(dx + c)}{d^3 e^3 x^2 + 2 c d^2 e^3 x + c^2 d e^3} \right) a b - \frac{1}{8} \left(d^2 \left(\frac{\log(dx + c + 1)}{d^3 e^3} - \frac{\log(dx + c - 1)}{d^3 e^3} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arctanh(d*x+c))^2/(d*e*x+c*e)^3,x, algorithm="maxima")

[Out]
$$-1/2*(d*(2/(d^3*e^3*x + c*d^2*e^3) - \log(d*x + c + 1)/(d^2*e^3) + \log(d*x + c - 1)/(d^2*e^3)) + 2*arctanh(d*x + c)/(d^3*e^3*x^2 + 2*c*d^2*e^3*x + c^2*d*e^3))*a*b - 1/8*(d^2*((\log(d*x + c + 1))^2 - 2*\log(d*x + c + 1)*\log(d*x + c - 1) + \log(d*x + c - 1)^2 + 4*\log(d*x + c - 1))/(d^3*e^3) + 4*\log(d*x + c + 1)/(d^3*e^3) - 8*\log(d*x + c)/(d^3*e^3)) + 4*d*(2/(d^3*e^3*x + c*d^2*e^3) - \log(d*x + c + 1)/(d^2*e^3) + \log(d*x + c - 1)/(d^2*e^3))*arctanh(d*x + c))*b^2 - 1/2*b^2*arctanh(d*x + c)^2/(d^3*e^3*x^2 + 2*c*d^2*e^3*x + c^2*d*e^3) - 1/2*a^2/(d^3*e^3*x^2 + 2*c*d^2*e^3*x + c^2*d*e^3)$$

mupad [B] time = 2.42, size = 776, normalized size = 6.52

$$\ln(1 - dx - c)^2 \left(\frac{b^2}{8 d e^3} - \frac{b^2}{2 d (4 c^2 e^3 + 8 c d e^3 x + 4 d^2 e^3 x^2)} \right) + \ln(c + dx + 1)^2 \left(\frac{b^2}{8 d e^3} - \frac{b^2}{8 d^2 e^3 (2 c x + d x^2 + c^2)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*atanh(c + d*x))^2/(c*e + d*e*x)^3,x)

[Out]
$$\log(1 - d*x - c)^2*(b^2/(8*d*e^3) - b^2/(2*d*(4*c^2*e^3 + 4*d^2*e^3*x^2 + 8*c*d*e^3*x))) + \log(c + d*x + 1)^2*(b^2/(8*d*e^3) - b^2/(8*d^2*e^3*(2*c*x + d*x^2 + c^2/d))) + \log(1 - d*x - c)*(\log(c + d*x + 1)*(b^2/(2*d*(2*c^2*e^3 + 2*d^2*e^3*x^2 + 4*c*d*e^3*x)) - (b^2*(c^2 + d^2*x^2 + 2*c*d*x))/(2*d*(2*c^2*e^3 + 2*d^2*e^3*x^2 + 4*c*d*e^3*x))) + b^2/(2*d*(4*c^2*e^3 + 4*d^2*e^3*x^2 + 8*c*d*e^3*x))) + \log(c + d*x + 1)*(\log(c + d*x + 1)*(b^2/(2*d*(2*c^2*e^3 + 2*d^2*e^3*x^2 + 4*c*d*e^3*x)) - (b^2*(c^2 + d^2*x^2 + 2*c*d*x))/(2*d*(2*c^2*e^3 + 2*d^2*e^3*x^2 + 4*c*d*e^3*x))) + b^2/(2*d*(4*c^2*e^3 + 4*d^2*e^3*x^2 + 8*c*d*e^3*x)))$$

$$x^2 + 8cd^3e^3x) + (b(4a - b))/(2d(4c^2e^3 + 4d^2e^3x^2 + 8cd^3e^3x)) - (b^2(x(4cd - d + d(2c - 1)) - c + c^2 + c(2c - 1) + 3d^2x^2 + 1))/(2d(4c^2e^3 + 4d^2e^3x^2 + 8cd^3e^3x)) + (b^2(x(2d^3e^3 + d(4c^2e^3 + 2e^3) + 8cd^3e^3) + 2c^2e^3 + 2e^3 + c(4c^2e^3 + 2e^3) + 2c^2e^3 + 6d^2e^3x^2))/(4d^3e^3(4c^2e^3 + 4d^2e^3x^2 + 8cd^3e^3x)) - ((a^2 + 2abc)/(2d) + abx)/(c^2e^3 + d^2e^3x^2 + 2cd^3e^3x) - (\log(c + dx + 1)(x((2b^2c + b^2)/(4d^3e^3) + (b^2c)/(4d^3e^3) - (b^2(3c - 1))/(4d^3e^3)) + (4ab + b^2c + b^2 + b^2c^2)/(8d^2e^3) - (b^2((c^2 - c + 1)/(2d) + (c(2c - 1))/(2d)))/(4d^3e^3) + (c(2b^2c + b^2))/(8d^2e^3)))/(2cx + dx^2 + c^2/d) + (b^2 \log(c + dx))/(d^3e^3) - (\log(c + dx - 1)(ab + b^2))/(2d^3e^3) + (\log(c + dx + 1)(ab - b^2))/(2d^3e^3)$$

sympy [A] time = 4.31, size = 1102, normalized size = 9.26

$$\left\{ \begin{array}{l} -\frac{a^2}{2c^2de^3+4cd^2e^3x+2d^3e^3x^2} + \frac{2abc^2 \operatorname{atanh}(c+dx)}{2c^2de^3+4cd^2e^3x+2d^3e^3x^2} + \frac{4abcdx \operatorname{atanh}(c+dx)}{2c^2de^3+4cd^2e^3x+2d^3e^3x^2} - \frac{2abc}{2c^2de^3+4cd^2e^3x+2d^3e^3x^2} + \frac{2abd^2x^2 \operatorname{atanh}(c+dx)}{2c^2de^3+4cd^2e^3x+2d^3e^3x^2} \\ \frac{x(a+b \operatorname{atanh}(c))^2}{c^3e^3} \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*atanh(d*x+c))**2/(d*e*x+c*e)**3,x)

[Out] Piecewise((-a**2/(2*c**2*d*e**3 + 4*c*d**2*e**3*x + 2*d**3*e**3*x**2) + 2*a*b*c**2*atanh(c + d*x)/(2*c**2*d*e**3 + 4*c*d**2*e**3*x + 2*d**3*e**3*x**2) + 4*a*b*c*d*x*atanh(c + d*x)/(2*c**2*d*e**3 + 4*c*d**2*e**3*x + 2*d**3*e**3*x**2) - 2*a*b*c/(2*c**2*d*e**3 + 4*c*d**2*e**3*x + 2*d**3*e**3*x**2) + 2*a*b*d**2*x**2*atanh(c + d*x)/(2*c**2*d*e**3 + 4*c*d**2*e**3*x + 2*d**3*e**3*x**2) - 2*a*b*d*x/(2*c**2*d*e**3 + 4*c*d**2*e**3*x + 2*d**3*e**3*x**2) - 2*a*b*atanh(c + d*x)/(2*c**2*d*e**3 + 4*c*d**2*e**3*x + 2*d**3*e**3*x**2) + 2*b**2*c**2*log(c/d + x)/(2*c**2*d*e**3 + 4*c*d**2*e**3*x + 2*d**3*e**3*x**2) - 2*b**2*c**2*log(c/d + x + 1/d)/(2*c**2*d*e**3 + 4*c*d**2*e**3*x + 2*d**3*e**3*x**2) + b**2*c**2*atanh(c + d*x)**2/(2*c**2*d*e**3 + 4*c*d**2*e**3*x + 2*d**3*e**3*x**2) + 2*b**2*c**2*atanh(c + d*x)/(2*c**2*d*e**3 + 4*c*d**2*e**3*x + 2*d**3*e**3*x**2) + 4*b**2*c*d*x*log(c/d + x)/(2*c**2*d*e**3 + 4*c*d**2*e**3*x + 2*d**3*e**3*x**2) - 4*b**2*c*d*x*log(c/d + x + 1/d)/(2*c**2*d*e**3 + 4*c*d**2*e**3*x + 2*d**3*e**3*x**2) + 2*b**2*c*d*x*atanh(c + d*x)**2/(2*c**2*d*e**3 + 4*c*d**2*e**3*x + 2*d**3*e**3*x**2) + 4*b**2*c*d*x*atanh(c + d*x)/(2*c**2*d*e**3 + 4*c*d**2*e**3*x + 2*d**3*e**3*x**2) - 2*b**2*c*atanh(c + d*x)/(2*c**2*d*e**3 + 4*c*d**2*e**3*x + 2*d**3*e**3*x**2) + 2*b**2*d**2*x**2*log(c/d + x)/(2*c**2*d*e**3 + 4*c*d**2*e**3*x + 2*d**3*e**3*x**2) - 2*b**2*d**2*x**2*log(c/d + x + 1/d)/(2*c**2*d*e**3 + 4*c*d**2*e**3*x + 2*d**3*e**3*x**2) + b**2*d**2*x**2*atanh(c + d*x)**2/(2*c**2*d*e**3 + 4*c*d**2*e**3*x + 2*d**3*e**3*x**2) + 2*b**2*d**2*x**2*atanh(c + d*x)/(2*c**2

```

*d**3 + 4*c*d**2*e**3*x + 2*d**3*e**3*x**2) - 2*b**2*d*x*atanh(c + d*x)/(
2*c**2*d*e**3 + 4*c*d**2*e**3*x + 2*d**3*e**3*x**2) - b**2*atanh(c + d*x)**
2/(2*c**2*d*e**3 + 4*c*d**2*e**3*x + 2*d**3*e**3*x**2), Ne(d, 0)), (x*(a +
b*atanh(c))**2/(c**3*e**3), True))

```

$$3.21 \quad \int \frac{(a+b \tanh^{-1}(c+dx))^2}{(ce+dex)^4} dx$$

Optimal. Leaf size=180

$$\frac{b(a+b \tanh^{-1}(c+dx))}{3de^4(c+dx)^2} - \frac{(a+b \tanh^{-1}(c+dx))^2}{3de^4(c+dx)^3} + \frac{(a+b \tanh^{-1}(c+dx))^2}{3de^4} + \frac{2b \log\left(2 - \frac{2}{c+dx+1}\right)(a+b \tanh^{-1}(c+dx))^2}{3de^4}$$

[Out] $-1/3*b^2/d/e^4/(d*x+c)+1/3*b^2*\arctanh(d*x+c)/d/e^4-1/3*b*(a+b*\arctanh(d*x+c))/d/e^4/(d*x+c)^2+1/3*(a+b*\arctanh(d*x+c))^2/d/e^4-1/3*(a+b*\arctanh(d*x+c))^2/d/e^4/(d*x+c)^3+2/3*b*(a+b*\arctanh(d*x+c))*\ln(2-2/(d*x+c+1))/d/e^4-1/3*b^2*\text{polylog}(2,-1+2/(d*x+c+1))/d/e^4$

Rubi [A] time = 0.27, antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {6107, 12, 5916, 5982, 325, 206, 5988, 5932, 2447}

$$\frac{b^2 \text{PolyLog}\left(2, \frac{2}{c+dx+1} - 1\right)}{3de^4} - \frac{b(a+b \tanh^{-1}(c+dx))}{3de^4(c+dx)^2} - \frac{(a+b \tanh^{-1}(c+dx))^2}{3de^4(c+dx)^3} + \frac{(a+b \tanh^{-1}(c+dx))^2}{3de^4} + \frac{2b \log\left(2 - \frac{2}{c+dx+1}\right)(a+b \tanh^{-1}(c+dx))^2}{3de^4}$$

Antiderivative was successfully verified.

[In] Int[(a + b*ArcTanh[c + d*x])^2/(c*e + d*e*x)^4, x]

[Out] $-b^2/(3*d*e^4*(c+d*x)) + (b^2*ArcTanh[c+d*x])/(3*d*e^4) - (b*(a+b*ArcTanh[c+d*x]))/(3*d*e^4*(c+d*x)^2) + (a+b*ArcTanh[c+d*x])^2/(3*d*e^4) - (a+b*ArcTanh[c+d*x])^2/(3*d*e^4*(c+d*x)^3) + (2*b*(a+b*ArcTanh[c+d*x])*Log[2-2/(1+c+d*x)])/(3*d*e^4) - (b^2*PolyLog[2,-1+2/(1+c+d*x)])/(3*d*e^4)$

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[((c*x)^(m+1)*(a + b*x^n)^(p+1))/(a*c*(m+1)), x] - Dist[(b*(m+n*(p+1)+1))/(a*c^n*(m+1)), Int[(c*x)^(m+n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2447

```
Int[Log[u]*(Pq_)^(m_.), x_Symbol] := With[{C = FullSimplify[(Pq^m*(1-u))/D[u, x]]}, Simp[C*PolyLog[2, 1-u], x] /; FreeQ[C, x] /; IntegerQ[m] && PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u, x][[2]], Expon[Pq, x]]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol] := Simp[((d*x)^(m+1)*(a + b*ArcTanh[c*x])^p)/(d*(m+1)), x] - Dist[(b*c*p)/(d*(m+1)), Int[((d*x)^(m+1)*(a + b*ArcTanh[c*x])^(p-1))/(1-c^2*x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || IntegerQ[m]) && NeQ[m, -1]
```

Rule 5932

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_Symbol] := Simp[((a + b*ArcTanh[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] - Dist[(b*c*p)/d, Int[((a + b*ArcTanh[c*x])^(p-1)*Log[2 - 2/(1 + (e*x)/d)])/((1 - c^2*x^2), x), x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0]
```

Rule 5982

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)/((d_) + (e_.)*(x_)^2), x_Symbol] := Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x], x] - Dist[e/(d*f^2), Int[((f*x)^(m+2)*(a + b*ArcTanh[c*x])^p)/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5988

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)), x_Symbol] := Simp[(a + b*ArcTanh[c*x])^(p+1)/(b*d*(p+1)), x] + Dist[1/d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 6107


```
Int[((a_.) + ArcTanh[(c_) + (d_.)*(x_)])*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] := Dist[1/d, Subst[Int[((f*x)/d)^m*(a + b*ArcTanh[x])^p, x]
, x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] &
& IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a + b \tanh^{-1}(c + dx))^2}{(ce + dex)^4} dx &= \frac{\text{Subst}\left(\int \frac{(a+b \tanh^{-1}(x))^2}{e^4 x^4} dx, x, c + dx\right)}{d} \\
&= \frac{\text{Subst}\left(\int \frac{(a+b \tanh^{-1}(x))^2}{x^4} dx, x, c + dx\right)}{de^4} \\
&= -\frac{(a + b \tanh^{-1}(c + dx))^2}{3de^4(c + dx)^3} + \frac{(2b) \text{Subst}\left(\int \frac{a+b \tanh^{-1}(x)}{x^3(1-x^2)} dx, x, c + dx\right)}{3de^4} \\
&= -\frac{(a + b \tanh^{-1}(c + dx))^2}{3de^4(c + dx)^3} + \frac{(2b) \text{Subst}\left(\int \frac{a+b \tanh^{-1}(x)}{x^3} dx, x, c + dx\right)}{3de^4} + \frac{(2b) \text{Subst}\left(\int \frac{a+b \tanh^{-1}(x)}{x^3(1-x^2)} dx, x, c + dx\right)}{3de^4} \\
&= -\frac{b(a + b \tanh^{-1}(c + dx))}{3de^4(c + dx)^2} + \frac{(a + b \tanh^{-1}(c + dx))^2}{3de^4} - \frac{(a + b \tanh^{-1}(c + dx))^2}{3de^4(c + dx)^3} \\
&= -\frac{b^2}{3de^4(c + dx)} - \frac{b(a + b \tanh^{-1}(c + dx))}{3de^4(c + dx)^2} + \frac{(a + b \tanh^{-1}(c + dx))^2}{3de^4} - \frac{(a + b \tanh^{-1}(c + dx))^2}{3de^4(c + dx)^3} \\
&= -\frac{b^2}{3de^4(c + dx)} + \frac{b^2 \tanh^{-1}(c + dx)}{3de^4} - \frac{b(a + b \tanh^{-1}(c + dx))}{3de^4(c + dx)^2} + \frac{(a + b \tanh^{-1}(c + dx))^2}{3de^4}
\end{aligned}$$

Mathematica [A] time = 0.54, size = 218, normalized size = 1.21

$$a^2 - ab \left((c + dx) \left(c^2 + 2cdx + 2(c + dx)^2 \log\left(\frac{c+dx}{\sqrt{1-(c+dx)^2}}\right) + d^2x^2 - 1 \right) - 2 \tanh^{-1}(c + dx) \right) + b^2 \left((c + dx)^3 \text{Li}_2\left(\frac{c+dx}{\sqrt{1-(c+dx)^2}}\right) \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b*ArcTanh[c + d*x])^2/(c*e + d*e*x)^4,x]

[Out] $-1/3*(a^2 - a*b*(-2*\text{ArcTanh}[c + d*x] + (c + d*x)*(-1 + c^2 + 2*c*d*x + d^2*x^2 + 2*(c + d*x)^2*\text{Log}[(c + d*x)/\text{Sqrt}[1 - (c + d*x)^2]])) + b^2*((c + d*x)^2 + (c + d*x)^2*\text{ArcTanh}[c + d*x]^2 + (1 - (c + d*x)^2)*\text{ArcTanh}[c + d*x]^2 + (c + d*x)*\text{ArcTanh}[c + d*x]*(1 - (c + d*x)^2 - (c + d*x)^2*\text{ArcTanh}[c + d*x]) - 2*(c + d*x)^2*\text{Log}[1 - E^{(-2*\text{ArcTanh}[c + d*x])}]) + (c + d*x)^3*\text{PolyLog}[2, E^{(-2*\text{ArcTanh}[c + d*x])}])]/(d*e^4*(c + d*x)^3)$

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^2 \operatorname{artanh}(dx + c)^2 + 2ab \operatorname{artanh}(dx + c) + a^2}{d^4 e^4 x^4 + 4cd^3 e^4 x^3 + 6c^2 d^2 e^4 x^2 + 4c^3 d e^4 x + c^4 e^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arctanh(d*x+c))^2/(d*e*x+c*e)^4,x, algorithm="fricas")`

[Out] `integral((b^2*arctanh(d*x + c)^2 + 2*a*b*arctanh(d*x + c) + a^2)/(d^4*e^4*x^4 + 4*c*d^3*e^4*x^3 + 6*c^2*d^2*e^4*x^2 + 4*c^3*d*e^4*x + c^4*e^4), x)`

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{artanh}(dx + c) + a)^2}{(dex + ce)^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arctanh(d*x+c))^2/(d*e*x+c*e)^4,x, algorithm="giac")`

[Out] `integrate((b*arctanh(d*x + c) + a)^2/(d*e*x + c*e)^4, x)`

maple [B] time = 0.08, size = 492, normalized size = 2.73

$$\frac{a^2}{3d e^4 (dx + c)^3} - \frac{b^2 \operatorname{arctanh}(dx + c)^2}{3d e^4 (dx + c)^3} - \frac{b^2 \operatorname{arctanh}(dx + c)}{3d e^4 (dx + c)^2} + \frac{2b^2 \ln(dx + c) \operatorname{arctanh}(dx + c)}{3d e^4} - \frac{b^2 \operatorname{arctanh}(dx + c)}{3d e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b*arctanh(d*x+c))^2/(d*e*x+c*e)^4,x)`

[Out] $-1/3/d*a^2/e^4/(d*x+c)^3-1/3/d*b^2/e^4/(d*x+c)^3*\operatorname{arctanh}(d*x+c)^2-1/3/d*b^2/e^4*\operatorname{arctanh}(d*x+c)/(d*x+c)^2+2/3/d*b^2/e^4*\ln(d*x+c)*\operatorname{arctanh}(d*x+c)-1/3/d*b^2/e^4*\operatorname{arctanh}(d*x+c)*\ln(d*x+c-1)-1/3/d*b^2/e^4*\operatorname{arctanh}(d*x+c)*\ln(d*x+c+1)-1/3*b^2/d/e^4/(d*x+c)-1/6/d*b^2/e^4*\ln(d*x+c-1)+1/6/d*b^2/e^4*\ln(d*x+c+1)-1/12/d*b^2/e^4*\ln(d*x+c-1)^2+1/3/d*b^2/e^4*\operatorname{dilog}(1/2+1/2*d*x+1/2*c)+1/6/d*b^2/e^4*\ln(d*x+c-1)*\ln(1/2+1/2*d*x+1/2*c)+1/12/d*b^2/e^4*\ln(d*x+c+1)^2-1/6/d*b^2/e^4*\ln(-1/2*d*x-1/2*c+1/2)*\ln(d*x+c+1)+1/6/d*b^2/e^4*\ln(-1/2*d*x-1/2*c$

$+1/2) \cdot \ln(1/2 + 1/2 \cdot dx + 1/2 \cdot c) - 1/3 \cdot d \cdot b^2 / e^4 \cdot \operatorname{dilog}(dx + c) - 1/3 \cdot d \cdot b^2 / e^4 \cdot \operatorname{dilog}(dx + c + 1) - 1/3 \cdot d \cdot b^2 / e^4 \cdot \ln(dx + c) \cdot \ln(dx + c + 1) - 2/3 \cdot d \cdot a \cdot b / e^4 \cdot (dx + c)^3 \cdot \operatorname{arctanh}(dx + c) - 1/3 \cdot d \cdot a \cdot b / e^4 \cdot (dx + c)^2 + 2/3 \cdot d \cdot a \cdot b / e^4 \cdot \ln(dx + c) - 1/3 \cdot d \cdot a \cdot b / e^4 \cdot \ln(dx + c - 1) - 1/3 \cdot d \cdot a \cdot b / e^4 \cdot \ln(dx + c + 1)$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{3} \left(d \left(\frac{1}{d^4 e^4 x^2 + 2 c d^3 e^4 x + c^2 d^2 e^4} + \frac{\log(dx + c + 1)}{d^2 e^4} - \frac{2 \log(dx + c)}{d^2 e^4} + \frac{\log(dx + c - 1)}{d^2 e^4} \right) + \frac{2 \operatorname{arctanh}(dx + c)}{d^4 e^4 x^3 + 3 c d^3 e^4 x^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arctanh(dx+c))^2/(d*e*x+c*e)^4,x, algorithm="maxima")

[Out] $-1/3 \cdot (d \cdot (1 / (d^4 \cdot e^4 \cdot x^2 + 2 \cdot c \cdot d^3 \cdot e^4 \cdot x + c^2 \cdot d^2 \cdot e^4) + \log(dx + c + 1) / (d^2 \cdot e^4) - 2 \cdot \log(dx + c) / (d^2 \cdot e^4) + \log(dx + c - 1) / (d^2 \cdot e^4)) + 2 \cdot \operatorname{arctanh}(dx + c) / (d^4 \cdot e^4 \cdot x^3 + 3 \cdot c \cdot d^3 \cdot e^4 \cdot x^2 + 3 \cdot c^2 \cdot d^2 \cdot e^4 \cdot x + c^3 \cdot d \cdot e^4)) \cdot a \cdot b - 1/12 \cdot b^2 \cdot (\log(-dx - c + 1))^2 / (d^4 \cdot e^4 \cdot x^3 + 3 \cdot c \cdot d^3 \cdot e^4 \cdot x^2 + 3 \cdot c^2 \cdot d^2 \cdot e^4 \cdot x + c^3 \cdot d \cdot e^4) + 3 \cdot \operatorname{integrate}(-1/3 \cdot (3 \cdot (dx + c - 1) \cdot \log(dx + c + 1)^2 + 2 \cdot (dx - 3 \cdot (dx + c - 1) \cdot \log(dx + c + 1) + c) \cdot \log(-dx - c + 1)) / (d^5 \cdot e^4 \cdot x^5 + c^5 \cdot e^4 - c^4 \cdot e^4 + (5 \cdot c \cdot d^4 \cdot e^4 - d^4 \cdot e^4) \cdot x^4 + 2 \cdot (5 \cdot c^2 \cdot d^3 \cdot e^4 - 2 \cdot c \cdot d^3 \cdot e^4) \cdot x^3 + 2 \cdot (5 \cdot c^3 \cdot d^2 \cdot e^4 - 3 \cdot c^2 \cdot d^2 \cdot e^4) \cdot x^2 + (5 \cdot c^4 \cdot d \cdot e^4 - 4 \cdot c^3 \cdot d \cdot e^4) \cdot x), x) - 1/3 \cdot a^2 / (d^4 \cdot e^4 \cdot x^3 + 3 \cdot c \cdot d^3 \cdot e^4 \cdot x^2 + 3 \cdot c^2 \cdot d^2 \cdot e^4 \cdot x + c^3 \cdot d \cdot e^4)$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + b \operatorname{atanh}(c + dx))^2}{(ce + dex)^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*atanh(c + dx))^2/(c*e + d*e*x)^4,x)

[Out] int((a + b*atanh(c + dx))^2/(c*e + d*e*x)^4, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a^2}{c^4 + 4c^3 dx + 6c^2 d^2 x^2 + 4cd^3 x^3 + d^4 x^4} dx + \int \frac{b^2 \operatorname{atanh}^2(c + dx)}{c^4 + 4c^3 dx + 6c^2 d^2 x^2 + 4cd^3 x^3 + d^4 x^4} dx + \int \frac{2ab \operatorname{atanh}(c + dx)}{c^4 + 4c^3 dx + 6c^2 d^2 x^2 + 4cd^3 x^3 + d^4 x^4} dx}{e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*atanh(dx+c))**2/(d*e*x+c*e)**4,x)

```
[Out] (Integral(a**2/(c**4 + 4*c**3*d*x + 6*c**2*d**2*x**2 + 4*c*d**3*x**3 + d**4*x**4), x) + Integral(b**2*atanh(c + d*x)**2/(c**4 + 4*c**3*d*x + 6*c**2*d**2*x**2 + 4*c*d**3*x**3 + d**4*x**4), x) + Integral(2*a*b*atanh(c + d*x)/(c**4 + 4*c**3*d*x + 6*c**2*d**2*x**2 + 4*c*d**3*x**3 + d**4*x**4), x))/e**4
```

$$3.22 \quad \int \frac{(a+b \tanh^{-1}(c+dx))^2}{(ce+dex)^5} dx$$

Optimal. Leaf size=172

$$\frac{b(a+b \tanh^{-1}(c+dx))}{2de^5(c+dx)} - \frac{b(a+b \tanh^{-1}(c+dx))}{6de^5(c+dx)^3} - \frac{(a+b \tanh^{-1}(c+dx))^2}{4de^5(c+dx)^4} + \frac{(a+b \tanh^{-1}(c+dx))^2}{4de^5} - \frac{(a+b \tanh^{-1}(c+dx))^2}{12de^5}$$

[Out] $-1/12*b^2/d/e^5/(d*x+c)^2-1/6*b*(a+b*\operatorname{arctanh}(d*x+c))/d/e^5/(d*x+c)^3-1/2*b*(a+b*\operatorname{arctanh}(d*x+c))/d/e^5/(d*x+c)+1/4*(a+b*\operatorname{arctanh}(d*x+c))^2/d/e^5-1/4*(a+b*\operatorname{arctanh}(d*x+c))^2/d/e^5/(d*x+c)^4+2/3*b^2*\ln(d*x+c)/d/e^5-1/3*b^2*\ln(1-(d*x+c)^2)/d/e^5$

Rubi [A] time = 0.26, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {6107, 12, 5916, 5982, 266, 44, 36, 31, 29, 5948}

$$\frac{b(a+b \tanh^{-1}(c+dx))}{2de^5(c+dx)} - \frac{b(a+b \tanh^{-1}(c+dx))}{6de^5(c+dx)^3} - \frac{(a+b \tanh^{-1}(c+dx))^2}{4de^5(c+dx)^4} + \frac{(a+b \tanh^{-1}(c+dx))^2}{4de^5} - \frac{(a+b \tanh^{-1}(c+dx))^2}{12de^5}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(a + b*\operatorname{ArcTanh}[c + d*x])^2/(c*e + d*e*x)^5, x]$

[Out] $-b^2/(12*d*e^5*(c + d*x)^2) - (b*(a + b*\operatorname{ArcTanh}[c + d*x]))/(6*d*e^5*(c + d*x)^3) - (b*(a + b*\operatorname{ArcTanh}[c + d*x]))/(2*d*e^5*(c + d*x)) + (a + b*\operatorname{ArcTanh}[c + d*x])^2/(4*d*e^5) - (a + b*\operatorname{ArcTanh}[c + d*x])^2/(4*d*e^5*(c + d*x)^4) + (2*b^2*\operatorname{Log}[c + d*x])/(3*d*e^5) - (b^2*\operatorname{Log}[1 - (c + d*x)^2])/(3*d*e^5)$

Rule 12

$\operatorname{Int}[(a_*)(u_), x_Symbol] \rightarrow \operatorname{Dist}[a, \operatorname{Int}[u, x], x] /; \operatorname{FreeQ}[a, x] \&\& \operatorname{!MatchQ}[u, (b_*)(v_)] /; \operatorname{FreeQ}[b, x]$

Rule 29

$\operatorname{Int}[(x_)^{-1}, x_Symbol] \rightarrow \operatorname{Simp}[\operatorname{Log}[x], x]$

Rule 31

$\operatorname{Int}[(a_*) + (b_*)(x_)^{-1}, x_Symbol] \rightarrow \operatorname{Simp}[\operatorname{Log}[\operatorname{RemoveContent}[a + b*x, x]]/b, x] /; \operatorname{FreeQ}[\{a, b\}, x]$

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] := Dist[b/(b*c
- a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:= Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] := Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 5982

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] := Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x]
, x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcTanh[c*x])^p)/(d + e*x^
2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 6107

```
Int[((a_.) + ArcTanh[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] := Dist[1/d, Subst[Int[((f*x)/d)^m*(a + b*ArcTanh[x])^p, x]
, x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] &
& IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a + b \tanh^{-1}(c + dx))^2}{(ce + dex)^5} dx &= \frac{\text{Subst} \left(\int \frac{(a+b \tanh^{-1}(x))^2}{e^5 x^5} dx, x, c + dx \right)}{d} \\
&= \frac{\text{Subst} \left(\int \frac{(a+b \tanh^{-1}(x))^2}{x^5} dx, x, c + dx \right)}{de^5} \\
&= -\frac{(a + b \tanh^{-1}(c + dx))^2}{4de^5(c + dx)^4} + \frac{b \text{Subst} \left(\int \frac{a+b \tanh^{-1}(x)}{x^4(1-x^2)} dx, x, c + dx \right)}{2de^5} \\
&= -\frac{(a + b \tanh^{-1}(c + dx))^2}{4de^5(c + dx)^4} + \frac{b \text{Subst} \left(\int \frac{a+b \tanh^{-1}(x)}{x^4} dx, x, c + dx \right)}{2de^5} + \frac{b \text{Subst} \left(\int \frac{a+b \tanh^{-1}(x)}{x^2} dx, x, c + dx \right)}{2de^5} \\
&= -\frac{b(a + b \tanh^{-1}(c + dx))}{6de^5(c + dx)^3} - \frac{(a + b \tanh^{-1}(c + dx))^2}{4de^5(c + dx)^4} + \frac{b \text{Subst} \left(\int \frac{a+b \tanh^{-1}(x)}{x^2} dx, x, c + dx \right)}{2de^5} \\
&= -\frac{b(a + b \tanh^{-1}(c + dx))}{6de^5(c + dx)^3} - \frac{b(a + b \tanh^{-1}(c + dx))}{2de^5(c + dx)} + \frac{(a + b \tanh^{-1}(c + dx))}{4de^5} \\
&= -\frac{b(a + b \tanh^{-1}(c + dx))}{6de^5(c + dx)^3} - \frac{b(a + b \tanh^{-1}(c + dx))}{2de^5(c + dx)} + \frac{(a + b \tanh^{-1}(c + dx))}{4de^5} \\
&= -\frac{b^2}{12de^5(c + dx)^2} - \frac{b(a + b \tanh^{-1}(c + dx))}{6de^5(c + dx)^3} - \frac{b(a + b \tanh^{-1}(c + dx))}{2de^5(c + dx)} + \frac{(a + b \tanh^{-1}(c + dx))}{4de^5} \\
&= -\frac{b^2}{12de^5(c + dx)^2} - \frac{b(a + b \tanh^{-1}(c + dx))}{6de^5(c + dx)^3} - \frac{b(a + b \tanh^{-1}(c + dx))}{2de^5(c + dx)} + \frac{(a + b \tanh^{-1}(c + dx))}{4de^5}
\end{aligned}$$

Mathematica [A] time = 0.26, size = 218, normalized size = 1.27

$$\frac{\frac{3a^2}{(c+dx)^4} + \frac{2b \tanh^{-1}(c+dx)(3a+b(3c^3+9c^2dx+9cd^2x^2+c+3d^3x^3+dx))}{(c+dx)^4}}{12de^5} + \frac{6ab}{c+dx} + \frac{2ab}{(c+dx)^3} + b(3a+4b) \log(-c-dx+1) - b(3a-b)$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*ArcTanh[c + d*x])^2/(c*e + d*e*x)^5,x]

[Out] $-1/12*((3*a^2)/(c + d*x)^4 + (2*a*b)/(c + d*x)^3 + b^2/(c + d*x)^2 + (6*a*b)/(c + d*x) + (2*b*(3*a + b*(c + 3*c^3 + d*x + 9*c^2*d*x + 9*c*d^2*x^2 + 3*d^3*x^3))*ArcTanh[c + d*x])/(c + d*x)^4 - (3*b^2*(-1 + c^4 + 4*c^3*d*x + 6*c^2*d^2*x^2 + 4*c*d^3*x^3 + d^4*x^4))*ArcTanh[c + d*x]^2/(c + d*x)^4 + b*(3*a + 4*b)*Log[1 - c - d*x] - 8*b^2*Log[c + d*x] - (3*a - 4*b)*b*Log[1 + c + d*x])/(d*e^5)$

fricas [B] time = 0.52, size = 547, normalized size = 3.18

$$24abd^3x^3 + 24abc^3 + 4(18abc + b^2)d^2x^2 + 4b^2c^2 + 8abc + 8(9abc^2 + b^2c + ab)dx - 3(b^2d^4x^4 + 4b^2cd^3x^3 +$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arctanh(d*x+c))^2/(d*e*x+c*e)^5,x, algorithm="fricas")

[Out] $-1/48*(24*a*b*d^3*x^3 + 24*a*b*c^3 + 4*(18*a*b*c + b^2)*d^2*x^2 + 4*b^2*c^2 + 8*a*b*c + 8*(9*a*b*c^2 + b^2*c + a*b)*d*x - 3*(b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*c^4 - b^2)*log(-(d*x + c + 1)/(d*x + c - 1))^2 + 12*a^2 - 4*((3*a*b - 4*b^2)*d^4*x^4 + 4*(3*a*b - 4*b^2)*c*d^3*x^3 + 6*(3*a*b - 4*b^2)*c^2*d^2*x^2 + 4*(3*a*b - 4*b^2)*c^3*d*x + (3*a*b - 4*b^2)*c^4)*log(d*x + c + 1) - 32*(b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*c^4)*log(d*x + c) + 4*((3*a*b + 4*b^2)*d^4*x^4 + 4*(3*a*b + 4*b^2)*c*d^3*x^3 + 6*(3*a*b + 4*b^2)*c^2*d^2*x^2 + 4*(3*a*b + 4*b^2)*c^3*d*x + (3*a*b + 4*b^2)*c^4)*log(d*x + c - 1) + 4*(3*b^2*d^3*x^3 + 9*b^2*c*d^2*x^2 + 3*b^2*c^3 + b^2*c + (9*b^2*c^2 + b^2)*d*x + 3*a*b)*log(-(d*x + c + 1)/(d*x + c - 1)))/(d^5*e^5*x^4 + 4*c*d^4*e^5*x^3 + 6*c^2*d^3*e^5*x^2 + 4*c^3*d^2*e^5*x + c^4*d*e^5)$

giac [B] time = 0.20, size = 758, normalized size = 4.41

$$\left(\frac{3(dx+c+1)^3 b^2 \log\left(-\frac{dx+c+1}{dx+c-1}\right)^2}{(dx+c-1)^3} + \frac{3(dx+c+1)b^2 \log\left(-\frac{dx+c+1}{dx+c-1}\right)^2}{dx+c-1} + \frac{4(dx+c+1)^4 b^2 \log\left(-\frac{dx+c+1}{dx+c-1}-1\right)}{(dx+c-1)^4} + \frac{16(dx+c+1)^3 b^2 \log\left(-\frac{dx+c+1}{dx+c-1}-1\right)}{(dx+c-1)^3} + \frac{24(dx+c+1)^2 b^2 \log\left(-\frac{dx+c+1}{dx+c-1}-1\right)^2}{(dx+c-1)^2} + \frac{24(dx+c+1)b^2 \log\left(-\frac{dx+c+1}{dx+c-1}-1\right)^2}{dx+c-1} + \frac{4(dx+c+1)^4 b^2 \log\left(-\frac{dx+c+1}{dx+c-1}-1\right)}{(dx+c-1)^4} + \frac{16(dx+c+1)^3 b^2 \log\left(-\frac{dx+c+1}{dx+c-1}-1\right)}{(dx+c-1)^3} + \frac{24(dx+c+1)^2 b^2 \log\left(-\frac{dx+c+1}{dx+c-1}-1\right)^2}{(dx+c-1)^2} + \frac{24(dx+c+1)b^2 \log\left(-\frac{dx+c+1}{dx+c-1}-1\right)^2}{dx+c-1} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arctanh(d*x+c))^2/(d*e*x+c*e)^5,x, algorithm="giac")

[Out] $1/12*(3*(d*x + c + 1)^3*b^2*log(-(d*x + c + 1)/(d*x + c - 1))^2/(d*x + c - 1)^3 + 3*(d*x + c + 1)*b^2*log(-(d*x + c + 1)/(d*x + c - 1))^2/(d*x + c - 1) + 4*(d*x + c + 1)^4*b^2*log(-(d*x + c + 1)/(d*x + c - 1) - 1)/(d*x + c - 1)^4 + 16*(d*x + c + 1)^3*b^2*log(-(d*x + c + 1)/(d*x + c - 1) - 1)/(d*x +$

$$\begin{aligned}
& (c-1)^3 + 24(d*x+c+1)^2*b^2*\log(-(d*x+c+1)/(d*x+c-1)-1)/(d*x+c-1)^2 \\
& + 16*(d*x+c+1)*b^2*\log(-(d*x+c+1)/(d*x+c-1)-1)/(d*x+c-1) \\
& + 4*b^2*\log(-(d*x+c+1)/(d*x+c-1)-1) + 12*(d*x+c+1)^3*a*b*\log(-(d*x+c+1)/(d*x+c-1)) \\
& /((d*x+c-1)^3 + 12*(d*x+c+1)*a*b*\log(-(d*x+c+1)/(d*x+c-1))/(d*x+c-1) - 4*(d*x+c+1)^4*b^2*\log(-(d*x+c+1)/(d*x+c-1)) \\
& /((d*x+c-1)^4 - 10*(d*x+c+1)^3*b^2*\log(-(d*x+c+1)/(d*x+c-1))/(d*x+c-1)^3 - 12*(d*x+c+1)^2*b^2*\log(-(d*x+c+1)/(d*x+c-1)) \\
& /((d*x+c-1)^2 - 6*(d*x+c+1)*b^2*\log(-(d*x+c+1)/(d*x+c-1))/(d*x+c-1) + 12*(d*x+c+1)^3*a^2/((d*x+c-1)^3 \\
& + 12*(d*x+c+1)*a^2/(d*x+c-1) + 12*(d*x+c+1)^3*a*b/(d*x+c-1)^3 + 24*(d*x+c+1)^2*a*b/(d*x+c-1)^2 \\
& + 20*(d*x+c+1)*a*b/(d*x+c-1) + 8*a*b + 2*(d*x+c+1)^3*b^2/(d*x+c-1)^3 + 4*(d*x+c+1)^2*b^2/(d*x+c-1)^2 \\
& + 2*(d*x+c+1)*b^2/(d*x+c-1))*((c+1)*d - (c-1)*d)/((d*x+c+1)^4*d^2*e^5/(d*x+c-1)^4 + 4*(d*x+c+1)^3*d^2*e^5/(d*x+c-1)^3 \\
& + 6*(d*x+c+1)^2*d^2*e^5/(d*x+c-1)^2 + 4*(d*x+c+1)*d^2*e^5/(d*x+c-1) + d^2*e^5)
\end{aligned}$$

maple [B] time = 0.08, size = 431, normalized size = 2.51

$$\frac{a^2}{4d e^5 (dx+c)^4} - \frac{b^2 \operatorname{arctanh}(dx+c)^2}{4d e^5 (dx+c)^4} - \frac{b^2 \operatorname{arctanh}(dx+c)}{6d e^5 (dx+c)^3} - \frac{b^2 \operatorname{arctanh}(dx+c)}{2d e^5 (dx+c)} - \frac{b^2 \operatorname{arctanh}(dx+c) \ln(dx+c)}{4d e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*arctanh(d*x+c))^2/(d*e*x+c*e)^5,x)

[Out] $-1/4/d*a^2/e^5/(d*x+c)^4 - 1/4/d*b^2/e^5/(d*x+c)^4*\operatorname{arctanh}(d*x+c)^2 - 1/6/d*b^2/e^5*\operatorname{arctanh}(d*x+c)/(d*x+c)^3 - 1/2/d*b^2/e^5*\operatorname{arctanh}(d*x+c)/(d*x+c) - 1/4/d*b^2/e^5*\operatorname{arctanh}(d*x+c)*\ln(d*x+c-1) + 1/4/d*b^2/e^5*\operatorname{arctanh}(d*x+c)*\ln(d*x+c+1) - 1/16/d*b^2/e^5*\ln(d*x+c-1)^2 + 1/8/d*b^2/e^5*\ln(d*x+c-1)*\ln(1/2+1/2*d*x+1/2*c) - 1/16/d*b^2/e^5*\ln(d*x+c+1)^2 + 1/8/d*b^2/e^5*\ln(-1/2*d*x-1/2*c+1/2)*\ln(d*x+c+1) - 1/8/d*b^2/e^5*\ln(-1/2*d*x-1/2*c+1/2)*\ln(1/2+1/2*d*x+1/2*c) - 1/12*b^2/d/e^5/(d*x+c)^2 + 2/3*b^2*\ln(d*x+c)/d/e^5 - 1/3/d*b^2/e^5*\ln(d*x+c-1) - 1/3/d*b^2/e^5*\ln(d*x+c+1) - 1/2/d*a*b/e^5/(d*x+c)^4*\operatorname{arctanh}(d*x+c) - 1/6/d*a*b/e^5/(d*x+c)^3 - 1/2/d*a*b/e^5/(d*x+c) - 1/4/d*a*b/e^5*\ln(d*x+c-1) + 1/4/d*a*b/e^5*\ln(d*x+c+1)$

maxima [B] time = 0.38, size = 613, normalized size = 3.56

$$-\frac{1}{12} \left(d \left(\frac{2(3d^2x^2 + 6cdx + 3c^2 + 1)}{d^5e^5x^3 + 3cd^4e^5x^2 + 3c^2d^3e^5x + c^3d^2e^5} - \frac{3 \log(dx+c+1)}{d^2e^5} + \frac{3 \log(dx+c-1)}{d^2e^5} \right) \right) + \frac{6}{d^5e^5x^4 + 4cd^4e^5x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arctanh(d*x+c))^2/(d*e*x+c*e)^5,x, algorithm="maxima")

```
[Out] -1/12*(d*(2*(3*d^2*x^2 + 6*c*d*x + 3*c^2 + 1)/(d^5*e^5*x^3 + 3*c*d^4*e^5*x^2 + 3*c^2*d^3*e^5*x + c^3*d^2*e^5) - 3*log(d*x + c + 1)/(d^2*e^5) + 3*log(d*x + c - 1)/(d^2*e^5)) + 6*arctanh(d*x + c)/(d^5*e^5*x^4 + 4*c*d^4*e^5*x^3 + 6*c^2*d^3*e^5*x^2 + 4*c^3*d^2*e^5*x + c^4*d*e^5))*a*b - 1/48*(d^2*((3*(d^2*x^2 + 2*c*d*x + c^2)*log(d*x + c + 1)^2 + 3*(d^2*x^2 + 2*c*d*x + c^2)*log(d*x + c - 1)^2 + 2*(8*d^2*x^2 + 16*c*d*x + 8*c^2 - 3*(d^2*x^2 + 2*c*d*x + c^2)*log(d*x + c - 1))*log(d*x + c + 1) + 16*(d^2*x^2 + 2*c*d*x + c^2)*log(d*x + c - 1) + 4)/(d^5*e^5*x^2 + 2*c*d^4*e^5*x + c^2*d^3*e^5) - 32*log(d*x + c)/(d^3*e^5)) + 4*d*(2*(3*d^2*x^2 + 6*c*d*x + 3*c^2 + 1)/(d^5*e^5*x^3 + 3*c*d^4*e^5*x^2 + 3*c^2*d^3*e^5*x + c^3*d^2*e^5) - 3*log(d*x + c + 1)/(d^2*e^5) + 3*log(d*x + c - 1)/(d^2*e^5))*arctanh(d*x + c))*b^2 - 1/4*b^2*arctanh(d*x + c)^2/(d^5*e^5*x^4 + 4*c*d^4*e^5*x^3 + 6*c^2*d^3*e^5*x^2 + 4*c^3*d^2*e^5*x + c^4*d*e^5) - 1/4*a^2/(d^5*e^5*x^4 + 4*c*d^4*e^5*x^3 + 6*c^2*d^3*e^5*x^2 + 4*c^3*d^2*e^5*x + c^4*d*e^5)
```

mupad [B] time = 3.42, size = 2746, normalized size = 15.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c + d*x))^2/(c*e + d*e*x)^5,x)
```

```
[Out] log(1 - d*x - c)^2*(b^2/(16*d*e^5) - b^2/(4*d*(4*c^4*e^5 + 4*d^4*e^5*x^4 + 16*c*d^3*e^5*x^3 + 24*c^2*d^2*e^5*x^2 + 16*c^3*d*e^5*x))) + log(c + d*x + 1)^2*(b^2/(16*d*e^5) - b^2/(16*d^2*e^5*(4*c^3*x + c^4/d + d^3*x^4 + 6*c^2*d*x^2 + 4*c*d^2*x^3))) + log(1 - d*x - c)*(log(c + d*x + 1)*(b^2/(4*d*(2*c^4*e^5 + 2*d^4*e^5*x^4 + 8*c*d^3*e^5*x^3 + 12*c^2*d^2*e^5*x^2 + 8*c^3*d*e^5*x)) - (b^2*(c^4 + d^4*x^4 + 4*c*d^3*x^3 + 6*c^2*d^2*x^2 + 4*c^3*d*x))/(4*d*(2*c^4*e^5 + 2*d^4*e^5*x^4 + 8*c*d^3*e^5*x^3 + 12*c^2*d^2*e^5*x^2 + 8*c^3*d*e^5*x))) + (3*b^2)/(4*d*(24*c^4*e^5 + 24*d^4*e^5*x^4 + 96*c*d^3*e^5*x^3 + 144*c^2*d^2*e^5*x^2 + 96*c^3*d*e^5*x)) + (3*b*(8*a - b))/(4*d*(24*c^4*e^5 + 24*d^4*e^5*x^4 + 96*c*d^3*e^5*x^3 + 144*c^2*d^2*e^5*x^2 + 96*c^3*d*e^5*x)) - (b^2*(c*(2*c - 3*c^2 + 4*c^3 + c*(6*c^2 - 3*c + c*(12*c - 3) + 1) - 1) - 3*c + x^2*(d*(2*d - 6*c*d + 12*c^2*d + d*(6*c^2 - 3*c + c*(12*c - 3) + 1) + c*(24*c*d - 3*d + d*(12*c - 3))) - 9*c*d^2 + c*(30*c*d^2 - 3*d^2 + d*(24*c*d - 3*d + d*(12*c - 3))) + 3*d^2 + 18*c^2*d^2) + x*(d*(2*c - 3*c^2 + 4*c^3 + c*(6*c^2 - 3*c + c*(12*c - 3) + 1) - 1) - 3*d + 6*c*d + c*(2*d - 6*c*d + 12*c^2*d + d*(6*c^2 - 3*c + c*(12*c - 3) + 1) + c*(24*c*d - 3*d + d*(12*c - 3))) - 9*c^2*d + 12*c^3*d) + 3*c^2 - 3*c^3 + 3*c^4 + 25*d^4*x^4 + x^3*(34*c*d^3 + d*(30*c*d^2 - 3*d^2 + d*(24*c*d - 3*d + d*(12*c - 3))) - 3*d^3) + 3))/(4*d*(24*c^4*e^5 + 24*d^4*e^5*x^4 + 96*c*d^3*e^5*x^3 + 144*c^2*d^2*e^5*x^2 + 96*c^3*d*e^5*x)) + (b^2*(c*(c*(6*c*e^5 + 2*e^5 + c*(24*c*e^5 + 6*e^5) + 12*c^2*e^5) + 4*c*e^5 + 2*e^5 + 6*c^2*e^5 + 8*c^3*e^5) + x*(d*(c*(6*c*e^5 + 2*e^5 + c*(24*c*e^5 + 6*e^5) + 12*c^2*e^5) + 4*c*e^5 + 2*e^5 + 6*c^2*e^5 + 8*c^3*e^5) + 6*d*e^5 + c*(c*(6*d*e^5 + d*(24*c*e^5 + 6*e^5) + 48*c*d*e^5
```

$$\begin{aligned}
&) + d*(6*c*e^5 + 2*e^5 + c*(24*c*e^5 + 6*e^5) + 12*c^2*e^5) + 4*d*e^5 + 24*c^2*d*e^5 + 12*c*d*e^5) + 18*c^2*d*e^5 + 24*c^3*d*e^5 + 12*c*d*e^5) + x^3*(\\
& d*(d*(6*d*e^5 + d*(24*c*e^5 + 6*e^5) + 48*c*d*e^5) + 6*d^2*e^5 + 60*c*d^2*e^5) + 6*d^3*e^5 + 68*c*d^3*e^5) + 6*c*e^5 + 6*e^5 + 6*c^2*e^5 + 6*c^3*e^5 + \\
& 6*c^4*e^5 + x^2*(c*(d*(6*d*e^5 + d*(24*c*e^5 + 6*e^5) + 48*c*d*e^5) + 6*d^2*e^5 + 60*c*d^2*e^5) + d*(c*(6*d*e^5 + d*(24*c*e^5 + 6*e^5) + 48*c*d*e^5) \\
& + d*(6*c*e^5 + 2*e^5 + c*(24*c*e^5 + 6*e^5) + 12*c^2*e^5) + 4*d*e^5 + 24*c^2*d*e^5 + 12*c*d*e^5) + 6*d^2*e^5 + 18*c*d^2*e^5 + 36*c^2*d^2*e^5) + 50*d^4 \\
& *e^5*x^4)/(8*d*e^5*(24*c^4*e^5 + 24*d^4*e^5*x^4 + 96*c*d^3*e^5*x^3 + 144*c^2*d^2*e^5*x^2 + 96*c^3*d*e^5*x)) - (x^2*((b^2*d)/2 + 9*a*b*c*d) + x*(a*b \\
& + b^2*c + 9*a*b*c^2) + (3*a^2 + b^2*c^2 + 2*a*b*c + 6*a*b*c^3)/(2*d) + 3*a*b*d^2*x^3)/(6*c^4*e^5 + 6*d^4*e^5*x^4 + 24*c*d^3*e^5*x^3 + 36*c^2*d^2*e^5*x^2 + 24*c^3*d*e^5*x) - (log(c + d*x + 1)*(x^2*((3*b^2*c + b^2 + 6*b^2*c^2)/ \\
& (32*e^5) + d*(d*((3*b^2*c + b^2 + 6*b^2*c^2)/(96*d^2*e^5) + (c*(4*b^2*c + b^2))/(32*d^2*e^5))) + c*((4*b^2*c + b^2)/(16*d*e^5) + (b^2*c)/(8*d*e^5))) + (\\
& 3*b^2*c + b^2 + 6*b^2*c^2)/(48*d*e^5)) + c*(d*((4*b^2*c + b^2)/(16*d*e^5) + (b^2*c)/(8*d*e^5)) + (4*b^2*c + b^2)/(32*e^5) + (3*b^2*c)/(16*e^5)) - (b^2 \\
& *(d/4 - (3*c*d)/4 + (3*c^2*d)/2 + d*(d*((6*c^2 - 3*c + 1)/(12*d) + (c*(4*c - 1))/(4*d)) - c/2 + c^2 + c*(3*c - 1/2) + 1/6) + c*((5*c*d)/2 - d/4 + d*(3 \\
& *c - 1/2)))/(8*d*e^5) + x*(c*(d*((3*b^2*c + b^2 + 6*b^2*c^2)/(96*d^2*e^5) + (c*(4*b^2*c + b^2))/(32*d^2*e^5)) + c*((4*b^2*c + b^2)/(16*d*e^5) + (b^2 \\
& *c)/(8*d*e^5)) + (3*b^2*c + b^2 + 6*b^2*c^2)/(48*d*e^5) + d*(c*((3*b^2*c + b^2 + 6*b^2*c^2)/(96*d^2*e^5) + (c*(4*b^2*c + b^2))/(32*d^2*e^5)) + (2*b^2 \\
& *c + b^2 + 3*b^2*c^2 + 4*b^2*c^3)/(96*d^2*e^5)) + (2*b^2*c + b^2 + 3*b^2*c^2 + 4*b^2*c^3)/(32*d*e^5) - (b^2*(c/2 + c*(d*((6*c^2 - 3*c + 1)/(12*d) + (c \\
& *(4*c - 1))/(4*d)) - c/2 + c^2 + c*(3*c - 1/2) + 1/6) - (3*c^2)/4 + c^3 + d \\
& *(c*((6*c^2 - 3*c + 1)/(12*d) + (c*(4*c - 1))/(4*d)) + (2*c - 3*c^2 + 4*c^3 - 1)/(12*d) - 1/4))/(8*d*e^5) + x^3*(d*(d*((4*b^2*c + b^2)/(16*d*e^5) + \\
& (b^2*c)/(8*d*e^5)) + (4*b^2*c + b^2)/(32*e^5) + (3*b^2*c)/(16*e^5)) + (b^2*d + 4*b^2*c*d)/(32*e^5) - (b^2*((17*c*d^2)/6 - d^2/4 + d*((5*c*d)/2 - d/4 + \\
& d*(3*c - 1/2)))/(8*d*e^5) + (11*b^2*c*d)/(48*e^5) + c*(c*((3*b^2*c + b^2 + 6*b^2*c^2)/(96*d^2*e^5) + (c*(4*b^2*c + b^2))/(32*d^2*e^5)) + (2*b^2*c + \\
& b^2 + 3*b^2*c^2 + 4*b^2*c^3)/(96*d^2*e^5)) + (8*a*b + b^2*c + b^2 + b^2*c^2 + b^2*c^3 + b^2*c^4)/(32*d^2*e^5) - (b^2*(c*(c*((6*c^2 - 3*c + 1)/(12*d) \\
& + (c*(4*c - 1))/(4*d)) + (2*c - 3*c^2 + 4*c^3 - 1)/(12*d)) + (c^2 - c - c^3 + c^4 + 1)/(4*d)))/(8*d*e^5))/(4*c^3*x + c^4/d + d^3*x^4 + 6*c^2*d*x^2 + \\
& 4*c*d^2*x^3) + (2*b^2*log(c + d*x))/(3*d*e^5) - (log(c + d*x - 1)*(3*a*b + 4*b^2))/(12*d*e^5) + (log(c + d*x + 1)*(3*a*b - 4*b^2))/(12*d*e^5)
\end{aligned}$$

sympy [A] time = 10.59, size = 3516, normalized size = 20.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*atanh(d*x+c))**2/(d*e*x+c*e)**5,x)


```

**2*c**2*d**2*x**2*atanh(c + d*x)/(12*c**4*d*e**5 + 48*c**3*d**2*e**5*x + 7
2*c**2*d**3*e**5*x**2 + 48*c*d**4*e**5*x**3 + 12*d**5*e**5*x**4) - 18*b**2*
c**2*d*x*atanh(c + d*x)/(12*c**4*d*e**5 + 48*c**3*d**2*e**5*x + 72*c**2*d**
3*e**5*x**2 + 48*c*d**4*e**5*x**3 + 12*d**5*e**5*x**4) - b**2*c**2/(12*c**4
*d*e**5 + 48*c**3*d**2*e**5*x + 72*c**2*d**3*e**5*x**2 + 48*c*d**4*e**5*x**
3 + 12*d**5*e**5*x**4) + 32*b**2*c*d**3*x**3*log(c/d + x)/(12*c**4*d*e**5 +
48*c**3*d**2*e**5*x + 72*c**2*d**3*e**5*x**2 + 48*c*d**4*e**5*x**3 + 12*d
**5*e**5*x**4) - 32*b**2*c*d**3*x**3*log(c/d + x + 1/d)/(12*c**4*d*e**5 + 48
*c**3*d**2*e**5*x + 72*c**2*d**3*e**5*x**2 + 48*c*d**4*e**5*x**3 + 12*d**5
e**5*x**4) + 12*b**2*c*d**3*x**3*atanh(c + d*x)**2/(12*c**4*d*e**5 + 48*c**
3*d**2*e**5*x + 72*c**2*d**3*e**5*x**2 + 48*c*d**4*e**5*x**3 + 12*d**5*e**5
*x**4) + 32*b**2*c*d**3*x**3*atanh(c + d*x)/(12*c**4*d*e**5 + 48*c**3*d**2
e**5*x + 72*c**2*d**3*e**5*x**2 + 48*c*d**4*e**5*x**3 + 12*d**5*e**5*x**4)
- 18*b**2*c*d**2*x**2*atanh(c + d*x)/(12*c**4*d*e**5 + 48*c**3*d**2*e**5*x
+ 72*c**2*d**3*e**5*x**2 + 48*c*d**4*e**5*x**3 + 12*d**5*e**5*x**4) - 2*b**
2*c*d*x/(12*c**4*d*e**5 + 48*c**3*d**2*e**5*x + 72*c**2*d**3*e**5*x**2 + 48
*c*d**4*e**5*x**3 + 12*d**5*e**5*x**4) - 2*b**2*c*atanh(c + d*x)/(12*c**4*d
e**5 + 48*c**3*d**2*e**5*x + 72*c**2*d**3*e**5*x**2 + 48*c*d**4*e**5*x**3
+ 12*d**5*e**5*x**4) + 8*b**2*d**4*x**4*log(c/d + x)/(12*c**4*d*e**5 + 48*c
**3*d**2*e**5*x + 72*c**2*d**3*e**5*x**2 + 48*c*d**4*e**5*x**3 + 12*d**5*e
**5*x**4) - 8*b**2*d**4*x**4*log(c/d + x + 1/d)/(12*c**4*d*e**5 + 48*c**3*d
**2*e**5*x + 72*c**2*d**3*e**5*x**2 + 48*c*d**4*e**5*x**3 + 12*d**5*e**5*x**
4) + 3*b**2*d**4*x**4*atanh(c + d*x)**2/(12*c**4*d*e**5 + 48*c**3*d**2*e**5
*x + 72*c**2*d**3*e**5*x**2 + 48*c*d**4*e**5*x**3 + 12*d**5*e**5*x**4) + 8*
b**2*d**4*x**4*atanh(c + d*x)/(12*c**4*d*e**5 + 48*c**3*d**2*e**5*x + 72*c
**2*d**3*e**5*x**2 + 48*c*d**4*e**5*x**3 + 12*d**5*e**5*x**4) - 6*b**2*d**3*
x**3*atanh(c + d*x)/(12*c**4*d*e**5 + 48*c**3*d**2*e**5*x + 72*c**2*d**3*e
**5*x**2 + 48*c*d**4*e**5*x**3 + 12*d**5*e**5*x**4) - b**2*d**2*x**2/(12*c**
4*d*e**5 + 48*c**3*d**2*e**5*x + 72*c**2*d**3*e**5*x**2 + 48*c*d**4*e**5*x
**3 + 12*d**5*e**5*x**4) - 2*b**2*d*x*atanh(c + d*x)/(12*c**4*d*e**5 + 48*c
**3*d**2*e**5*x + 72*c**2*d**3*e**5*x**2 + 48*c*d**4*e**5*x**3 + 12*d**5*e
**5*x**4) - 3*b**2*atanh(c + d*x)**2/(12*c**4*d*e**5 + 48*c**3*d**2*e**5*x
+ 72*c**2*d**3*e**5*x**2 + 48*c*d**4*e**5*x**3 + 12*d**5*e**5*x**4), Ne(d, 0)
), (x*(a + b*atanh(c))**2/(c**5*e**5), True))

```

3.23 $\int (ce + dex)^2 (a + b \tanh^{-1}(c + dx))^3 dx$

Optimal. Leaf size=263

$$\frac{b^2 e^2 \operatorname{Li}_2\left(1 - \frac{2}{-c-dx+1}\right) (a + b \tanh^{-1}(c + dx))}{d} + ab^2 e^2 x - \frac{b e^2 (a + b \tanh^{-1}(c + dx))^2}{2d} + \frac{b e^2 (c + dx)^2 (a + b \tanh^{-1}(c + dx))}{2d}$$

[Out] $a*b^2*e^2*x + b^3*e^2*(d*x+c)*\operatorname{arctanh}(d*x+c)/d - 1/2*b*e^2*(a+b*\operatorname{arctanh}(d*x+c))^2/d + 1/2*b*e^2*(d*x+c)^2*(a+b*\operatorname{arctanh}(d*x+c))^2/d + 1/3*e^2*(a+b*\operatorname{arctanh}(d*x+c))^3/d + 1/3*e^2*(d*x+c)^3*(a+b*\operatorname{arctanh}(d*x+c))^3/d - b*e^2*(a+b*\operatorname{arctanh}(d*x+c))^2*\ln(2/(-d*x-c+1))/d + 1/2*b^3*e^2*\ln(1-(d*x+c)^2)/d - b^2*e^2*(a+b*\operatorname{arctanh}(d*x+c))*\operatorname{polylog}(2, 1-2/(-d*x-c+1))/d + 1/2*b^3*e^2*\operatorname{polylog}(3, 1-2/(-d*x-c+1))/d$

Rubi [A] time = 0.47, antiderivative size = 263, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {6107, 12, 5916, 5980, 5910, 260, 5948, 5984, 5918, 6058, 6610}

$$\frac{b^2 e^2 \operatorname{PolyLog}\left(2, 1 - \frac{2}{-c-dx+1}\right) (a + b \tanh^{-1}(c + dx))}{d} + \frac{b^3 e^2 \operatorname{PolyLog}\left(3, 1 - \frac{2}{-c-dx+1}\right)}{2d} + ab^2 e^2 x - \frac{b e^2 (a + b \tanh^{-1}(c + dx))^2}{2d}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(c*e + d*e*x)^2*(a + b*\operatorname{ArcTanh}[c + d*x])^3, x]$

[Out] $a*b^2*e^2*x + (b^3*e^2*(c + d*x)*\operatorname{ArcTanh}[c + d*x])/d - (b*e^2*(a + b*\operatorname{ArcTanh}[c + d*x])^2)/(2*d) + (b*e^2*(c + d*x)^2*(a + b*\operatorname{ArcTanh}[c + d*x])^2)/(2*d) + (e^2*(a + b*\operatorname{ArcTanh}[c + d*x])^3)/(3*d) + (e^2*(c + d*x)^3*(a + b*\operatorname{ArcTanh}[c + d*x])^3)/(3*d) - (b*e^2*(a + b*\operatorname{ArcTanh}[c + d*x])^2*\operatorname{Log}[2/(1 - c - d*x)])/d + (b^3*e^2*\operatorname{Log}[1 - (c + d*x)^2])/(2*d) - (b^2*e^2*(a + b*\operatorname{ArcTanh}[c + d*x])*\operatorname{PolyLog}[2, 1 - 2/(1 - c - d*x)])/d + (b^3*e^2*\operatorname{PolyLog}[3, 1 - 2/(1 - c - d*x)])/d$

Rule 12

$\operatorname{Int}[(a_*)*(u_), x_Symbol] \rightarrow \operatorname{Dist}[a, \operatorname{Int}[u, x], x] /; \operatorname{FreeQ}[a, x] \ \&\& \ !\operatorname{MatchQ}[u, (b_*)*(v_)] /; \operatorname{FreeQ}[b, x]$

Rule 260

$\operatorname{Int}[(x_)^{(m_.)}/((a_) + (b_.)*(x_)^{(n_.)}), x_Symbol] \rightarrow \operatorname{Simp}[\operatorname{Log}[\operatorname{RemoveContent}[a + b*x^n, x]]/(b*n), x] /; \operatorname{FreeQ}[\{a, b, m, n\}, x] \ \&\& \ \operatorname{EqQ}[m, n - 1]$

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_.)]*(b_.))^(p_.), x_Symbol] := Simp[x*(a + b*ArcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_.)]*(b_.))^(p_.)*((d_.)*(x_.))^(m_.), x_Symbol] := Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || IntegerQ[m]) && NeQ[m, -1]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_.)]*(b_.))^(p_.)/((d_.) + (e_.)*(x_.)), x_Symbol] := -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/((1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_.)]*(b_.))^(p_.)/((d_.) + (e_.)*(x_.)^2), x_Symbol] := Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 5980

```
Int[((a_.) + ArcTanh[(c_.)*(x_.)]*(b_.))^(p_.)*((f_.)*(x_.))^(m_.)/((d_.) + (e_.)*(x_.)^2), x_Symbol] := Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x])^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTanh[c*x])^p)/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 5984

```
Int[((a_.) + ArcTanh[(c_.)*(x_.)]*(b_.))^(p_.)*(x_.)/((d_.) + (e_.)*(x_.)^2), x_Symbol] := Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 6058

```
Int[(Log[u]*((a_.) + ArcTanh[(c_.)*(x_.)]*(b_.))^(p_.))/((d_.) + (e_.)*(x_.)^2), x_Symbol] := -Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
```

```
x] + Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6107

```
Int[((a_.) + ArcTanh[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((f*x)/d)^m*(a + b*ArcTanh[x])^p, x]
, x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] &
& IGtQ[p, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /; !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\begin{aligned}
\int (ce + dex)^2 (a + b \tanh^{-1}(c + dx))^3 dx &= \frac{\text{Subst}\left(\int e^2 x^2 (a + b \tanh^{-1}(x))^3 dx, x, c + dx\right)}{d} \\
&= \frac{e^2 \text{Subst}\left(\int x^2 (a + b \tanh^{-1}(x))^3 dx, x, c + dx\right)}{d} \\
&= \frac{e^2 (c + dx)^3 (a + b \tanh^{-1}(c + dx))^3}{3d} - \frac{(be^2) \text{Subst}\left(\int \frac{x^3 (a + b \tanh^{-1}(x))}{1-x^2} dx, x, c + dx\right)}{d} \\
&= \frac{e^2 (c + dx)^3 (a + b \tanh^{-1}(c + dx))^3}{3d} + \frac{(be^2) \text{Subst}\left(\int x (a + b \tanh^{-1}(x)) dx, x, c + dx\right)}{d} \\
&= \frac{be^2 (c + dx)^2 (a + b \tanh^{-1}(c + dx))^2}{2d} + \frac{e^2 (a + b \tanh^{-1}(c + dx))^3}{3d} + \dots \\
&= \frac{be^2 (c + dx)^2 (a + b \tanh^{-1}(c + dx))^2}{2d} + \frac{e^2 (a + b \tanh^{-1}(c + dx))^3}{3d} + \dots \\
&= ab^2 e^2 x - \frac{be^2 (a + b \tanh^{-1}(c + dx))^2}{2d} + \frac{be^2 (c + dx)^2 (a + b \tanh^{-1}(c + dx))^2}{2d} \\
&= ab^2 e^2 x + \frac{b^3 e^2 (c + dx) \tanh^{-1}(c + dx)}{d} - \frac{be^2 (a + b \tanh^{-1}(c + dx))^2}{2d} \\
&= ab^2 e^2 x + \frac{b^3 e^2 (c + dx) \tanh^{-1}(c + dx)}{d} - \frac{be^2 (a + b \tanh^{-1}(c + dx))^2}{2d}
\end{aligned}$$

Mathematica [A] time = 0.74, size = 336, normalized size = 1.28

$$e^2 \left(2a^3 (c + dx)^3 + 3a^2 b (c + dx)^2 + 3a^2 b \log(1 - (c + dx)^2) + 6a^2 b (c + dx)^3 \tanh^{-1}(c + dx) + 6ab^2 \left(\text{Li}_2 \left(-e^{-2 \tanh^{-1}(c + dx)} \right) \right) \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[(c*e + d*e*x)^2*(a + b*ArcTanh[c + d*x])^3,x]

[Out] (e^2*(3*a^2*b*(c + d*x)^2 + 2*a^3*(c + d*x)^3 + 6*a^2*b*(c + d*x)^3*ArcTanh[c + d*x] + 3*a^2*b*Log[1 - (c + d*x)^2] + 6*a*b^2*(c + d*x - ArcTanh[c + d*x])^2)

*x] + (c + d*x)^2*ArcTanh[c + d*x] - ArcTanh[c + d*x]^2 + (c + d*x)^3*ArcTanh[c + d*x]^2 - 2*ArcTanh[c + d*x]*Log[1 + E^(-2*ArcTanh[c + d*x])] + PolyLog[2, -E^(-2*ArcTanh[c + d*x])] + b^3*(6*(c + d*x)*ArcTanh[c + d*x] - 3*(1 - (c + d*x)^2)*ArcTanh[c + d*x]^2 - 2*ArcTanh[c + d*x]^3 + 2*(c + d*x)*ArcTanh[c + d*x]^3 - 2*(c + d*x)*(1 - (c + d*x)^2)*ArcTanh[c + d*x]^3 - 6*ArcTanh[c + d*x]^2*Log[1 + E^(-2*ArcTanh[c + d*x])] - 6*Log[1/Sqrt[1 - (c + d*x)^2]]) + 6*ArcTanh[c + d*x]*PolyLog[2, -E^(-2*ArcTanh[c + d*x])] + 3*PolyLog[3, -E^(-2*ArcTanh[c + d*x])])/(6*d)

fricas [F] time = 0.65, size = 0, normalized size = 0.00

integral(a^3*d^2*e^2*x^2 + 2*a^3*c*d*e^2*x + a^3*c^2*e^2 + (b^3*d^2*e^2*x^2 + 2*b^3*c*d*e^2*x + b^3*c^2*e^2) artanh(dx + c)^3 + 3(ab^2*d^2*e^2*x^2 + 2*ab

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*e*x+c*e)^2*(a+b*arctanh(d*x+c))^3,x, algorithm="fricas")

[Out] integral(a^3*d^2*e^2*x^2 + 2*a^3*c*d*e^2*x + a^3*c^2*e^2 + (b^3*d^2*e^2*x^2 + 2*b^3*c*d*e^2*x + b^3*c^2*e^2)*arctanh(d*x + c)^3 + 3*(a*b^2*d^2*e^2*x^2 + 2*a*b^2*c*d*e^2*x + a*b^2*c^2*e^2)*arctanh(d*x + c)^2 + 3*(a^2*b*d^2*e^2*x^2 + 2*a^2*b*c*d*e^2*x + a^2*b*c^2*e^2)*arctanh(d*x + c), x)

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*e*x+c*e)^2*(a+b*arctanh(d*x+c))^3,x, algorithm="giac")

[Out] Timed out

maple [C] time = 0.86, size = 1768, normalized size = 6.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*e*x+c*e)^2*(a+b*arctanh(d*x+c))^3,x)

[Out] -1/2*I/d*e^2*b^3*arctanh(d*x+c)^2*Pi+x*a^2*b*c*e^2+a*b^2*e^2*x+1/4*I/d*e^2*b^3*arctanh(d*x+c)^2*Pi*csgn(I/(1+(d*x+c+1)^2/(1-(d*x+c)^2)))*csgn(I*(d*x+c+1)^2/((d*x+c)^2-1))*csgn(I*(d*x+c+1)^2/((d*x+c)^2-1)/(1+(d*x+c+1)^2/(1-(d*x+c)^2)))+1/3/d*arctanh(d*x+c)^3*b^3*c^3*e^2+1/3*d^2*arctanh(d*x+c)^3*x^3*b^3*e^2+1/2*d*arctanh(d*x+c)^2*x^2*b^3*e^2-1/4*I/d*e^2*b^3*arctanh(d*x+c)^2*Pi*csgn(I*(d*x+c+1)^2/((d*x+c)^2-1))*csgn(I*(d*x+c+1)/(1-(d*x+c)^2)^(1/2))^2-1/4*I/d*e^2*b^3*arctanh(d*x+c)^2*Pi*csgn(I/(1+(d*x+c+1)^2/(1-(d*x+c)^2)))

```

*csgn(I*(d*x+c+1)^2/((d*x+c)^2-1)/(1+(d*x+c+1)^2/(1-(d*x+c)^2)))^2+1/d*arct
anh(d*x+c)^2*a*b^2*c^3*e^2+1/d*arctanh(d*x+c)*a*b^2*c^2*e^2+1/d*arctanh(d*x
+c)*a^2*b*c^3*e^2+1/d*e^2*a*b^2*arctanh(d*x+c)*ln(d*x+c-1)+1/d*e^2*a*b^2*ar
ctanh(d*x+c)*ln(d*x+c+1)+1/2/d*e^2*a*b^2*ln(-1/2*d*x-1/2*c+1/2)*ln(d*x+c+1)
-1/2/d*e^2*a*b^2*ln(-1/2*d*x-1/2*c+1/2)*ln(1/2+1/2*d*x+1/2*c)-1/2/d*e^2*a*b
^2*ln(d*x+c-1)*ln(1/2+1/2*d*x+1/2*c)+d*arctanh(d*x+c)^3*x^2*b^3*c*e^2+d^2*a
rctanh(d*x+c)^2*x^3*a*b^2*e^2+d*arctanh(d*x+c)*x^2*a*b^2*e^2+d^2*arctanh(d*
x+c)*x^3*a^2*b*e^2+3*arctanh(d*x+c)^2*x*a*b^2*c^2*e^2+2*arctanh(d*x+c)*x*a*
b^2*c*e^2+3*arctanh(d*x+c)*x*a^2*b*c^2*e^2-1/2*I/d*e^2*b^3*arctanh(d*x+c)^2
*Pi*csgn(I*(d*x+c+1)^2/((d*x+c)^2-1))^2*csgn(I*(d*x+c+1)/(1-(d*x+c)^2)^(1/2
))+1/4*I/d*e^2*b^3*arctanh(d*x+c)^2*Pi*csgn(I*(d*x+c+1)^2/((d*x+c)^2-1))*cs
gn(I*(d*x+c+1)^2/((d*x+c)^2-1)/(1+(d*x+c+1)^2/(1-(d*x+c)^2)))^2+3*d*arctanh
(d*x+c)^2*x^2*a*b^2*c*e^2+3*d*arctanh(d*x+c)*x^2*a^2*b*c*e^2-1/2*I/d*e^2*b^
3*arctanh(d*x+c)^2*Pi*csgn(I/(1+(d*x+c+1)^2/(1-(d*x+c)^2)))^3-1/4*I/d*e^2*b^
3*arctanh(d*x+c)^2*Pi*csgn(I*(d*x+c+1)^2/((d*x+c)^2-1))^3-1/4*I/d*e^2*b^3*
arctanh(d*x+c)^2*Pi*csgn(I*(d*x+c+1)^2/((d*x+c)^2-1)/(1+(d*x+c+1)^2/(1-(d*x
+c)^2)))^3+1/2*I/d*e^2*b^3*arctanh(d*x+c)^2*Pi*csgn(I/(1+(d*x+c+1)^2/(1-(d*
x+c)^2)))^2+1/3/d*a^3*c^3*e^2+1/3*d^2*x^3*a^3*e^2-1/d*e^2*b^3*ln(1+(d*x+c+1
)^2/(1-(d*x+c)^2))+1/2/d*e^2*b^3*polylog(3,-(d*x+c+1)^2/(1-(d*x+c)^2))+1/3/
d*e^2*b^3*arctanh(d*x+c)^3-1/2/d*e^2*b^3*arctanh(d*x+c)^2+1/d*e^2*b^3*arcta
nh(d*x+c)+arctanh(d*x+c)*x*b^3*e^2+x*a^3*c^2*e^2+1/d*a*b^2*c*e^2-1/d*e^2*b^
3*arctanh(d*x+c)^2*ln((d*x+c+1)/(1-(d*x+c)^2)^(1/2))-1/d*e^2*b^3*arctanh(d*
x+c)*polylog(2,-(d*x+c+1)^2/(1-(d*x+c)^2))+1/2/d*e^2*a*b^2*ln(d*x+c-1)-1/2/
d*e^2*a*b^2*ln(d*x+c+1)+1/4/d*e^2*a*b^2*ln(d*x+c-1)^2-1/d*e^2*a*b^2*dilog(1
/2+1/2*d*x+1/2*c)-1/4/d*e^2*a*b^2*ln(d*x+c+1)^2+1/2/d*e^2*a^2*b*ln(d*x+c-1)
+1/2/d*e^2*a^2*b*ln(d*x+c+1)-1/d*e^2*b^3*arctanh(d*x+c)^2*ln(2)+1/2/d*arcta
nh(d*x+c)^2*b^3*c^2*e^2+1/d*arctanh(d*x+c)*b^3*c*e^2+d*x^2*a^3*c*e^2+1/2*d*
x^2*a^2*b*e^2+arctanh(d*x+c)^3*x*b^3*c^2*e^2+arctanh(d*x+c)^2*x*b^3*c*e^2+1
/2/d*e^2*b^3*arctanh(d*x+c)^2*ln(d*x+c-1)+1/2/d*e^2*b^3*arctanh(d*x+c)^2*ln
(d*x+c+1)+1/2/d*a^2*b*c^2*e^2

```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{3} a^3 d^2 e^2 x^3 + a^3 c d e^2 x^2 + \frac{3}{2} \left(2 x^2 \operatorname{artanh}(d x + c) + d \left(\frac{2 x}{d^2} - \frac{(c^2 + 2 c + 1) \log(d x + c + 1)}{d^3} + \frac{(c^2 - 2 c + 1) \log(d x - 1)}{d^3} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*e*x+c*e)^2*(a+b*arctanh(d*x+c))^3,x, algorithm="maxima")

[Out] 1/3*a^3*d^2*e^2*x^3 + a^3*c*d*e^2*x^2 + 3/2*(2*x^2*arctanh(d*x + c) + d*(2*x/d^2 - (c^2 + 2*c + 1)*log(d*x + c + 1)/d^3 + (c^2 - 2*c + 1)*log(d*x + c - 1)/d^3))*a^2*b*c*d*e^2 + 1/2*(2*x^3*arctanh(d*x + c) + d*((d*x^2 - 4*c*x)/d^3 + (c^3 + 3*c^2 + 3*c + 1)*log(d*x + c + 1)/d^4 - (c^3 - 3*c^2 + 3*c - 1)*log(d*x + c - 1)/d^4))*a^2*b*d^2*e^2 + a^3*c^2*e^2*x + 3/2*(2*(d*x + c)*

```

arctanh(d*x + c) + log(-(d*x + c)^2 + 1))*a^2*b*c^2*e^2/d - 1/24*((b^3*d^3*
e^2*x^3 + 3*b^3*c*d^2*e^2*x^2 + 3*b^3*c^2*d*e^2*x + (c^3*e^2 - e^2)*b^3)*lo
g(-d*x - c + 1)^3 - 3*(2*a*b^2*d^3*e^2*x^3 + (6*a*b^2*c*d^2*e^2 + b^3*d^2*e
^2)*x^2 + 2*(3*a*b^2*c^2*d*e^2 + b^3*c*d*e^2)*x + (b^3*d^3*e^2*x^3 + 3*b^3*
c*d^2*e^2*x^2 + 3*b^3*c^2*d*e^2*x + (c^3*e^2 + e^2)*b^3)*log(d*x + c + 1))*
log(-d*x - c + 1)^2)/d - integrate(-1/8*((b^3*d^3*e^2*x^3 + (3*c*d^2*e^2 -
d^2*e^2)*b^3*x^2 + (3*c^2*d*e^2 - 2*c*d*e^2)*b^3*x + (c^3*e^2 - c^2*e^2)*b^
3)*log(d*x + c + 1)^3 + 6*(a*b^2*d^3*e^2*x^3 + (3*c*d^2*e^2 - d^2*e^2)*a*b^
2*x^2 + (3*c^2*d*e^2 - 2*c*d*e^2)*a*b^2*x + (c^3*e^2 - c^2*e^2)*a*b^2)*log(
d*x + c + 1)^2 - (4*a*b^2*d^3*e^2*x^3 + 2*(6*a*b^2*c*d^2*e^2 + b^3*d^2*e^2)
*x^2 + 3*(b^3*d^3*e^2*x^3 + (3*c*d^2*e^2 - d^2*e^2)*b^3*x^2 + (3*c^2*d*e^2
- 2*c*d*e^2)*b^3*x + (c^3*e^2 - c^2*e^2)*b^3)*log(d*x + c + 1)^2 + 4*(3*a*b
^2*c^2*d*e^2 + b^3*c*d*e^2)*x + 2*(6*(c^3*e^2 - c^2*e^2)*a*b^2 + (c^3*e^2 +
e^2)*b^3 + (6*a*b^2*d^3*e^2 + b^3*d^3*e^2)*x^3 + 3*(b^3*c*d^2*e^2 + 2*(3*c
*d^2*e^2 - d^2*e^2)*a*b^2)*x^2 + 3*(b^3*c^2*d*e^2 + 2*(3*c^2*d*e^2 - 2*c*d*
e^2)*a*b^2)*x)*log(d*x + c + 1))*log(-d*x - c + 1))/(d*x + c - 1), x)

```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (ce + dex)^2 (a + b \operatorname{atanh}(c + dx))^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*e + d*e*x)^2*(a + b*atanh(c + d*x))^3,x)

[Out] int((c*e + d*e*x)^2*(a + b*atanh(c + d*x))^3, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$e^2 \left(\int a^3 c^2 dx + \int a^3 d^2 x^2 dx + \int b^3 c^2 \operatorname{atanh}^3(c + dx) dx + \int 3ab^2 c^2 \operatorname{atanh}^2(c + dx) dx + \int 3a^2 bc^2 \operatorname{atanh}(c + dx) dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*e*x+c*e)**2*(a+b*atanh(d*x+c))**3,x)

[Out] e**2*(Integral(a**3*c**2, x) + Integral(a**3*d**2*x**2, x) + Integral(b**3*c**2*atanh(c + d*x)**3, x) + Integral(3*a*b**2*c**2*atanh(c + d*x)**2, x) + Integral(3*a**2*b*c**2*atanh(c + d*x), x) + Integral(2*a**3*c*d*x, x) + Integral(b**3*d**2*x**2*atanh(c + d*x)**3, x) + Integral(3*a*b**2*d**2*x**2*atanh(c + d*x)**2, x) + Integral(3*a**2*b*d**2*x**2*atanh(c + d*x), x) + Integral(2*b**3*c*d*x*atanh(c + d*x)**3, x) + Integral(6*a*b**2*c*d*x*atanh(c + d*x)**2, x) + Integral(6*a**2*b*c*d*x*atanh(c + d*x), x))

3.24 $\int (ce + dex) \left(a + b \tanh^{-1}(c + dx) \right)^3 dx$

Optimal. Leaf size=160

$$\frac{3b^2e \log\left(\frac{2}{-c-dx+1}\right) \left(a + b \tanh^{-1}(c + dx)\right)}{d} + \frac{3be \left(a + b \tanh^{-1}(c + dx)\right)^2}{2d} + \frac{3be(c + dx) \left(a + b \tanh^{-1}(c + dx)\right)^2}{2d} +$$

[Out] $3/2*b*e*(a+b*\operatorname{arctanh}(d*x+c))^{2/d}+3/2*b*e*(d*x+c)*(a+b*\operatorname{arctanh}(d*x+c))^{2/d-1}/2*e*(a+b*\operatorname{arctanh}(d*x+c))^{3/d}+1/2*e*(d*x+c)^2*(a+b*\operatorname{arctanh}(d*x+c))^{3/d}-3*b^2*e*(a+b*\operatorname{arctanh}(d*x+c))*\ln(2/(-d*x-c+1))/d-3/2*b^3*e*\operatorname{polylog}(2,(-d*x-c-1)/(-d*x-c+1))/d$

Rubi [A] time = 0.26, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {6107, 12, 5916, 5980, 5910, 5984, 5918, 2402, 2315, 5948}

$$\frac{3b^3e \operatorname{PolyLog}\left(2, -\frac{c+dx+1}{-c-dx+1}\right)}{2d} - \frac{3b^2e \log\left(\frac{2}{-c-dx+1}\right) \left(a + b \tanh^{-1}(c + dx)\right)}{d} + \frac{3be \left(a + b \tanh^{-1}(c + dx)\right)^2}{2d} + \frac{3be(c + dx) \left(a + b \tanh^{-1}(c + dx)\right)^2}{2d} +$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(c*e + d*e*x)*(a + b*\operatorname{ArcTanh}[c + d*x])^3, x]$

[Out] $(3*b*e*(a + b*\operatorname{ArcTanh}[c + d*x])^2)/(2*d) + (3*b*e*(c + d*x)*(a + b*\operatorname{ArcTanh}[c + d*x])^2)/(2*d) - (e*(a + b*\operatorname{ArcTanh}[c + d*x])^3)/(2*d) + (e*(c + d*x)^2*(a + b*\operatorname{ArcTanh}[c + d*x])^3)/(2*d) - (3*b^2*e*(a + b*\operatorname{ArcTanh}[c + d*x])*Log[2/(1 - c - d*x)])/d - (3*b^3*e*\operatorname{PolyLog}[2, -((1 + c + d*x)/(1 - c - d*x))])/(2*d)$

Rule 12

$\operatorname{Int}[(a_*)(u_), x_Symbol] \rightarrow \operatorname{Dist}[a, \operatorname{Int}[u, x], x] /; \operatorname{FreeQ}[a, x] \&\& !\operatorname{Match}Q[u, (b_*)(v_)] /; \operatorname{FreeQ}[b, x]$

Rule 2315

$\operatorname{Int}[\operatorname{Log}[(c_*)(x_)]/((d_*) + (e_*)(x_)), x_Symbol] \rightarrow -\operatorname{Simp}[\operatorname{PolyLog}[2, 1 - c*x]/e, x] /; \operatorname{FreeQ}\{c, d, e\}, x \&\& \operatorname{EqQ}[e + c*d, 0]$

Rule 2402

$\operatorname{Int}[\operatorname{Log}[(c_*)/((d_*) + (e_*)(x_))]/((f_*) + (g_*)(x_)^2), x_Symbol] \rightarrow -\operatorname{Dist}[e/g, \operatorname{Subst}[\operatorname{Int}[\operatorname{Log}[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; \operatorname{FreeQ}\{$

c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]

Rule 5910

Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^ (p_.), x_Symbol] := Simp[x*(a + b*ArcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]

Rule 5916

Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^ (p_.)*((d_.)*(x_))^(m_.), x_Symbol] := Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || IntegerQ[m]) && NeQ[m, -1]

Rule 5918

Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^ (p_.)/((d_) + (e_.)*(x_)), x_Symbol] := -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/ (1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0]

Rule 5948

Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^ (p_.)/((d_) + (e_.)*(x_)^2), x_Symbol] := Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]

Rule 5980

Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^ (p_.)*((f_.)*(x_))^(m_))/((d_) + (e_.)*(x_)^2), x_Symbol] := Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x])^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTanh[c*x])^p)/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]

Rule 5984

Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^ (p_.)*(x_))/((d_) + (e_.)*(x_)^2), x_Symbol] := Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]

Rule 6107

Int[((a_.) + ArcTanh[(c_) + (d_.)*(x_)])*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m_.), x_Symbol] := Dist[1/d, Subst[Int[((f*x)/d)^m*(a + b*ArcTanh[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] & IGtQ[p, 0]

Rubi steps

$$\begin{aligned}
 \int (ce + dex) (a + b \tanh^{-1}(c + dx))^3 dx &= \frac{\text{Subst}\left(\int ex (a + b \tanh^{-1}(x))^3 dx, x, c + dx\right)}{d} \\
 &= \frac{e \text{Subst}\left(\int x (a + b \tanh^{-1}(x))^3 dx, x, c + dx\right)}{d} \\
 &= \frac{e(c + dx)^2 (a + b \tanh^{-1}(c + dx))^3}{2d} - \frac{(3be) \text{Subst}\left(\int \frac{x^2 (a + b \tanh^{-1}(x))^2}{1-x^2} dx, x, c + dx\right)}{2d} \\
 &= \frac{e(c + dx)^2 (a + b \tanh^{-1}(c + dx))^3}{2d} + \frac{(3be) \text{Subst}\left(\int (a + b \tanh^{-1}(x))^2 dx, x, c + dx\right)}{2d} \\
 &= \frac{3be(c + dx) (a + b \tanh^{-1}(c + dx))^2}{2d} - \frac{e (a + b \tanh^{-1}(c + dx))^3}{2d} + \frac{e(c + dx) (a + b \tanh^{-1}(c + dx))^2}{2d} \\
 &= \frac{3be (a + b \tanh^{-1}(c + dx))^2}{2d} + \frac{3be(c + dx) (a + b \tanh^{-1}(c + dx))^2}{2d} \\
 &= \frac{3be (a + b \tanh^{-1}(c + dx))^2}{2d} + \frac{3be(c + dx) (a + b \tanh^{-1}(c + dx))^2}{2d} \\
 &= \frac{3be (a + b \tanh^{-1}(c + dx))^2}{2d} + \frac{3be(c + dx) (a + b \tanh^{-1}(c + dx))^2}{2d} \\
 &= \frac{3be (a + b \tanh^{-1}(c + dx))^2}{2d} + \frac{3be(c + dx) (a + b \tanh^{-1}(c + dx))^2}{2d}
 \end{aligned}$$

Mathematica [A] time = 1.13, size = 213, normalized size = 1.33

$$\frac{e \left(a \left(-3ab(c^2 - 1) \log(-c - dx + 1) + 3ab(c^2 - 1) \log(c + dx + 1) + 2adx(2ac + adx + 3b) - 12b^2 \log\left(\frac{1}{\sqrt{1-(c+dx)}}\right) \right) \right)}{2d}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(c*e + d*e*x)*(a + b*ArcTanh[c + d*x])^3,x]

[Out] (e*(6*b^2*(-1 + c + d*x)*(b + a*(1 + c + d*x))*ArcTanh[c + d*x]^2 + 2*b^3*(-1 + c^2 + 2*c*d*x + d^2*x^2)*ArcTanh[c + d*x]^3 + 6*b*ArcTanh[c + d*x]*(a*(2*b*(c + d*x) + a*d*x*(2*c + d*x)) - 2*b^2*Log[1 + E^(-2*ArcTanh[c + d*x])]) + a*(2*a*d*x*(3*b + 2*a*c + a*d*x) - 3*a*b*(-1 + c^2)*Log[1 - c - d*x] + 3*a*b*(-1 + c^2)*Log[1 + c + d*x] - 12*b^2*Log[1/Sqrt[1 - (c + d*x)^2]]) + 6*b^3*PolyLog[2, -E^(-2*ArcTanh[c + d*x])]))/(4*d)

fricas [F] time = 0.46, size = 0, normalized size = 0.00

integral(a^3dex + a^3ce + (b^3dex + b^3ce) artanh(dx + c)^3 + 3(ab^2dex + ab^2ce) artanh(dx + c)^2 + 3(a^2bdex + a^2bce) artanh(dx + c), x)

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*e*x+c*e)*(a+b*arctanh(d*x+c))^3,x, algorithm="fricas")

[Out] integral(a^3*d*e*x + a^3*c*e + (b^3*d*e*x + b^3*c*e)*arctanh(d*x + c)^3 + 3*(a*b^2*d*e*x + a*b^2*c*e)*arctanh(d*x + c)^2 + 3*(a^2*b*d*e*x + a^2*b*c*e)*arctanh(d*x + c), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dex + ce)(b \operatorname{artanh}(dx + c) + a)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*e*x+c*e)*(a+b*arctanh(d*x+c))^3,x, algorithm="giac")

[Out] integrate((d*e*x + c*e)*(b*arctanh(d*x + c) + a)^3, x)

maple [C] time = 0.61, size = 6834, normalized size = 42.71

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*e*x+c*e)*(a+b*arctanh(d*x+c))^3,x)

[Out] result too large to display

maxima [B] time = 0.64, size = 629, normalized size = 3.93

$$\frac{1}{2} a^3 dex^2 + \frac{3}{4} \left(2x^2 \operatorname{artanh}(dx + c) + d \left(\frac{2x}{d^2} - \frac{(c^2 + 2c + 1) \log(dx + c + 1)}{d^3} + \frac{(c^2 - 2c + 1) \log(dx + c - 1)}{d^3} \right) \right) a^2 b$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*e*x+c*e)*(a+b*arctanh(d*x+c))^3,x, algorithm="maxima")

[Out] $\frac{1}{2}a^3d^2e^2x^2 + \frac{3}{4}(2x^2\operatorname{arctanh}(dx+c) + d(2x/d^2 - (c^2 + 2c + 1)\log(dx+c+1)/d^3 + (c^2 - 2c + 1)\log(dx+c-1)/d^3))a^2b^2d^2e + a^3c^2e^2x + \frac{3}{2}(2(dx+c)\operatorname{arctanh}(dx+c) + \log(-(dx+c)^2 + 1))a^2b^2c^2e/d + \frac{3}{2}(\log(dx+c+1)\log(-1/2dx - 1/2c + 1/2) + \operatorname{dilog}(1/2dx + 1/2c + 1/2))b^3e/d + \frac{3}{2}(c^2e + e)a^2b^2\log(dx+c+1)/d - \frac{3}{2}(c^2e - e)a^2b^2\log(dx+c-1)/d + \frac{1}{16}(24a^2b^2d^2e^2x\log(dx+c+1) + (b^3d^2e^2x^2 + 2b^3c^2d^2e^2x + (c^2e - e)b^3)\log(dx+c+1)^3 - (b^3d^2e^2x^2 + 2b^3c^2d^2e^2x + (c^2e - e)b^3)\log(-dx-c+1)^3 + 6(a^2b^2d^2e^2x^2 + (c^2e - e)a^2b^2 + (c^2e + e)b^3 + (2a^2b^2c^2d^2e + b^3d^2e)x)\log(dx+c+1)^2 + 3(2a^2b^2d^2e^2x^2 + 2(c^2e - e)a^2b^2 + 2(c^2e - e)b^3 + 2(2a^2b^2c^2d^2e + b^3d^2e)x + (b^3d^2e^2x^2 + 2b^3c^2d^2e^2x + (c^2e - e)b^3)\log(dx+c+1))\log(-dx-c+1)^2 - 3(8a^2b^2d^2e^2x + (b^3d^2e^2x^2 + 2b^3c^2d^2e^2x + (c^2e - e)b^3)\log(dx+c+1)^2 + 4(a^2b^2d^2e^2x^2 + (c^2e - e)a^2b^2 + (c^2e + e)b^3 + (2a^2b^2c^2d^2e + b^3d^2e)x)\log(dx+c+1))\log(-dx-c+1))/d$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (ce + dex) (a + b \operatorname{atanh}(c + dx))^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*e + d*e*x)*(a + b*atanh(c + d*x))^3,x)

[Out] int((c*e + d*e*x)*(a + b*atanh(c + d*x))^3, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$e \left(\int a^3 c dx + \int a^3 dx dx + \int b^3 c \operatorname{atanh}^3(c + dx) dx + \int 3ab^2 c \operatorname{atanh}^2(c + dx) dx + \int 3a^2 b c \operatorname{atanh}(c + dx) dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*e*x+c*e)*(a+b*atanh(d*x+c))**3,x)

[Out] $e * (\operatorname{Integral}(a^{**3}c, x) + \operatorname{Integral}(a^{**3}d*x, x) + \operatorname{Integral}(b^{**3}c*\operatorname{atanh}(c + d*x)^{**3}, x) + \operatorname{Integral}(3*a*b^{**2}c*\operatorname{atanh}(c + d*x)^{**2}, x) + \operatorname{Integral}(3*a^{**2}b*c*\operatorname{atanh}(c + d*x), x) + \operatorname{Integral}(b^{**3}d*x*\operatorname{atanh}(c + d*x)^{**3}, x) + \operatorname{Integral}(3*a*b^{**2}d*x*\operatorname{atanh}(c + d*x)^{**2}, x) + \operatorname{Integral}(3*a^{**2}b*d*x*\operatorname{atanh}(c + d*x), x))$

$$3.25 \quad \int \frac{(a+b \tanh^{-1}(c+dx))^3}{ce+dex} dx$$

Optimal. Leaf size=257

$$\frac{3b^2 \text{Li}_3\left(1 - \frac{2}{-c-dx+1}\right)(a+b \tanh^{-1}(c+dx))}{2de} - \frac{3b^2 \text{Li}_3\left(\frac{2}{-c-dx+1} - 1\right)(a+b \tanh^{-1}(c+dx))}{2de} - \frac{3b \text{Li}_2\left(1 - \frac{2}{-c-dx+1}\right)(a+b \tanh^{-1}(c+dx))}{2a}$$

[Out] $-2*(a+b*\text{arctanh}(d*x+c))^3*\text{arctanh}(-1+2/(-d*x-c+1))/d/e-3/2*b*(a+b*\text{arctanh}(d*x+c))^2*\text{polylog}(2,1-2/(-d*x-c+1))/d/e+3/2*b*(a+b*\text{arctanh}(d*x+c))^2*\text{polylog}(2,-1+2/(-d*x-c+1))/d/e+3/2*b^2*(a+b*\text{arctanh}(d*x+c))*\text{polylog}(3,1-2/(-d*x-c+1))/d/e-3/2*b^2*(a+b*\text{arctanh}(d*x+c))*\text{polylog}(3,-1+2/(-d*x-c+1))/d/e-3/4*b^3*\text{polylog}(4,1-2/(-d*x-c+1))/d/e+3/4*b^3*\text{polylog}(4,-1+2/(-d*x-c+1))/d/e$

Rubi [A] time = 0.49, antiderivative size = 257, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {6107, 12, 5914, 6052, 5948, 6058, 6062, 6610}

$$\frac{3b^2 \text{PolyLog}\left(3, 1 - \frac{2}{-c-dx+1}\right)(a+b \tanh^{-1}(c+dx))}{2de} - \frac{3b^2 \text{PolyLog}\left(3, \frac{2}{-c-dx+1} - 1\right)(a+b \tanh^{-1}(c+dx))}{2de} - \frac{3b \text{PolyLog}\left(2, 1 - \frac{2}{-c-dx+1}\right)(a+b \tanh^{-1}(c+dx))}{2a}$$

Antiderivative was successfully verified.

[In] Int[(a + b*ArcTanh[c + d*x])^3/(c*e + d*e*x), x]

[Out] $(2*(a + b*\text{ArcTanh}[c + d*x])^3*\text{ArcTanh}[1 - 2/(1 - c - d*x)])/(d*e) - (3*b*(a + b*\text{ArcTanh}[c + d*x])^2*\text{PolyLog}[2, 1 - 2/(1 - c - d*x)])/(2*d*e) + (3*b*(a + b*\text{ArcTanh}[c + d*x])^2*\text{PolyLog}[2, -1 + 2/(1 - c - d*x)])/(2*d*e) + (3*b^2*(a + b*\text{ArcTanh}[c + d*x])*\text{PolyLog}[3, 1 - 2/(1 - c - d*x)])/(2*d*e) - (3*b^2*(a + b*\text{ArcTanh}[c + d*x])*\text{PolyLog}[3, -1 + 2/(1 - c - d*x)])/(2*d*e) - (3*b^3*\text{PolyLog}[4, 1 - 2/(1 - c - d*x)])/(4*d*e) + (3*b^3*\text{PolyLog}[4, -1 + 2/(1 - c - d*x)])/(4*d*e)$

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 5914

Int[((a_.) + ArcTanh[(c_.)*(x_.)]*(b_.))^p/(x_), x_Symbol] := Simp[2*(a + b*ArcTanh[c*x])^p*ArcTanh[1 - 2/(1 - c*x)], x] - Dist[2*b*c^p, Int[(a + b*ArcTanh[c*x])^(p-1)*ArcTanh[1 - 2/(1 - c*x)]/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 1]

Rule 5948

Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]

Rule 6052

Int[(ArcTanh[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2), x_Symbol] :> Dist[1/2, Int[(Log[1 + u]*(a + b*ArcTanh[c*x])^p)/(d + e*x^2), x], x] - Dist[1/2, Int[(Log[1 - u]*(a + b*ArcTanh[c*x])^p)/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e, 0] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]

Rule 6058

Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2), x_Symbol] :> -Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d), x] + Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]

Rule 6062

Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_]/((d_) + (e_.)*(x_)^2), x_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*PolyLog[k + 1, u])/(2*c*d), x] - Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[k + 1, u])/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] && EqQ[c^2*d + e, 0] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]

Rule 6107

Int[((a_.) + ArcTanh[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m_.), x_Symbol] :> Dist[1/d, Subst[Int[((f*x)/d)^m*(a + b*ArcTanh[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] && IGtQ[p, 0]

Rule 6610

Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v, x]}, Simp[w*PolyLog[n + 1, v], x] /; !FalseQ[w]] /; FreeQ[n, x]

Rubi steps

$$\begin{aligned}
\int \frac{(a + b \tanh^{-1}(c + dx))^3}{ce + dex} dx &= \frac{\text{Subst} \left(\int \frac{(a+b \tanh^{-1}(x))^3}{ex} dx, x, c + dx \right)}{d} \\
&= \frac{\text{Subst} \left(\int \frac{(a+b \tanh^{-1}(x))^3}{x} dx, x, c + dx \right)}{de} \\
&= \frac{2(a + b \tanh^{-1}(c + dx))^3 \tanh^{-1} \left(1 - \frac{2}{1-c-dx} \right)}{de} - \frac{(6b) \text{Subst} \left(\int \frac{\tanh^{-1} \left(1 - \frac{2}{1-x} \right) (a+b \tanh^{-1}(x))}{1-x^2} dx, x, c + dx \right)}{de} \\
&= \frac{2(a + b \tanh^{-1}(c + dx))^3 \tanh^{-1} \left(1 - \frac{2}{1-c-dx} \right)}{de} - \frac{(3b) \text{Subst} \left(\int \frac{(a+b \tanh^{-1}(x))^2 \log(2)}{1-x^2} dx, x, c + dx \right)}{de} \\
&= \frac{2(a + b \tanh^{-1}(c + dx))^3 \tanh^{-1} \left(1 - \frac{2}{1-c-dx} \right)}{de} - \frac{3b(a + b \tanh^{-1}(c + dx))^2 \text{Li}_2 \left(\frac{2}{1-c-dx} \right)}{2de} \\
&= \frac{2(a + b \tanh^{-1}(c + dx))^3 \tanh^{-1} \left(1 - \frac{2}{1-c-dx} \right)}{de} - \frac{3b(a + b \tanh^{-1}(c + dx))^2 \text{Li}_2 \left(\frac{2}{1-c-dx} \right)}{2de} \\
&= \frac{2(a + b \tanh^{-1}(c + dx))^3 \tanh^{-1} \left(1 - \frac{2}{1-c-dx} \right)}{de} - \frac{3b(a + b \tanh^{-1}(c + dx))^2 \text{Li}_2 \left(\frac{2}{1-c-dx} \right)}{2de}
\end{aligned}$$

Mathematica [C] time = 0.51, size = 599, normalized size = 2.33

$$64a^3 \log(c + dx) - 96ia^2b \left(-i \text{Li}_2 \left(e^{-2 \tanh^{-1}(c+dx)} \right) \right) - i \text{Li}_2 \left(-e^{2 \tanh^{-1}(c+dx)} \right) - \frac{1}{4}i \left(\pi - 2i \tanh^{-1}(c + dx) \right)^2 + i \tanh^{-1}(c + dx)$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b*ArcTanh[c + d*x])^3/(c*e + d*e*x), x]

[Out] (64*a^3*Log[c + d*x] + 192*a^2*b*ArcTanh[c + d*x]*(-Log[1/Sqrt[1 - (c + d*x)^2]] + Log[(I*(c + d*x))/Sqrt[1 - (c + d*x)^2]]) - (96*I)*a^2*b*((-1/4*I)*(Pi - (2*I)*ArcTanh[c + d*x])^2 + I*ArcTanh[c + d*x]^2 + (2*I)*ArcTanh[c + d*x]*Log[1 - E^(-2*ArcTanh[c + d*x])]) + (Pi - (2*I)*ArcTanh[c + d*x])*Log[1 + E^(2*ArcTanh[c + d*x])] - (Pi - (2*I)*ArcTanh[c + d*x])*Log[2/Sqrt[1 - (c + d*x)^2]] - (2*I)*ArcTanh[c + d*x]*Log[((2*I)*(c + d*x))/Sqrt[1 - (c + d*x)^2]]

$x^2]] - I \cdot \text{PolyLog}[2, E^{(-2 \cdot \text{ArcTanh}[c + d \cdot x])}] - I \cdot \text{PolyLog}[2, -E^{(2 \cdot \text{ArcTanh}[c + d \cdot x])}] + 8 \cdot a \cdot b^2 \cdot (I \cdot \text{Pi}^3 - 16 \cdot \text{ArcTanh}[c + d \cdot x]^3 - 24 \cdot \text{ArcTanh}[c + d \cdot x]^2 \cdot \text{Log}[1 + E^{(-2 \cdot \text{ArcTanh}[c + d \cdot x])}] + 24 \cdot \text{ArcTanh}[c + d \cdot x]^2 \cdot \text{Log}[1 - E^{(2 \cdot \text{ArcTanh}[c + d \cdot x])}] + 24 \cdot \text{ArcTanh}[c + d \cdot x] \cdot \text{PolyLog}[2, -E^{(-2 \cdot \text{ArcTanh}[c + d \cdot x])}] + 24 \cdot \text{ArcTanh}[c + d \cdot x] \cdot \text{PolyLog}[2, E^{(2 \cdot \text{ArcTanh}[c + d \cdot x])}] + 12 \cdot \text{PolyLog}[3, -E^{(-2 \cdot \text{ArcTanh}[c + d \cdot x])}] - 12 \cdot \text{PolyLog}[3, E^{(2 \cdot \text{ArcTanh}[c + d \cdot x])}] + b^3 \cdot (\text{Pi}^4 - 32 \cdot \text{ArcTanh}[c + d \cdot x]^4 - 64 \cdot \text{ArcTanh}[c + d \cdot x]^3 \cdot \text{Log}[1 + E^{(-2 \cdot \text{ArcTanh}[c + d \cdot x])}] + 64 \cdot \text{ArcTanh}[c + d \cdot x]^3 \cdot \text{Log}[1 - E^{(2 \cdot \text{ArcTanh}[c + d \cdot x])}] + 96 \cdot \text{ArcTanh}[c + d \cdot x]^2 \cdot \text{PolyLog}[2, -E^{(-2 \cdot \text{ArcTanh}[c + d \cdot x])}] + 96 \cdot \text{ArcTanh}[c + d \cdot x]^2 \cdot \text{PolyLog}[2, E^{(2 \cdot \text{ArcTanh}[c + d \cdot x])}] + 96 \cdot \text{ArcTanh}[c + d \cdot x] \cdot \text{PolyLog}[3, -E^{(-2 \cdot \text{ArcTanh}[c + d \cdot x])}] - 96 \cdot \text{ArcTanh}[c + d \cdot x] \cdot \text{PolyLog}[3, E^{(2 \cdot \text{ArcTanh}[c + d \cdot x])}] + 48 \cdot \text{PolyLog}[4, -E^{(-2 \cdot \text{ArcTanh}[c + d \cdot x])}] + 48 \cdot \text{PolyLog}[4, E^{(2 \cdot \text{ArcTanh}[c + d \cdot x])}])) / (64 \cdot d \cdot e)$

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^3 \operatorname{artanh}(dx + c)^3 + 3ab^2 \operatorname{artanh}(dx + c)^2 + 3a^2b \operatorname{artanh}(dx + c) + a^3}{dex + ce}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arctanh(d*x+c))^3/(d*e*x+c*e),x, algorithm="fricas")

[Out] integral((b^3*arctanh(d*x + c)^3 + 3*a*b^2*arctanh(d*x + c)^2 + 3*a^2*b*arctanh(d*x + c) + a^3)/(d*e*x + c*e), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{artanh}(dx + c) + a)^3}{dex + ce} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arctanh(d*x+c))^3/(d*e*x+c*e),x, algorithm="giac")

[Out] integrate((b*arctanh(d*x + c) + a)^3/(d*e*x + c*e), x)

maple [C] time = 0.18, size = 1842, normalized size = 7.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*arctanh(d*x+c))^3/(d*e*x+c*e),x)

[Out] 1/d*b^3/e*arctanh(d*x+c)^3*ln(1-(d*x+c+1)/(1-(d*x+c)^2)^(1/2))-1/d*b^3/e*arctanh(d*x+c)^3*ln((d*x+c+1)^2/(1-(d*x+c)^2)-1)+3/2/d*b^3/e*arctanh(d*x+c)*p

$$\begin{aligned} & \text{polylog}(3, -(d*x+c+1)^2/(1-(d*x+c)^2)) - 3/2/d*b^3/e*\text{arctanh}(d*x+c)^2*\text{polylog}(2, \\ & -(d*x+c+1)^2/(1-(d*x+c)^2)) + 3/d*b^3/e*\text{arctanh}(d*x+c)^2*\text{polylog}(2, (d*x+c+1) \\ & / (1-(d*x+c)^2)^{(1/2)}) - 6/d*b^3/e*\text{arctanh}(d*x+c)*\text{polylog}(3, (d*x+c+1)/(1-(d*x+ \\ & c)^2)^{(1/2)}) + 1/d*b^3/e*\text{arctanh}(d*x+c)^3*\ln(1+(d*x+c+1)/(1-(d*x+c)^2)^{(1/2)}) \\ & + 3/d*b^3/e*\text{arctanh}(d*x+c)^2*\text{polylog}(2, -(d*x+c+1)/(1-(d*x+c)^2)^{(1/2)}) - 6/d*b \\ & ^3/e*\text{arctanh}(d*x+c)*\text{polylog}(3, -(d*x+c+1)/(1-(d*x+c)^2)^{(1/2)}) + 3/2/d*a*b^2/e \\ & *\text{polylog}(3, -(d*x+c+1)^2/(1-(d*x+c)^2)) - 6/d*a*b^2/e*\text{polylog}(3, (d*x+c+1)/(1-(\\ & d*x+c)^2)^{(1/2)}) - 6/d*a*b^2/e*\text{polylog}(3, -(d*x+c+1)/(1-(d*x+c)^2)^{(1/2)}) - 3/2/ \\ & d*a^2*b/e*\text{dilog}(d*x+c) - 3/2/d*a^2*b/e*\text{dilog}(d*x+c+1) + 1/d*b^3/e*\ln(d*x+c)*\text{arc} \\ & \text{tanh}(d*x+c)^3 - 1/2*I/d*b^3/e*Pi*\text{csgn}(I*((d*x+c+1)^2/(1-(d*x+c)^2)-1))*\text{csgn}(I \\ & *((d*x+c+1)^2/(1-(d*x+c)^2)-1)/(1+(d*x+c+1)^2/(1-(d*x+c)^2)))^2*\text{arctanh}(d*x \\ & +c)^3 - 1/2*I/d*b^3/e*Pi*\text{csgn}(I/(1+(d*x+c+1)^2/(1-(d*x+c)^2)))*\text{csgn}(I*((d*x+c \\ & +1)^2/(1-(d*x+c)^2)-1)/(1+(d*x+c+1)^2/(1-(d*x+c)^2)))^2*\text{arctanh}(d*x+c)^3 + 6/ \\ & d*b^3/e*\text{polylog}(4, -(d*x+c+1)/(1-(d*x+c)^2)^{(1/2)}) + 1/d*a^3/e*\ln(d*x+c) - 3/4/d \\ & *b^3/e*\text{polylog}(4, -(d*x+c+1)^2/(1-(d*x+c)^2)) + 6/d*b^3/e*\text{polylog}(4, (d*x+c+1)/ \\ & (1-(d*x+c)^2)^{(1/2)}) + 3/d*a*b^2/e*\ln(d*x+c)*\text{arctanh}(d*x+c)^2 + 3/d*a*b^2/e*\text{arc} \\ & \text{tanh}(d*x+c)^2*\ln(1+(d*x+c+1)/(1-(d*x+c)^2)^{(1/2)}) + 6/d*a*b^2/e*\text{arctanh}(d*x+c \\ &)*\text{polylog}(2, -(d*x+c+1)/(1-(d*x+c)^2)^{(1/2)}) + 3/d*a^2*b/e*\ln(d*x+c)*\text{arctanh}(d \\ & *x+c) - 3/2/d*a^2*b/e*\ln(d*x+c)*\ln(d*x+c+1) - 3/d*a*b^2/e*\text{arctanh}(d*x+c)*\text{polylo} \\ & \text{g}(2, -(d*x+c+1)^2/(1-(d*x+c)^2)) - 3/d*a*b^2/e*\text{arctanh}(d*x+c)^2*\ln((d*x+c+1)^2 \\ & / (1-(d*x+c)^2)-1) + 3/d*a*b^2/e*\text{arctanh}(d*x+c)^2*\ln(1-(d*x+c+1)/(1-(d*x+c)^2) \\ & ^{(1/2)}) - 3/2*I/d*a*b^2/e*Pi*\text{csgn}(I/(1+(d*x+c+1)^2/(1-(d*x+c)^2)))*\text{csgn}(I*((d \\ & *x+c+1)^2/(1-(d*x+c)^2)-1)/(1+(d*x+c+1)^2/(1-(d*x+c)^2)))^2*\text{arctanh}(d*x+c)^ \\ & 2 - 3/2*I/d*a*b^2/e*Pi*\text{csgn}(I*((d*x+c+1)^2/(1-(d*x+c)^2)-1))*\text{csgn}(I*((d*x+c+1 \\ &)^2/(1-(d*x+c)^2)-1)/(1+(d*x+c+1)^2/(1-(d*x+c)^2)))^2*\text{arctanh}(d*x+c)^2 + 3/2* \\ & I/d*a*b^2/e*Pi*\text{csgn}(I*((d*x+c+1)^2/(1-(d*x+c)^2)-1))*\text{csgn}(I/(1+(d*x+c+1)^2/ \\ & (1-(d*x+c)^2)))*\text{csgn}(I*((d*x+c+1)^2/(1-(d*x+c)^2)-1)/(1+(d*x+c+1)^2/(1-(d*x \\ & +c)^2)))*\text{arctanh}(d*x+c)^2 + 1/2*I/d*b^3/e*Pi*\text{csgn}(I*((d*x+c+1)^2/(1-(d*x+c)^2) \\ &)-1))*\text{csgn}(I/(1+(d*x+c+1)^2/(1-(d*x+c)^2)))*\text{csgn}(I*((d*x+c+1)^2/(1-(d*x+c)^ \\ & 2)-1)/(1+(d*x+c+1)^2/(1-(d*x+c)^2)))*\text{arctanh}(d*x+c)^3 + 1/2*I/d*b^3/e*Pi*\text{csgn} \\ & (I*((d*x+c+1)^2/(1-(d*x+c)^2)-1)/(1+(d*x+c+1)^2/(1-(d*x+c)^2)))^3*\text{arctanh}(d \\ & *x+c)^3 + 3/2*I/d*a*b^2/e*Pi*\text{csgn}(I*((d*x+c+1)^2/(1-(d*x+c)^2)-1)/(1+(d*x+c+1 \\ &)^2/(1-(d*x+c)^2)))^3*\text{arctanh}(d*x+c)^2 + 6/d*a*b^2/e*\text{arctanh}(d*x+c)*\text{polylog}(2 \\ & , (d*x+c+1)/(1-(d*x+c)^2)^{(1/2)}) \end{aligned}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{a^3 \log(dx + ce)}{de} + \int \frac{b^3 (\log(dx + c + 1) - \log(-dx - c + 1))^3}{8(dx + ce)} + \frac{3ab^2 (\log(dx + c + 1) - \log(-dx - c + 1))^2}{4(dx + ce)} + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arctanh(d*x+c))^3/(d*e*x+c*e), x, algorithm="maxima")

[Out] a^3*log(d*e*x + c*e)/(d*e) + integrate(1/8*b^3*(log(d*x + c + 1) - log(-d*x - c + 1))^3/(d*e*x + c*e) + 3/4*a*b^2*(log(d*x + c + 1) - log(-d*x - c + 1)

$)^2/(d*e*x + c*e) + 3/2*a^2*b*(\log(d*x + c + 1) - \log(-d*x - c + 1))/(d*e*x + c*e), x)$

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + b \operatorname{atanh}(c + dx))^3}{ce + dex} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b*atanh(c + d*x))^3/(c*e + d*e*x), x)`

[Out] `int((a + b*atanh(c + d*x))^3/(c*e + d*e*x), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a^3}{c+dx} dx + \int \frac{b^3 \operatorname{atanh}^3(c+dx)}{c+dx} dx + \int \frac{3ab^2 \operatorname{atanh}^2(c+dx)}{c+dx} dx + \int \frac{3a^2b \operatorname{atanh}(c+dx)}{c+dx} dx}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*atanh(d*x+c))**3/(d*e*x+c*e), x)`

[Out] `(Integral(a**3/(c + d*x), x) + Integral(b**3*atanh(c + d*x)**3/(c + d*x), x) + Integral(3*a*b**2*atanh(c + d*x)**2/(c + d*x), x) + Integral(3*a**2*b*a tanh(c + d*x)/(c + d*x), x))/e`

$$3.26 \quad \int \frac{(a+b \tanh^{-1}(c+dx))^3}{(ce+dex)^2} dx$$

Optimal. Leaf size=143

$$\frac{3b^2 \operatorname{Li}_2\left(\frac{2}{c+dx+1} - 1\right) (a+b \tanh^{-1}(c+dx))}{de^2} - \frac{(a+b \tanh^{-1}(c+dx))^3}{de^2(c+dx)} + \frac{(a+b \tanh^{-1}(c+dx))^3}{de^2} + \frac{3b \log\left(2 - \frac{2}{c+dx}\right)}{de^2}$$

[Out] (a+b*arctanh(d*x+c))^3/d/e^2-(a+b*arctanh(d*x+c))^3/d/e^2/(d*x+c)+3*b*(a+b*arctanh(d*x+c))^2*ln(2-2/(d*x+c+1))/d/e^2-3*b^2*(a+b*arctanh(d*x+c))*polylog(2,-1+2/(d*x+c+1))/d/e^2-3/2*b^3*polylog(3,-1+2/(d*x+c+1))/d/e^2

Rubi [A] time = 0.30, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {6107, 12, 5916, 5988, 5932, 5948, 6056, 6610}

$$\frac{3b^2 \operatorname{PolyLog}\left(2, \frac{2}{c+dx+1} - 1\right) (a+b \tanh^{-1}(c+dx))}{de^2} - \frac{3b^3 \operatorname{PolyLog}\left(3, \frac{2}{c+dx+1} - 1\right) (a+b \tanh^{-1}(c+dx))^3}{2de^2} - \frac{(a+b \tanh^{-1}(c+dx))^3}{de^2(c+dx)} + \frac{(a+b \tanh^{-1}(c+dx))^3}{de^2}$$

Antiderivative was successfully verified.

[In] Int[(a + b*ArcTanh[c + d*x])^3/(c*e + d*e*x)^2,x]

[Out] (a + b*ArcTanh[c + d*x])^3/(d*e^2) - (a + b*ArcTanh[c + d*x])^3/(d*e^2*(c + d*x)) + (3*b*(a + b*ArcTanh[c + d*x])^2*Log[2 - 2/(1 + c + d*x)])/(d*e^2) - (3*b^2*(a + b*ArcTanh[c + d*x])*PolyLog[2, -1 + 2/(1 + c + d*x)])/(d*e^2) - (3*b^3*PolyLog[3, -1 + 2/(1 + c + d*x)])/(2*d*e^2)

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 5916

Int[((a_.) + ArcTanh[(c_.)*(x_)])*(b_.))^p*((d_.)*(x_))^(m_.), x_Symbol] := Simp[((d*x)^(m+1)*(a + b*ArcTanh[c*x])^p)/(d*(m+1)), x] - Dist[(b*c*p)/(d*(m+1)), Int[((d*x)^(m+1)*(a + b*ArcTanh[c*x])^(p-1))/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || IntegerQ[m]) && NeQ[m, -1]

Rule 5932


```
Int[((a_.) + ArcTanh[(c_.)*(x_)])*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] := Simp[((a + b*ArcTanh[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] -
Dist[(b*c*p)/d, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])
/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)])*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] := Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 5988

```
Int[((a_.) + ArcTanh[(c_.)*(x_)])*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] := Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 6056

```
Int[(Log[u_] * ((a_.) + ArcTanh[(c_.)*(x_)])*(b_.))^(p_.)/((d_) + (e_.)*(x_)^
2), x_Symbol] := Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d), x
] - Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d +
e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e
, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6107

```
Int[((a_.) + ArcTanh[(c_) + (d_.)*(x_)])*(b_.))^(p_.)*((e_.) + (f_.)*(x_)^(
m_.), x_Symbol] := Dist[1/d, Subst[Int[((f*x)/d)^m*(a + b*ArcTanh[x])^p, x]
, x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] &
& IGtQ[p, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] := With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /; !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a + b \tanh^{-1}(c + dx))^3}{(ce + dex)^2} dx &= \frac{\text{Subst} \left(\int \frac{(a+b \tanh^{-1}(x))^3}{e^2 x^2} dx, x, c + dx \right)}{d} \\
&= \frac{\text{Subst} \left(\int \frac{(a+b \tanh^{-1}(x))^3}{x^2} dx, x, c + dx \right)}{de^2} \\
&= -\frac{(a + b \tanh^{-1}(c + dx))^3}{de^2(c + dx)} + \frac{(3b) \text{Subst} \left(\int \frac{(a+b \tanh^{-1}(x))^2}{x(1-x^2)} dx, x, c + dx \right)}{de^2} \\
&= \frac{(a + b \tanh^{-1}(c + dx))^3}{de^2} - \frac{(a + b \tanh^{-1}(c + dx))^3}{de^2(c + dx)} + \frac{(3b) \text{Subst} \left(\int \frac{(a+b \tanh^{-1}(x))}{x(1+x)} dx, x, c + dx \right)}{de^2} \\
&= \frac{(a + b \tanh^{-1}(c + dx))^3}{de^2} - \frac{(a + b \tanh^{-1}(c + dx))^3}{de^2(c + dx)} + \frac{3b(a + b \tanh^{-1}(c + dx))^2}{de^2} \\
&= \frac{(a + b \tanh^{-1}(c + dx))^3}{de^2} - \frac{(a + b \tanh^{-1}(c + dx))^3}{de^2(c + dx)} + \frac{3b(a + b \tanh^{-1}(c + dx))^2}{de^2} \\
&= \frac{(a + b \tanh^{-1}(c + dx))^3}{de^2} - \frac{(a + b \tanh^{-1}(c + dx))^3}{de^2(c + dx)} + \frac{3b(a + b \tanh^{-1}(c + dx))^2}{de^2}
\end{aligned}$$

Mathematica [C] time = 0.61, size = 248, normalized size = 1.73

$$-\frac{2a^3}{c+dx} - 3a^2b \log(-c^2 - 2cdx - d^2x^2 + 1) + 6a^2b \log(c + dx) - \frac{6a^2b \tanh^{-1}(c+dx)}{c+dx} + 6ab^2 \left(\tanh^{-1}(c + dx) \left(\left(1 - \frac{1}{c+dx} \right) \right) \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b*ArcTanh[c + d*x])^3/(c*e + d*e*x)^2,x]

[Out] ((-2*a^3)/(c + d*x) - (6*a^2*b*ArcTanh[c + d*x])/(c + d*x) + 6*a^2*b*Log[c + d*x] - 3*a^2*b*Log[1 - c^2 - 2*c*d*x - d^2*x^2] + 6*a*b^2*(ArcTanh[c + d*x]*((1 - (c + d*x)^(-1))*ArcTanh[c + d*x] + 2*Log[1 - E^(-2*ArcTanh[c + d*x]])]) - PolyLog[2, E^(-2*ArcTanh[c + d*x]])] + 2*b^3*((I/8)*Pi^3 - ArcTanh[c + d*x]^3 - ArcTanh[c + d*x]^3/(c + d*x) + 3*ArcTanh[c + d*x]^2*Log[1 - E^(-

$2*\text{ArcTanh}[c + d*x]) + 3*\text{ArcTanh}[c + d*x]*\text{PolyLog}[2, E^{(2*\text{ArcTanh}[c + d*x])}] - (3*\text{PolyLog}[3, E^{(2*\text{ArcTanh}[c + d*x])}]/2))/(2*d*e^2)$

fricas [F] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^3 \operatorname{artanh}(dx + c)^3 + 3ab^2 \operatorname{artanh}(dx + c)^2 + 3a^2b \operatorname{artanh}(dx + c) + a^3}{d^2e^2x^2 + 2cde^2x + c^2e^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arctanh(d*x+c))^3/(d*e*x+c*e)^2,x, algorithm="fricas")

[Out] integral((b^3*arctanh(d*x + c)^3 + 3*a*b^2*arctanh(d*x + c)^2 + 3*a^2*b*arctanh(d*x + c) + a^3)/(d^2*e^2*x^2 + 2*c*d*e^2*x + c^2*e^2), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{artanh}(dx + c) + a)^3}{(dex + ce)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arctanh(d*x+c))^3/(d*e*x+c*e)^2,x, algorithm="giac")

[Out] integrate((b*arctanh(d*x + c) + a)^3/(d*e*x + c*e)^2, x)

maple [C] time = 0.51, size = 2001, normalized size = 13.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*arctanh(d*x+c))^3/(d*e*x+c*e)^2,x)

[Out] $\frac{3}{2}*\frac{I}{d*b^3/e^2*Pi}*csgn(I*(d*x+c+1)/(1-(d*x+c)^2)^{(1/2)})*csgn(I*(d*x+c+1)^2/((d*x+c)^2-1))^2*\operatorname{arctanh}(d*x+c)^2-1/d*b^3/e^2*\operatorname{arctanh}(d*x+c)^3-6/d*b^3/e^2*\operatorname{polylog}(3, -(d*x+c+1)/(1-(d*x+c)^2)^{(1/2)})-6/d*b^3/e^2*\operatorname{polylog}(3, (d*x+c+1)/(1-(d*x+c)^2)^{(1/2)})-1/d*a^3/e^2/(d*x+c)-3/2*I/d*b^3/e^2*Pi*csgn(I/(1+(d*x+c+1)^2/(1-(d*x+c)^2)))^2*\operatorname{arctanh}(d*x+c)^2+3/4*I/d*b^3/e^2*Pi*csgn(I*(d*x+c+1)^2/((d*x+c)^2-1)/(1+(d*x+c+1)^2/(1-(d*x+c)^2)))^3*\operatorname{arctanh}(d*x+c)^2+3/2*I/d*b^3/e^2*Pi*csgn(I*((d*x+c+1)^2/(1-(d*x+c)^2)-1)/(1+(d*x+c+1)^2/(1-(d*x+c)^2)))^3*\operatorname{arctanh}(d*x+c)^2+3/4*I/d*b^3/e^2*Pi*csgn(I*(d*x+c+1)^2/((d*x+c)^2-1))^3*\operatorname{arctanh}(d*x+c)^2+3/2*I/d*b^3/e^2*Pi*csgn(I/(1+(d*x+c+1)^2/(1-(d*x+c)^2)))^3*\operatorname{arctanh}(d*x+c)^2+6/d*b^3/e^2*\operatorname{arctanh}(d*x+c)*\operatorname{polylog}(2, -(d*x+c+1)/(1-(d*x+c)^2)^{(1/2)})-3/2/d*b^3/e^2*\operatorname{arctanh}(d*x+c)^2*\ln(d*x+c-1)-3/2/d*b^3/e^2*a*\operatorname{rctanh}(d*x+c)^2*\ln(d*x+c+1)+3/d*b^3/e^2*\ln(d*x+c)*\operatorname{arctanh}(d*x+c)^2+3/d*b^3/e^2*\operatorname{arctanh}(d*x+c)^2*\ln(1-(d*x+c+1)/(1-(d*x+c)^2)^{(1/2)})+6/d*b^3/e^2*\operatorname{arctan}$

$$\begin{aligned}
& h(dx+c) \cdot \text{polylog}(2, (dx+c+1)/(1-(dx+c)^2)^{1/2}) - 3/2/d \cdot a^2 \cdot b/e^2 \cdot \ln(dx+c-1) \\
& - 3/2/d \cdot a^2 \cdot b/e^2 \cdot \ln(dx+c+1) - 3/4/d \cdot a \cdot b^2/e^2 \cdot \ln(dx+c-1)^2 + 3/d \cdot a \cdot b^2/e^2 \cdot \text{dilog}(1/2 + 1/2 \cdot dx + 1/2 \cdot c) \\
& + 3/4/d \cdot a \cdot b^2/e^2 \cdot \ln(dx+c+1)^2 - 3/d \cdot a \cdot b^2/e^2 \cdot \text{dilog}(dx+c) - 3/d \cdot a \cdot b^2/e^2 \cdot \text{dilog}(dx+c+1) \\
& + 3/d \cdot b^3/e^2 \cdot \text{arctanh}(dx+c)^2 \cdot \ln(2) + 3/d \cdot a^2 \cdot b/e^2 \cdot \ln(dx+c) - 3/2 \cdot I/d \cdot b^3/e^2 \cdot \text{Pi} \cdot \text{csgn}(I \cdot ((dx+c+1)^2/(1-(dx+c)^2) - 1)) \\
& \cdot \text{csgn}(I \cdot ((dx+c+1)^2/(1-(dx+c)^2) - 1)/(1+(dx+c+1)^2/(1-(dx+c)^2)))^2 \cdot \text{arctanh}(dx+c)^2 \\
& + 3/4 \cdot I/d \cdot b^3/e^2 \cdot \text{Pi} \cdot \text{csgn}(I/(1+(dx+c+1)^2/(1-(dx+c)^2))) \cdot \text{csgn}(I \cdot (dx+c+1)^2/((dx+c)^2 - 1)/(1+(dx+c+1)^2/(1-(dx+c)^2)))^2 \\
& \cdot \text{arctanh}(dx+c)^2 + 3/4 \cdot I/d \cdot b^3/e^2 \cdot \text{Pi} \cdot \text{csgn}(I \cdot (dx+c+1)/(1-(dx+c)^2)^{1/2})^2 \cdot \text{csgn}(I \cdot (dx+c+1)^2/((dx+c)^2 - 1)) \\
& \cdot \text{arctanh}(dx+c)^2 - 3/d \cdot a \cdot b^2/e^2 \cdot \text{arctanh}(dx+c) \cdot \ln(dx+c+1) + 3/2/d \cdot a \cdot b^2/e^2 \cdot \ln(dx+c-1) \cdot \ln(1/2 + 1/2 \cdot dx + 1/2 \cdot c) \\
& - 3/d \cdot a^2 \cdot b/e^2/(dx+c) \cdot \text{arctanh}(dx+c) - 3/2/d \cdot a \cdot b^2/e^2 \cdot \ln(-1/2 \cdot dx - 1/2 \cdot c + 1/2) \cdot \ln(dx+c+1) + 3/2/d \cdot a \cdot b^2/e^2 \cdot \ln(-1/2 \cdot dx - 1/2 \cdot c + 1/2) \cdot \ln(1/2 + 1/2 \cdot dx + 1/2 \cdot c) \\
& - 3/d \cdot a \cdot b^2/e^2 \cdot \ln(dx+c) \cdot \ln(dx+c+1) - 3/d \cdot a \cdot b^2/e^2/(dx+c) \cdot \text{arctanh}(dx+c)^2 + 6/d \cdot a \cdot b^2/e^2 \cdot \text{arctanh}(dx+c) \cdot \ln(dx+c) - 3/d \cdot a \cdot b^2/e^2 \cdot \text{arctanh}(dx+c) \cdot \ln(dx+c-1) \\
& + 3/2 \cdot I/d \cdot b^3/e^2 \cdot \text{Pi} \cdot \text{arctanh}(dx+c)^2 - 3/2 \cdot I/d \cdot b^3/e^2 \cdot \text{Pi} \cdot \text{csgn}(I/(1+(dx+c+1)^2/(1-(dx+c)^2))) \cdot \text{csgn}(I \cdot ((dx+c+1)^2/(1-(dx+c)^2) - 1)/(1+(dx+c+1)^2/(1-(dx+c)^2)))^2 \\
& \cdot \text{arctanh}(dx+c)^2 - 3/4 \cdot I/d \cdot b^3/e^2 \cdot \text{Pi} \cdot \text{csgn}(I/(1+(dx+c+1)^2/(1-(dx+c)^2))) \cdot \text{csgn}(I \cdot (dx+c+1)^2/((dx+c)^2 - 1)) \cdot \text{csgn}(I \cdot (dx+c+1)^2/((dx+c)^2 - 1)/(1+(dx+c+1)^2/(1-(dx+c)^2))) \\
& \cdot \text{arctanh}(dx+c)^2 + 3/2 \cdot I/d \cdot b^3/e^2 \cdot \text{Pi} \cdot \text{csgn}(I \cdot ((dx+c+1)^2/(1-(dx+c)^2) - 1)) \cdot \text{csgn}(I/(1+(dx+c+1)^2/(1-(dx+c)^2))) \cdot \text{csgn}(I \cdot ((dx+c+1)^2/(1-(dx+c)^2) - 1)/(1+(dx+c+1)^2/(1-(dx+c)^2))) \\
& \cdot \text{arctanh}(dx+c)^2 - 3/4 \cdot I/d \cdot b^3/e^2 \cdot \text{Pi} \cdot \text{csgn}(I \cdot (dx+c+1)^2/((dx+c)^2 - 1)) \cdot \text{csgn}(I \cdot (dx+c+1)^2/((dx+c)^2 - 1)/(1+(dx+c+1)^2/(1-(dx+c)^2)))^2 \cdot \text{arctanh}(dx+c)^2 - 1/d \cdot b^3/e^2/(dx+c) \cdot \text{arctanh}(dx+c)^3 \\
& + 3/d \cdot b^3/e^2 \cdot \text{arctanh}(dx+c)^2 \cdot \ln((dx+c+1)/(1-(dx+c)^2)^{1/2}) - 3/d \cdot b^3/e^2 \cdot \text{arctanh}(dx+c)^2 \cdot \ln((dx+c+1)^2/(1-(dx+c)^2) - 1) + 3/d \cdot b^3/e^2 \cdot \text{arctanh}(dx+c)^2 \cdot \ln(1+(dx+c+1)/(1-(dx+c)^2)^{1/2})
\end{aligned}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{3}{2} \left(d \left(\frac{\log(dx+c+1)}{d^2 e^2} - \frac{2 \log(dx+c)}{d^2 e^2} + \frac{\log(dx+c-1)}{d^2 e^2} \right) + \frac{2 \operatorname{artanh}(dx+c)}{d^2 e^2 x + c d e^2} \right) a^2 b - \frac{a^3}{d^2 e^2 x + c d e^2} - \frac{(b^3 dx + b^3)}{d^2 e^2 x + c d e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arctanh(dx+c))^3/(d*e*x+c*e)^2,x, algorithm="maxima")

[Out]
$$\begin{aligned}
& -3/2 \cdot (d \cdot (\log(dx+c+1)/(d^2 \cdot e^2) - 2 \cdot \log(dx+c)/(d^2 \cdot e^2) + \log(dx+c-1)/(d^2 \cdot e^2)) + 2 \cdot \text{arctanh}(dx+c)/(d^2 \cdot e^2 \cdot x + c \cdot d \cdot e^2)) \cdot a^2 \cdot b - a^3/(d^2 \cdot e^2 \cdot x + c \cdot d \cdot e^2) \\
& - 1/8 \cdot ((b^3 \cdot dx + b^3 \cdot (c-1)) \cdot \log(-dx-c+1)^3 + 3 \cdot (2 \cdot a \cdot b^2 + (b^3 \cdot dx + b^3 \cdot (c+1)) \cdot \log(dx+c+1)) \cdot \log(-dx-c+1)^2)/(d^2 \cdot e^2 \cdot x + c \cdot d \cdot e^2) \\
& - \text{integrate}(-1/8 \cdot ((b^3 \cdot dx + b^3 \cdot (c-1)) \cdot \log(dx+c+1)^3 + 6 \cdot (a \cdot b^2 \cdot dx + a \cdot b^2 \cdot (c-1)) \cdot \log(dx+c+1)^2 + 3 \cdot (4 \cdot a \cdot b^2 \cdot dx + 4 \cdot a \cdot b^2 \cdot c - (b^3 \cdot dx + b^3 \cdot (c-1)) \cdot \log(dx+c+1)^2 + 2 \cdot (b^3 \cdot d^2 \cdot x^2 + (c^2 + c) \cdot b^3 - 2 \cdot a \cdot b^2 \cdot (c-1) + ((2 \cdot c \cdot d + d) \cdot b^3 - 2 \cdot a \cdot b^2 \cdot d) \cdot x) \cdot \log(dx+c+1)^2)
\end{aligned}$$

$x + c + 1)) \cdot \log(-d \cdot x - c + 1)) / (d^3 \cdot e^{2 \cdot x^3} + c^3 \cdot e^2 - c^2 \cdot e^2 + (3 \cdot c \cdot d^2 \cdot e^2 - d^2 \cdot e^2) \cdot x^2 + (3 \cdot c^2 \cdot d \cdot e^2 - 2 \cdot c \cdot d \cdot e^2) \cdot x), x)$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + b \operatorname{atanh}(c + d x))^3}{(c e + d e x)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b*atanh(c + d*x))^3/(c*e + d*e*x)^2,x)`

[Out] `int((a + b*atanh(c + d*x))^3/(c*e + d*e*x)^2, x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a^3}{c^2+2cdx+d^2x^2} dx + \int \frac{b^3 \operatorname{atanh}^3(c+dx)}{c^2+2cdx+d^2x^2} dx + \int \frac{3ab^2 \operatorname{atanh}^2(c+dx)}{c^2+2cdx+d^2x^2} dx + \int \frac{3a^2b \operatorname{atanh}(c+dx)}{c^2+2cdx+d^2x^2} dx}{e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*atanh(d*x+c))**3/(d*e*x+c*e)**2,x)`

[Out] `(Integral(a**3/(c**2 + 2*c*d*x + d**2*x**2), x) + Integral(b**3*atanh(c + d*x)**3/(c**2 + 2*c*d*x + d**2*x**2), x) + Integral(3*a*b**2*atanh(c + d*x)**2/(c**2 + 2*c*d*x + d**2*x**2), x) + Integral(3*a**2*b*atanh(c + d*x)/(c**2 + 2*c*d*x + d**2*x**2), x))/e**2`

$$3.27 \quad \int \frac{(a+b \tanh^{-1}(c+dx))^3}{(ce+dex)^3} dx$$

Optimal. Leaf size=166

$$\frac{3b^2 \log\left(2 - \frac{2}{c+dx+1}\right) (a+b \tanh^{-1}(c+dx))}{de^3} - \frac{3b (a+b \tanh^{-1}(c+dx))^2}{2de^3(c+dx)} + \frac{3b (a+b \tanh^{-1}(c+dx))^2}{2de^3} - \frac{(a+b \tanh^{-1}(c+dx))^3}{2de^3}$$

[Out] $3/2*b*(a+b*\operatorname{arctanh}(d*x+c))^2/d/e^3-3/2*b*(a+b*\operatorname{arctanh}(d*x+c))^2/d/e^3/(d*x+c)+1/2*(a+b*\operatorname{arctanh}(d*x+c))^3/d/e^3-1/2*(a+b*\operatorname{arctanh}(d*x+c))^3/d/e^3/(d*x+c)^2+3*b^2*(a+b*\operatorname{arctanh}(d*x+c))*\ln(2-2/(d*x+c+1))/d/e^3-3/2*b^3*\operatorname{polylog}(2,-1+2/(d*x+c+1))/d/e^3$

Rubi [A] time = 0.34, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {6107, 12, 5916, 5982, 5988, 5932, 2447, 5948}

$$-\frac{3b^3 \operatorname{PolyLog}\left(2, \frac{2}{c+dx+1} - 1\right)}{2de^3} + \frac{3b^2 \log\left(2 - \frac{2}{c+dx+1}\right) (a+b \tanh^{-1}(c+dx))}{de^3} - \frac{3b (a+b \tanh^{-1}(c+dx))^2}{2de^3(c+dx)} + \frac{3b (a+b \tanh^{-1}(c+dx))^2}{2de^3}$$

Antiderivative was successfully verified.

[In] Int[(a + b*ArcTanh[c + d*x])^3/(c*e + d*e*x)^3,x]

[Out] $(3*b*(a + b*\operatorname{ArcTanh}[c + d*x])^2)/(2*d*e^3) - (3*b*(a + b*\operatorname{ArcTanh}[c + d*x])^2)/(2*d*e^3*(c + d*x)) + (a + b*\operatorname{ArcTanh}[c + d*x])^3/(2*d*e^3) - (a + b*\operatorname{ArcTanh}[c + d*x])^3/(2*d*e^3*(c + d*x)^2) + (3*b^2*(a + b*\operatorname{ArcTanh}[c + d*x])*Log[2 - 2/(1 + c + d*x)])/(d*e^3) - (3*b^3*\operatorname{PolyLog}[2, -1 + 2/(1 + c + d*x)])/(2*d*e^3)$

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 2447

Int[Log[u]*(Pq_)^(m_), x_Symbol] := With[{C = FullSimplify[(Pq^m*(1 - u))/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x] /; IntegerQ[m] && PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u, x][[2]], Expon[Pq, x]]

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
  :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5932

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] -
Dist[(b*c*p)/d, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)]]
/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 5982

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x]
, x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcTanh[c*x])^p)/(d + e*x^
2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5988

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 6107

```
Int[((a_.) + ArcTanh[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((f*x)/d)^m*(a + b*ArcTanh[x])^p, x]
, x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] &
& IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a + b \tanh^{-1}(c + dx))^3}{(ce + dex)^3} dx &= \frac{\text{Subst} \left(\int \frac{(a+b \tanh^{-1}(x))^3}{e^3 x^3} dx, x, c + dx \right)}{d} \\
&= \frac{\text{Subst} \left(\int \frac{(a+b \tanh^{-1}(x))^3}{x^3} dx, x, c + dx \right)}{de^3} \\
&= -\frac{(a + b \tanh^{-1}(c + dx))^3}{2de^3(c + dx)^2} + \frac{(3b) \text{Subst} \left(\int \frac{(a+b \tanh^{-1}(x))^2}{x^2(1-x^2)} dx, x, c + dx \right)}{2de^3} \\
&= -\frac{(a + b \tanh^{-1}(c + dx))^3}{2de^3(c + dx)^2} + \frac{(3b) \text{Subst} \left(\int \frac{(a+b \tanh^{-1}(x))^2}{x^2} dx, x, c + dx \right)}{2de^3} + \frac{(3b) \text{Subst} \left(\int \frac{(a+b \tanh^{-1}(x))^2}{1-x^2} dx, x, c + dx \right)}{2de^3} \\
&= -\frac{3b(a + b \tanh^{-1}(c + dx))^2}{2de^3(c + dx)} + \frac{(a + b \tanh^{-1}(c + dx))^3}{2de^3} - \frac{(a + b \tanh^{-1}(c + dx))^2}{2de^3(c + dx)^2} \\
&= \frac{3b(a + b \tanh^{-1}(c + dx))^2}{2de^3} - \frac{3b(a + b \tanh^{-1}(c + dx))^2}{2de^3(c + dx)} + \frac{(a + b \tanh^{-1}(c + dx))^3}{2de^3} \\
&= \frac{3b(a + b \tanh^{-1}(c + dx))^2}{2de^3} - \frac{3b(a + b \tanh^{-1}(c + dx))^2}{2de^3(c + dx)} + \frac{(a + b \tanh^{-1}(c + dx))^3}{2de^3} \\
&= \frac{3b(a + b \tanh^{-1}(c + dx))^2}{2de^3} - \frac{3b(a + b \tanh^{-1}(c + dx))^2}{2de^3(c + dx)} + \frac{(a + b \tanh^{-1}(c + dx))^3}{2de^3}
\end{aligned}$$

Mathematica [C] time = 1.18, size = 335, normalized size = 2.02

$$-4a^3 - 12a^2bc - 12a^2bdx + 12b \tanh^{-1}(c + dx) \left(a \left(a \left(c^2 + 2cdx + d^2x^2 - 1 \right) - 2b(c + dx) \right) + 2b^2(c + dx)^2 \log \left(1 - \frac{dx}{c + dx} \right) \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b*ArcTanh[c + d*x])^3/(c*e + d*e*x)^3,x]

[Out] (-4*a^3 - 12*a^2*b*c + I*b^3*c^3*Pi^3 - 12*a^2*b*d*x + (2*I)*b^3*c^2*d*Pi^3*x + I*b^3*c*d^2*Pi^3*x^2 + 12*b^2*(-1 + c + d*x)*(b*(c + d*x) + a*(1 + c + d*x))*ArcTanh[c + d*x]^2 + 4*b^3*(-1 + c^2 + 2*c*d*x + d^2*x^2)*ArcTanh[c + d*x])/(c + d*x)^3

+ d*x]^3 + 12*b*ArcTanh[c + d*x]*(a*(-2*b*(c + d*x) + a*(-1 + c^2 + 2*c*d*x + d^2*x^2)) + 2*b^2*(c + d*x)^2*Log[1 - E^(-2*ArcTanh[c + d*x])]) + 24*a*b^2*c^2*Log[(c + d*x)/Sqrt[1 - (c + d*x)^2]] + 48*a*b^2*c*d*x*Log[(c + d*x)/Sqrt[1 - (c + d*x)^2]] + 24*a*b^2*d^2*x^2*Log[(c + d*x)/Sqrt[1 - (c + d*x)^2]] - 12*b^3*(c + d*x)^2*PolyLog[2, E^(-2*ArcTanh[c + d*x])])/(8*d*e^3*(c + d*x)^2)

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^3 \operatorname{artanh}(dx + c)^3 + 3ab^2 \operatorname{artanh}(dx + c)^2 + 3a^2b \operatorname{artanh}(dx + c) + a^3}{d^3e^3x^3 + 3cd^2e^3x^2 + 3c^2de^3x + c^3e^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arctanh(d*x+c))^3/(d*e*x+c*e)^3,x, algorithm="fricas")

[Out] integral((b^3*arctanh(d*x + c)^3 + 3*a*b^2*arctanh(d*x + c)^2 + 3*a^2*b*arctanh(d*x + c) + a^3)/(d^3*e^3*x^3 + 3*c*d^2*e^3*x^2 + 3*c^2*d*e^3*x + c^3*e^3), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{artanh}(dx + c) + a)^3}{(dex + ce)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arctanh(d*x+c))^3/(d*e*x+c*e)^3,x, algorithm="giac")

[Out] integrate((b*arctanh(d*x + c) + a)^3/(d*e*x + c*e)^3, x)

maple [C] time = 0.68, size = 5796, normalized size = 34.92

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*arctanh(d*x+c))^3/(d*e*x+c*e)^3,x)

[Out] result too large to display

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{3}{4} \left(d \left(\frac{2}{d^3e^3x + cd^2e^3} - \frac{\log(dx + c + 1)}{d^2e^3} + \frac{\log(dx + c - 1)}{d^2e^3} \right) + \frac{2 \operatorname{artanh}(dx + c)}{d^3e^3x^2 + 2cd^2e^3x + c^2de^3} \right) a^2b - \frac{3}{8} \left(d^2 \left(\frac{\log(dx + c + 1)}{d^3e^3x + cd^2e^3} - \frac{\log(dx + c - 1)}{d^3e^3x + cd^2e^3} \right) + \frac{2 \operatorname{artanh}(dx + c)}{d^3e^3x^2 + 2cd^2e^3x + c^2de^3} \right) a^2b$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arctanh(d*x+c))^3/(d*e*x+c*e)^3,x, algorithm="maxima")

[Out]
$$-3/4*(d*(2/(d^3*e^3*x + c*d^2*e^3) - \log(d*x + c + 1)/(d^2*e^3) + \log(d*x + c - 1)/(d^2*e^3)) + 2*\arctanh(d*x + c)/(d^3*e^3*x^2 + 2*c*d^2*e^3*x + c^2*d*e^3))*a^2*b - 3/8*(d^2*((\log(d*x + c + 1)^2 - 2*\log(d*x + c + 1)*\log(d*x + c - 1) + \log(d*x + c - 1)^2 + 4*\log(d*x + c - 1))/(d^3*e^3) + 4*\log(d*x + c + 1)/(d^3*e^3) - 8*\log(d*x + c)/(d^3*e^3)) + 4*d*(2/(d^3*e^3*x + c*d^2*e^3) - \log(d*x + c + 1)/(d^2*e^3) + \log(d*x + c - 1)/(d^2*e^3))*\arctanh(d*x + c))*a*b^2 - 1/16*b^3*((d^2*x^2 + 2*c*d*x + c^2 - 1)*\log(-d*x - c + 1)^3 + 3*(2*d*x - (d^2*x^2 + 2*c*d*x + c^2 - 1)*\log(d*x + c + 1) + 2*c)*\log(-d*x - c + 1)^2)/(d^3*e^3*x^2 + 2*c*d^2*e^3*x + c^2*d*e^3) + 2*\integrate(-((d*x + c - 1)*\log(d*x + c + 1)^3 + 3*(2*d^2*x^2 + 4*c*d*x - (d*x + c - 1)*\log(d*x + c + 1)^2 + 2*c^2 - (d^3*x^3 + 3*c*d^2*x^2 + c^3 + (3*c^2*d - d)*x - c)*\log(d*x + c + 1))*\log(-d*x - c + 1))/(d^4*e^3*x^4 + c^4*e^3 - c^3*e^3 + (4*c*d^3*e^3 - d^3*e^3)*x^3 + 3*(2*c^2*d^2*e^3 - c*d^2*e^3)*x^2 + (4*c^3*d*e^3 - 3*c^2*d*e^3)*x), x) - 3/2*a*b^2*\arctanh(d*x + c)^2/(d^3*e^3*x^2 + 2*c*d^2*e^3*x + c^2*d*e^3) - 1/2*a^3/(d^3*e^3*x^2 + 2*c*d^2*e^3*x + c^2*d*e^3)$$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + b \operatorname{atanh}(c + dx))^3}{(ce + dex)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*atanh(c + d*x))^3/(c*e + d*e*x)^3,x)

[Out] int((a + b*atanh(c + d*x))^3/(c*e + d*e*x)^3, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a^3}{c^3+3c^2dx+3cd^2x^2+d^3x^3} dx + \int \frac{b^3 \operatorname{atanh}^3(c+dx)}{c^3+3c^2dx+3cd^2x^2+d^3x^3} dx + \int \frac{3ab^2 \operatorname{atanh}^2(c+dx)}{c^3+3c^2dx+3cd^2x^2+d^3x^3} dx + \int \frac{3a^2b \operatorname{atanh}(c+dx)}{c^3+3c^2dx+3cd^2x^2+d^3x^3} dx}{e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*atanh(d*x+c))**3/(d*e*x+c*e)**3,x)

[Out]
$$(\operatorname{Integral}(a**3/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3), x) + \operatorname{Integral}(b**3*\operatorname{atanh}(c + d*x)**3/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3), x) + \operatorname{Integral}(3*a*b**2*\operatorname{atanh}(c + d*x)**2/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3), x) + \operatorname{Integral}(3*a**2*b*\operatorname{atanh}(c + d*x)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3), x))/e**3$$

$$3.28 \quad \int \frac{(a+b \tanh^{-1}(c+dx))^3}{(ce+dex)^4} dx$$

Optimal. Leaf size=269

$$\frac{b^2 \operatorname{Li}_2\left(\frac{2}{c+dx+1} - 1\right) (a+b \tanh^{-1}(c+dx))}{de^4} - \frac{b^2 (a+b \tanh^{-1}(c+dx))}{de^4(c+dx)} - \frac{b (a+b \tanh^{-1}(c+dx))^2}{2de^4(c+dx)^2} + \frac{b (a+b \tanh^{-1}(c+dx))}{2de^4}$$

[Out] $-b^2*(a+b*\operatorname{arctanh}(d*x+c))/d/e^4/(d*x+c)+1/2*b*(a+b*\operatorname{arctanh}(d*x+c))^2/d/e^4-1/2*b*(a+b*\operatorname{arctanh}(d*x+c))^2/d/e^4/(d*x+c)^2+1/3*(a+b*\operatorname{arctanh}(d*x+c))^3/d/e^4-1/3*(a+b*\operatorname{arctanh}(d*x+c))^3/d/e^4/(d*x+c)^3+b^3*\ln(d*x+c)/d/e^4-1/2*b^3*1/n(1-(d*x+c)^2)/d/e^4+b*(a+b*\operatorname{arctanh}(d*x+c))^2*\ln(2-2/(d*x+c+1))/d/e^4-b^2*(a+b*\operatorname{arctanh}(d*x+c))*\operatorname{polylog}(2,-1+2/(d*x+c+1))/d/e^4-1/2*b^3*\operatorname{polylog}(3,-1+2/(d*x+c+1))/d/e^4$

Rubi [A] time = 0.51, antiderivative size = 269, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 13, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.565$, Rules used = {6107, 12, 5916, 5982, 266, 36, 31, 29, 5948, 5988, 5932, 6056, 6610}

$$\frac{b^2 \operatorname{PolyLog}\left(2, \frac{2}{c+dx+1} - 1\right) (a+b \tanh^{-1}(c+dx))}{de^4} - \frac{b^3 \operatorname{PolyLog}\left(3, \frac{2}{c+dx+1} - 1\right)}{2de^4} - \frac{b^2 (a+b \tanh^{-1}(c+dx))}{de^4(c+dx)} - \frac{b (a+b \tanh^{-1}(c+dx))}{2de^4}$$

Antiderivative was successfully verified.

[In] `Int[(a + b*ArcTanh[c + d*x])^3/(c*e + d*e*x)^4,x]`

[Out] $-((b^2*(a + b*\operatorname{ArcTanh}[c + d*x]))/(d*e^4*(c + d*x))) + (b*(a + b*\operatorname{ArcTanh}[c + d*x])^2)/(2*d*e^4) - (b*(a + b*\operatorname{ArcTanh}[c + d*x])^2)/(2*d*e^4*(c + d*x)^2) + (a + b*\operatorname{ArcTanh}[c + d*x])^3/(3*d*e^4) - (a + b*\operatorname{ArcTanh}[c + d*x])^3/(3*d*e^4*(c + d*x)^3) + (b^3*\operatorname{Log}[c + d*x])/(d*e^4) - (b^3*\operatorname{Log}[1 - (c + d*x)^2])/(2*d*e^4) + (b*(a + b*\operatorname{ArcTanh}[c + d*x])^2*\operatorname{Log}[2 - 2/(1 + c + d*x)])/(d*e^4) - (b^2*(a + b*\operatorname{ArcTanh}[c + d*x])*PolyLog[2, -1 + 2/(1 + c + d*x)])/(d*e^4) - (b^3*PolyLog[3, -1 + 2/(1 + c + d*x)])/(2*d*e^4)$

Rule 12

`Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]`

Rule 29

`Int[(x_)^(-1), x_Symbol] := Simp[Log[x], x]`

Rule 31

```
Int[((a_) + (b_)*(x_))^-1, x_Symbol] := Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_) + (b_)*(x_))*((c_) + (d_)*(x_))), x_Symbol] := Dist[b/(b*c
- a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 266

```
Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5916

```
Int[((a_) + ArcTanh[(c_)*(x_)]*(b_))^(p_)*((d_)*(x_))^(m_), x_Symbol]
:= Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5932

```
Int[((a_) + ArcTanh[(c_)*(x_)]*(b_))^(p_)/((d_) + (e_)*(x_)^2), x
_Symbol] := Simp[((a + b*ArcTanh[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] -
Dist[(b*c*p)/d, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])/
(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 5948

```
Int[((a_) + ArcTanh[(c_)*(x_)]*(b_))^(p_)/((d_) + (e_)*(x_)^2), x_Symb
ol] := Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 5982

```
Int[((a_) + ArcTanh[(c_)*(x_)]*(b_))^(p_)*((f_)*(x_))^(m_)/((d_) + (
e_)*(x_)^2), x_Symbol] := Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x]
, x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcTanh[c*x])^p)/(d + e*x^
```

2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]

Rule 5988

Int[((a_.) + ArcTanh[(c_.)*(x_.)]*(b_.))^(p_.)/((x_.)*((d_.) + (e_.)*(x_.)^2)), x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]

Rule 6056

Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_.)]*(b_.))^(p_.))/((d_.) + (e_.)*(x_.)^2), x_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d), x] - Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]

Rule 6107

Int[((a_.) + ArcTanh[(c_.) + (d_.)*(x_.)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_.))^(m_.), x_Symbol] :> Dist[1/d, Subst[Int[((f*x)/d)^m*(a + b*ArcTanh[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] && IGtQ[p, 0]

Rule 6610

Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v, x]}, Simp[w*PolyLog[n + 1, v], x] /; !FalseQ[w]] /; FreeQ[n, x]

Rubi steps

$$\begin{aligned}
\int \frac{(a + b \tanh^{-1}(c + dx))^3}{(ce + dex)^4} dx &= \frac{\text{Subst} \left(\int \frac{(a+b \tanh^{-1}(x))^3}{e^4 x^4} dx, x, c + dx \right)}{d} \\
&= \frac{\text{Subst} \left(\int \frac{(a+b \tanh^{-1}(x))^3}{x^4} dx, x, c + dx \right)}{de^4} \\
&= -\frac{(a + b \tanh^{-1}(c + dx))^3}{3de^4(c + dx)^3} + \frac{b \text{Subst} \left(\int \frac{(a+b \tanh^{-1}(x))^2}{x^3(1-x^2)} dx, x, c + dx \right)}{de^4} \\
&= -\frac{(a + b \tanh^{-1}(c + dx))^3}{3de^4(c + dx)^3} + \frac{b \text{Subst} \left(\int \frac{(a+b \tanh^{-1}(x))^2}{x^3} dx, x, c + dx \right)}{de^4} + \frac{b \text{Subst} \left(\int \frac{(a+b \tanh^{-1}(x))^2}{1-x^2} dx, x, c + dx \right)}{de^4} \\
&= -\frac{b(a + b \tanh^{-1}(c + dx))^2}{2de^4(c + dx)^2} + \frac{(a + b \tanh^{-1}(c + dx))^3}{3de^4} - \frac{(a + b \tanh^{-1}(c + dx))^3}{3de^4(c + dx)^3} \\
&= -\frac{b(a + b \tanh^{-1}(c + dx))^2}{2de^4(c + dx)^2} + \frac{(a + b \tanh^{-1}(c + dx))^3}{3de^4} - \frac{(a + b \tanh^{-1}(c + dx))^3}{3de^4(c + dx)^3} \\
&= -\frac{b^2(a + b \tanh^{-1}(c + dx))}{de^4(c + dx)} + \frac{b(a + b \tanh^{-1}(c + dx))^2}{2de^4} - \frac{b(a + b \tanh^{-1}(c + dx))}{2de^4(c + dx)^2} \\
&= -\frac{b^2(a + b \tanh^{-1}(c + dx))}{de^4(c + dx)} + \frac{b(a + b \tanh^{-1}(c + dx))^2}{2de^4} - \frac{b(a + b \tanh^{-1}(c + dx))}{2de^4(c + dx)^2} \\
&= -\frac{b^2(a + b \tanh^{-1}(c + dx))}{de^4(c + dx)} + \frac{b(a + b \tanh^{-1}(c + dx))^2}{2de^4} - \frac{b(a + b \tanh^{-1}(c + dx))}{2de^4(c + dx)^2} \\
&= -\frac{b^2(a + b \tanh^{-1}(c + dx))}{de^4(c + dx)} + \frac{b(a + b \tanh^{-1}(c + dx))^2}{2de^4} - \frac{b(a + b \tanh^{-1}(c + dx))}{2de^4(c + dx)^2}
\end{aligned}$$

Mathematica [C] time = 1.33, size = 393, normalized size = 1.46

$$-\frac{2a^3}{(c+dx)^3} - 3a^2b \log(-c^2 - 2cdx - d^2x^2 + 1) - \frac{3a^2b}{(c+dx)^2} + 6a^2b \log(c + dx) - \frac{6a^2b \tanh^{-1}(c+dx)}{(c+dx)^3} + 6ab^2 \left(-\text{Li}_2 \left(e^{-2 \tanh^{-1}(c+dx)} \right) \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b*ArcTanh[c + d*x])^3/(c*e + d*e*x)^4,x]

[Out]
$$\frac{((-2*a^3)/(c + d*x)^3 - (3*a^2*b)/(c + d*x)^2 - (6*a^2*b*ArcTanh[c + d*x])/(c + d*x)^3 + 6*a^2*b*Log[c + d*x] - 3*a^2*b*Log[1 - c^2 - 2*c*d*x - d^2*x^2] + 6*a*b^2*(-((c + d*x)^2 + ArcTanh[c + d*x]^2)/(c + d*x)^3) + ArcTanh[c + d*x]*(-((1 - (c + d*x)^2)/(c + d*x)^2) + ArcTanh[c + d*x] + 2*Log[1 - E^(-2*ArcTanh[c + d*x])]) - PolyLog[2, E^(-2*ArcTanh[c + d*x])]) + 6*b^3*((I/24)*Pi^3 - ArcTanh[c + d*x]/(c + d*x) - ((1 - (c + d*x)^2)*ArcTanh[c + d*x]^2)/(2*(c + d*x)^2) - ArcTanh[c + d*x]^3/3 - ArcTanh[c + d*x]^3/(3*(c + d*x))) - ((1 - (c + d*x)^2)*ArcTanh[c + d*x]^3)/(3*(c + d*x)^3) + ArcTanh[c + d*x]^2*Log[1 - E^(2*ArcTanh[c + d*x])] + Log[(c + d*x)/Sqrt[1 - (c + d*x)^2]] + ArcTanh[c + d*x]*PolyLog[2, E^(2*ArcTanh[c + d*x])] - PolyLog[3, E^(2*ArcTanh[c + d*x])]/2))/(6*d*e^4)$$

fricas [F] time = 0.52, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^3 \operatorname{artanh}(dx + c)^3 + 3ab^2 \operatorname{artanh}(dx + c)^2 + 3a^2b \operatorname{artanh}(dx + c) + a^3}{d^4e^4x^4 + 4cd^3e^4x^3 + 6c^2d^2e^4x^2 + 4c^3de^4x + c^4e^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arctanh(d*x+c))^3/(d*e*x+c*e)^4,x, algorithm="fricas")

[Out]
$$\text{integral}((b^3 \operatorname{arctanh}(d*x + c)^3 + 3*a*b^2 \operatorname{arctanh}(d*x + c)^2 + 3*a^2*b \operatorname{arctanh}(d*x + c) + a^3)/(d^4*e^4*x^4 + 4*c*d^3*e^4*x^3 + 6*c^2*d^2*e^4*x^2 + 4*c^3*d*e^4*x + c^4*e^4), x)$$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{artanh}(dx + c) + a)^3}{(dex + ce)^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arctanh(d*x+c))^3/(d*e*x+c*e)^4,x, algorithm="giac")

[Out] integrate((b*arctanh(d*x + c) + a)^3/(d*e*x + c*e)^4, x)

maple [C] time = 0.98, size = 2247, normalized size = 8.35

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*arctanh(d*x+c))^3/(d*e*x+c*e)^4,x)

```
[Out] 1/2*I/d*b^3/e^4*arctanh(d*x+c)^2*Pi*csgn(I/(1+(d*x+c+1)^2/(1-(d*x+c)^2)))*c
sgn(I*((d*x+c+1)^2/(1-(d*x+c)^2)-1)/(1+(d*x+c+1)^2/(1-(d*x+c)^2)))*csgn(I*(
(d*x+c+1)^2/(1-(d*x+c)^2)-1))+1/4*I/d*b^3/e^4*arctanh(d*x+c)^2*Pi*csgn(I*(d
*x+c+1)^2/((d*x+c)^2-1)/(1+(d*x+c+1)^2/(1-(d*x+c)^2)))^3+1/4*I/d*b^3/e^4*ar
ctanh(d*x+c)^2*Pi*csgn(I*(d*x+c+1)^2/((d*x+c)^2-1))^3+1/2*I/d*b^3/e^4*arcta
nh(d*x+c)^2*Pi*csgn(I*((d*x+c+1)^2/(1-(d*x+c)^2)-1)/(1+(d*x+c+1)^2/(1-(d*x+
c)^2)))^3-1/2*I/d*b^3/e^4*arctanh(d*x+c)^2*Pi*csgn(I/(1+(d*x+c+1)^2/(1-(d*x
+c)^2)))^2+1/2*I/d*b^3/e^4*arctanh(d*x+c)^2*Pi*csgn(I/(1+(d*x+c+1)^2/(1-(d*
x+c)^2)))^3-1/d*a*b^2/e^4/(d*x+c)-1/2/d*a^2*b/e^4/(d*x+c)^2+1/d*b^3/e^4*arc
tanh(d*x+c)^2*ln(2)-1/2/d*a*b^2/e^4*ln(d*x+c-1)+1/2/d*a*b^2/e^4*ln(d*x+c+1)
-1/4/d*a*b^2/e^4*ln(d*x+c-1)^2-1/4*I/d*b^3/e^4*arctanh(d*x+c)^2*Pi*csgn(I/(
1+(d*x+c+1)^2/(1-(d*x+c)^2)))*csgn(I*(d*x+c+1)^2/((d*x+c)^2-1)/(1+(d*x+c+1)
^2/(1-(d*x+c)^2)))*csgn(I*(d*x+c+1)^2/((d*x+c)^2-1))+1/d*a*b^2/e^4*dilog(1/
2+1/2*d*x+1/2*c)+1/4/d*a*b^2/e^4*ln(d*x+c+1)^2-1/d*a*b^2/e^4*dilog(d*x+c)-1
/d*a*b^2/e^4*dilog(d*x+c+1)+1/d*a^2*b/e^4*ln(d*x+c)-1/2/d*a^2*b/e^4*ln(d*x+
c-1)-1/2/d*a^2*b/e^4*ln(d*x+c+1)+2/d*b^3/e^4*arctanh(d*x+c)*polylog(2,-(d*x
+c+1)/(1-(d*x+c)^2)^(1/2))+1/d*b^3/e^4*ln(d*x+c)*arctanh(d*x+c)^2-1/2/d*b^3
/e^4*arctanh(d*x+c)^2*ln(d*x+c-1)-1/2/d*b^3/e^4*arctanh(d*x+c)^2*ln(d*x+c+1
)+2/d*b^3/e^4*arctanh(d*x+c)*polylog(2,(d*x+c+1)/(1-(d*x+c)^2)^(1/2))+1/d*b
^3/e^4*arctanh(d*x+c)^2*ln(1+(d*x+c+1)/(1-(d*x+c)^2)^(1/2))-1/d*b^3/e^4*arc
tanh(d*x+c)/(d*x+c)-1/d*b^3/e^4*arctanh(d*x+c)^2*ln((d*x+c+1)^2/(1-(d*x+c)^
2)-1)+1/d*b^3/e^4*arctanh(d*x+c)^2*ln((d*x+c+1)/(1-(d*x+c)^2)^(1/2))+1/d*b^
3/e^4*arctanh(d*x+c)^2*ln(1-(d*x+c+1)/(1-(d*x+c)^2)^(1/2))-1/3/d*b^3/e^4/(d
*x+c)^3*arctanh(d*x+c)^3-1/2/d*b^3/e^4*arctanh(d*x+c)^2/(d*x+c)^2-1/3/d*a^3
/e^4/(d*x+c)^3+1/2/d*b^3/e^4*arctanh(d*x+c)^2-1/3/d*b^3/e^4*arctanh(d*x+c)^
3+1/d*b^3/e^4*ln(1+(d*x+c+1)/(1-(d*x+c)^2)^(1/2))+1/d*b^3/e^4*ln((d*x+c+1)/
(1-(d*x+c)^2)^(1/2))-1)-2/d*b^3/e^4*polylog(3,(d*x+c+1)/(1-(d*x+c)^2)^(1/2))
-2/d*b^3/e^4*polylog(3,-(d*x+c+1)/(1-(d*x+c)^2)^(1/2))-1/d*b^3/e^4*arctanh(
d*x+c)-1/d*a*b^2/e^4*arctanh(d*x+c)*ln(d*x+c+1)+1/2/d*a*b^2/e^4*ln(d*x+c-1)
*ln(1/2+1/2*d*x+1/2*c)-1/2/d*a*b^2/e^4*ln(-1/2*d*x-1/2*c+1/2)*ln(d*x+c+1)-1
/d*a^2*b/e^4/(d*x+c)^3*arctanh(d*x+c)-1/d*a*b^2/e^4/(d*x+c)^3*arctanh(d*x+c
)^2-1/d*a*b^2/e^4*arctanh(d*x+c)/(d*x+c)^2+2/d*a*b^2/e^4*arctanh(d*x+c)*ln(
d*x+c)-1/d*a*b^2/e^4*arctanh(d*x+c)*ln(d*x+c-1)-1/d*a*b^2/e^4*ln(d*x+c)*ln(
d*x+c+1)+1/2/d*a*b^2/e^4*ln(-1/2*d*x-1/2*c+1/2)*ln(1/2+1/2*d*x+1/2*c)+1/2*I
/d*b^3/e^4*arctanh(d*x+c)^2*Pi-1/4*I/d*b^3/e^4*arctanh(d*x+c)^2*Pi*csgn(I*(
d*x+c+1)^2/((d*x+c)^2-1)/(1+(d*x+c+1)^2/(1-(d*x+c)^2)))^2*csgn(I*(d*x+c+1)^
2/((d*x+c)^2-1))+1/4*I/d*b^3/e^4*arctanh(d*x+c)^2*Pi*csgn(I*(d*x+c+1)/(1-(d
*x+c)^2)^(1/2))^2*csgn(I*(d*x+c+1)^2/((d*x+c)^2-1))-1/2*I/d*b^3/e^4*arctanh
(d*x+c)^2*Pi*csgn(I*((d*x+c+1)^2/(1-(d*x+c)^2)-1)/(1+(d*x+c+1)^2/(1-(d*x+c)
^2)))^2*csgn(I*((d*x+c+1)^2/(1-(d*x+c)^2)-1))+1/2*I/d*b^3/e^4*arctanh(d*x+c
)^2*Pi*csgn(I*(d*x+c+1)/(1-(d*x+c)^2)^(1/2))*csgn(I*(d*x+c+1)^2/((d*x+c)^2-
1))^2-1/2*I/d*b^3/e^4*arctanh(d*x+c)^2*Pi*csgn(I/(1+(d*x+c+1)^2/(1-(d*x+c)^
2)))^2*csgn(I*((d*x+c+1)^2/(1-(d*x+c)^2)-1)/(1+(d*x+c+1)^2/(1-(d*x+c)^2)))^2+
1/4*I/d*b^3/e^4*arctanh(d*x+c)^2*Pi*csgn(I/(1+(d*x+c+1)^2/(1-(d*x+c)^2)))^2*
sgn(I*(d*x+c+1)^2/((d*x+c)^2-1)/(1+(d*x+c+1)^2/(1-(d*x+c)^2)))^2
```


maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{2} \left(d \left(\frac{1}{d^4 e^4 x^2 + 2 c d^3 e^4 x + c^2 d^2 e^4} + \frac{\log(dx + c + 1)}{d^2 e^4} - \frac{2 \log(dx + c)}{d^2 e^4} + \frac{\log(dx + c - 1)}{d^2 e^4} \right) + \frac{2 \operatorname{arctan}}{d^4 e^4 x^3 + 3 c d^3 e^4 x^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arctanh(d*x+c))^3/(d*e*x+c*e)^4,x, algorithm="maxima")

[Out]
$$-1/2*(d*(1/(d^4*e^4*x^2 + 2*c*d^3*e^4*x + c^2*d^2*e^4) + \log(d*x + c + 1)/(d^2*e^4) - 2*\log(d*x + c)/(d^2*e^4) + \log(d*x + c - 1)/(d^2*e^4)) + 2*\operatorname{arctanh}(d*x + c)/(d^4*e^4*x^3 + 3*c*d^3*e^4*x^2 + 3*c^2*d^2*e^4*x + c^3*d*e^4)) * a^2*b - 1/3*a^3/(d^4*e^4*x^3 + 3*c*d^3*e^4*x^2 + 3*c^2*d^2*e^4*x + c^3*d*e^4) - 1/24*((b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + (c^3 - 1)*b^3)*\log(-d*x - c + 1)^3 + 3*(b^3*d*x + b^3*c + 2*a*b^2 + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + (c^3 + 1)*b^3)*\log(d*x + c + 1))*\log(-d*x - c + 1)^2)/(d^4*e^4*x^3 + 3*c*d^3*e^4*x^2 + 3*c^2*d^2*e^4*x + c^3*d*e^4) - \operatorname{integrate}(-1/8*((b^3*d*x + b^3*(c - 1))*\log(d*x + c + 1)^3 + 6*(a*b^2*d*x + a*b^2*(c - 1))*\log(d*x + c + 1)^2 + (2*b^3*d^2*x^2 + 2*b^3*c^2 + 4*a*b^2*c - 3*(b^3*d*x + b^3*(c - 1))*\log(d*x + c + 1)^2 + 4*(b^3*c*d + a*b^2*d)*x + 2*(b^3*d^4*x^4 + 4*b^3*c*d^3*x^3 + 6*b^3*c^2*d^2*x^2 + (c^4 + c)*b^3 - 6*a*b^2*(c - 1) + ((4*c^3*d + d)*b^3 - 6*a*b^2*d)*x)*\log(d*x + c + 1))*\log(-d*x - c + 1))/(d^5*e^4*x^5 + c^5*e^4 - c^4*e^4 + (5*c*d^4*e^4 - d^4*e^4)*x^4 + 2*(5*c^2*d^3*e^4 - 2*c*d^3*e^4)*x^3 + 2*(5*c^3*d^2*e^4 - 3*c^2*d^2*e^4)*x^2 + (5*c^4*d*e^4 - 4*c^3*d*e^4)*x), x)$$

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + b \operatorname{atanh}(c + dx))^3}{(ce + dex)^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*atanh(c + d*x))^3/(c*e + d*e*x)^4,x)

[Out] int((a + b*atanh(c + d*x))^3/(c*e + d*e*x)^4, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a^3}{c^4+4c^3dx+6c^2d^2x^2+4cd^3x^3+d^4x^4} dx + \int \frac{b^3 \operatorname{atanh}^3(c+dx)}{c^4+4c^3dx+6c^2d^2x^2+4cd^3x^3+d^4x^4} dx + \int \frac{3ab^2 \operatorname{atanh}^2(c+dx)}{c^4+4c^3dx+6c^2d^2x^2+4cd^3x^3+d^4x^4} dx + \int \frac{3a^2 b \operatorname{atanh}(c+dx)}{c^4+4c^3dx+6c^2d^2x^2+4cd^3x^3+d^4x^4} dx}{e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*atanh(d*x+c))**3/(d*e*x+c*e)**4,x)

```
[Out] (Integral(a**3/(c**4 + 4*c**3*d*x + 6*c**2*d**2*x**2 + 4*c*d**3*x**3 + d**4*x**4), x) + Integral(b**3*atanh(c + d*x)**3/(c**4 + 4*c**3*d*x + 6*c**2*d**2*x**2 + 4*c*d**3*x**3 + d**4*x**4), x) + Integral(3*a*b**2*atanh(c + d*x)**2/(c**4 + 4*c**3*d*x + 6*c**2*d**2*x**2 + 4*c*d**3*x**3 + d**4*x**4), x) + Integral(3*a**2*b*atanh(c + d*x)/(c**4 + 4*c**3*d*x + 6*c**2*d**2*x**2 + 4*c*d**3*x**3 + d**4*x**4), x))/e**4
```

$$3.29 \quad \int \frac{\tanh^{-1}(1+x)}{2+2x} dx$$

Optimal. Leaf size=21

$$\frac{\text{Li}_2(x+1)}{4} - \frac{\text{Li}_2(-x-1)}{4}$$

[Out] -1/4*polylog(2,-1-x)+1/4*polylog(2,1+x)

Rubi [A] time = 0.02, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {6107, 12, 5912}

$$\frac{1}{4}\text{PolyLog}(2, x+1) - \frac{1}{4}\text{PolyLog}(2, -x-1)$$

Antiderivative was successfully verified.

[In] Int[ArcTanh[1 + x]/(2 + 2*x), x]

[Out] -PolyLog[2, -1 - x]/4 + PolyLog[2, 1 + x]/4

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 5912

Int[((a_.) + ArcTanh[(c_.)*(x_)])*(b_.))/(x_), x_Symbol] := Simp[a*Log[x], x] + (-Simp[(b*PolyLog[2, -(c*x)])]/2, x] + Simp[(b*PolyLog[2, c*x])/2, x] /; FreeQ[{a, b, c}, x]

Rule 6107

Int[((a_.) + ArcTanh[(c_.) + (d_.)*(x_)])*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m_.), x_Symbol] := Dist[1/d, Subst[Int[((f*x)/d)^m*(a + b*ArcTanh[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] & IGtQ[p, 0]

Rubi steps

$$\begin{aligned} \int \frac{\tanh^{-1}(1+x)}{2+2x} dx &= \text{Subst} \left(\int \frac{\tanh^{-1}(x)}{2x} dx, x, 1+x \right) \\ &= \frac{1}{2} \text{Subst} \left(\int \frac{\tanh^{-1}(x)}{x} dx, x, 1+x \right) \\ &= -\frac{1}{4} \text{Li}_2(-1-x) + \frac{\text{Li}_2(1+x)}{4} \end{aligned}$$

Mathematica [A] time = 0.00, size = 31, normalized size = 1.48

$$\frac{1}{4} \text{Li}_2 \left(\frac{1}{2}(2x+2) \right) - \frac{1}{4} \text{Li}_2 \left(\frac{1}{2}(-2x-2) \right)$$

Antiderivative was successfully verified.

[In] Integrate[ArcTanh[1 + x]/(2 + 2*x), x]

[Out] -1/4*PolyLog[2, (-2 - 2*x)/2] + PolyLog[2, (2 + 2*x)/2]/4

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral} \left(\frac{\text{artanh}(x+1)}{2(x+1)}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctanh(1+x)/(2+2*x), x, algorithm="fricas")

[Out] integral(1/2*arctanh(x + 1)/(x + 1), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{artanh}(x+1)}{2(x+1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctanh(1+x)/(2+2*x), x, algorithm="giac")

[Out] integrate(1/2*arctanh(x + 1)/(x + 1), x)

maple [A] time = 0.05, size = 34, normalized size = 1.62

$$\frac{\ln(1+x) \arctanh(1+x)}{2} - \frac{\text{dilog}(1+x)}{4} - \frac{\text{dilog}(x+2)}{4} - \frac{\ln(1+x) \ln(x+2)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(arctanh(1+x)/(2+2*x),x)`

[Out] `1/2*ln(1+x)*arctanh(1+x)-1/4*dilog(1+x)-1/4*dilog(x+2)-1/4*ln(1+x)*ln(x+2)`

maxima [B] time = 0.32, size = 58, normalized size = 2.76

$$-\frac{1}{4}(\log(x+2) - \log(x)) \log(x+1) + \frac{1}{2} \operatorname{artanh}(x+1) \log(x+1) - \frac{1}{4} \log(x+1) \log(x) + \frac{1}{4} \log(x+2) \log(-x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arctanh(1+x)/(2+2*x),x, algorithm="maxima")`

[Out] `-1/4*(log(x + 2) - log(x))*log(x + 1) + 1/2*arctanh(x + 1)*log(x + 1) - 1/4*log(x + 1)*log(x) + 1/4*log(x + 2)*log(-x - 1) - 1/4*dilog(-x) + 1/4*dilog(x + 2)`

mupad [F] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{\operatorname{atanh}(x+1)}{2x+2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(atanh(x + 1)/(2*x + 2),x)`

[Out] `int(atanh(x + 1)/(2*x + 2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\operatorname{atanh}(x+1)}{x+1} dx}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(atanh(1+x)/(2+2*x),x)`

[Out] `Integral(atanh(x + 1)/(x + 1), x)/2`

$$3.30 \quad \int \frac{\tanh^{-1}(a+bx)}{\frac{ad}{b}+dx} dx$$

Optimal. Leaf size=32

$$\frac{\text{Li}_2(a+bx)}{2d} - \frac{\text{Li}_2(-a-bx)}{2d}$$

[Out] -1/2*polylog(2,-b*x-a)/d+1/2*polylog(2,b*x+a)/d

Rubi [A] time = 0.03, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {6107, 12, 5912}

$$\frac{\text{PolyLog}(2, a+bx)}{2d} - \frac{\text{PolyLog}(2, -a-bx)}{2d}$$

Antiderivative was successfully verified.

[In] Int[ArcTanh[a + b*x]/((a*d)/b + d*x), x]

[Out] -PolyLog[2, -a - b*x]/(2*d) + PolyLog[2, a + b*x]/(2*d)

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 5912

Int[((a_.) + ArcTanh[(c_.)*(x_.)]*(b_.))/(x_), x_Symbol] := Simp[a*Log[x], x] + (-Simp[(b*PolyLog[2, -(c*x)])/2, x] + Simp[(b*PolyLog[2, c*x])/2, x]) /; FreeQ[{a, b, c}, x]

Rule 6107

Int[((a_.) + ArcTanh[(c_.) + (d_.)*(x_.)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_.))^(m_.), x_Symbol] := Dist[1/d, Subst[Int[((f*x)/d)^m*(a + b*ArcTanh[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] & IGtQ[p, 0]

Rubi steps

$$\begin{aligned} \int \frac{\tanh^{-1}(a+bx)}{\frac{ad}{b} + dx} dx &= \frac{\text{Subst}\left(\int \frac{b \tanh^{-1}(x)}{dx} dx, x, a+bx\right)}{b} \\ &= \frac{\text{Subst}\left(\int \frac{\tanh^{-1}(x)}{x} dx, x, a+bx\right)}{d} \\ &= -\frac{\text{Li}_2(-a-bx)}{2d} + \frac{\text{Li}_2(a+bx)}{2d} \end{aligned}$$

Mathematica [A] time = 0.01, size = 52, normalized size = 1.62

$$b \left(\frac{\text{Li}_2\left(\frac{ad+bx}{d}\right)}{2bd} - \frac{\text{Li}_2\left(-\frac{ad+bx}{d}\right)}{2bd} \right)$$

Antiderivative was successfully verified.

[In] Integrate[ArcTanh[a + b*x]/((a*d)/b + d*x), x]

[Out] b*(-1/2*PolyLog[2, -((a*d + b*d*x)/d)]/(b*d) + PolyLog[2, (a*d + b*d*x)/d]/(2*b*d))

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b \operatorname{artanh}(bx+a)}{bdx+ad}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctanh(b*x+a)/(a*d/b+d*x), x, algorithm="fricas")

[Out] integral(b*arctanh(b*x + a)/(b*d*x + a*d), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{artanh}(bx+a)}{dx + \frac{ad}{b}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctanh(b*x+a)/(a*d/b+d*x), x, algorithm="giac")

[Out] integrate(arctanh(b*x + a)/(d*x + a*d/b), x)

maple [B] time = 0.05, size = 59, normalized size = 1.84

$$\frac{\ln(bx + a) \operatorname{arctanh}(bx + a)}{d} - \frac{\operatorname{dilog}(bx + a)}{2d} - \frac{\operatorname{dilog}(bx + a + 1)}{2d} - \frac{\ln(bx + a) \ln(bx + a + 1)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arctanh(b*x+a)/(a*d/b+d*x), x)

[Out] 1/d*ln(b*x+a)*arctanh(b*x+a)-1/2/d*dilog(b*x+a)-1/2/d*dilog(b*x+a+1)-1/2/d*ln(b*x+a)*ln(b*x+a+1)

maxima [B] time = 0.34, size = 132, normalized size = 4.12

$$-\frac{1}{2} b \left(\frac{\log(bx + a) \log(bx + a - 1) + \operatorname{Li}_2(-bx - a + 1)}{bd} - \frac{\log(bx + a + 1) \log(-bx - a) + \operatorname{Li}_2(bx + a + 1)}{bd} \right) - b \left(\frac{\log(bx + a) \log(bx + a - 1) + \operatorname{Li}_2(-bx - a + 1)}{bd} - \frac{\log(bx + a + 1) \log(-bx - a) + \operatorname{Li}_2(bx + a + 1)}{bd} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctanh(b*x+a)/(a*d/b+d*x), x, algorithm="maxima")

[Out] -1/2*b*((log(b*x + a)*log(b*x + a - 1) + dilog(-b*x - a + 1))/(b*d) - (log(b*x + a + 1)*log(-b*x - a) + dilog(b*x + a + 1))/(b*d)) - 1/2*b*(log(b*x + a + 1)/b - log(b*x + a - 1)/b)*log(d*x + a*d/b)/d + arctanh(b*x + a)*log(d*x + a*d/b)/d

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\operatorname{atanh}\left(\frac{a + bx}{d x + \frac{a d}{b}}\right) dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(atanh(a + b*x)/(d*x + (a*d)/b), x)

[Out] int(atanh(a + b*x)/(d*x + (a*d)/b), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{b \int \frac{\operatorname{atanh}\left(\frac{a + bx}{d x + \frac{a d}{b}}\right) dx}{d}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(atanh(b*x+a)/(a*d/b+d*x), x)

[Out] b*Integral(atanh(a + b*x)/(a + b*x), x)/d

3.31 $\int (e + fx)^3 \left(a + b \tanh^{-1}(c + dx) \right) dx$

Optimal. Leaf size=168

$$\frac{(e + fx)^4 (a + b \tanh^{-1}(c + dx))}{4f} + \frac{bfx \left((6c^2 + 1)f^2 - 12cdef + 6d^2e^2 \right)}{4d^3} + \frac{bf^2(c + dx)^2(de - cf)}{2d^4} - \frac{b(-cf + de - f)}{8}$$

[Out] $\frac{1}{4} b f (6 d^2 e^2 - 12 c d e f + (6 c^2 + 1) f^2) x / d^3 + \frac{1}{2} b f^2 (-c f + d e) (d x + c)^2 / d^4 + \frac{1}{12} b f^3 (d x + c)^3 / d^4 + \frac{1}{4} (f x + e)^4 (a + b \operatorname{arctanh}(d x + c)) / f + \frac{1}{8} b (-c f + d e + f)^4 \ln(-d x - c + 1) / d^4 - \frac{1}{8} b (-c f + d e - f)^4 \ln(d x + c + 1) / d^4$

Rubi [A] time = 0.33, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {6111, 5926, 702, 633, 31}

$$\frac{(e + fx)^4 (a + b \tanh^{-1}(c + dx))}{4f} + \frac{bfx \left((6c^2 + 1)f^2 - 12cdef + 6d^2e^2 \right)}{4d^3} + \frac{bf^2(c + dx)^2(de - cf)}{2d^4} - \frac{b(-cf + de - f)}{8}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(e + f*x)^3*(a + b*\text{ArcTanh}[c + d*x]), x]$

[Out] $(b*f*(6*d^2*e^2 - 12*c*d*e*f + (1 + 6*c^2)*f^2)*x)/(4*d^3) + (b*f^2*(d*e - c*f)*(c + d*x)^2)/(2*d^4) + (b*f^3*(c + d*x)^3)/(12*d^4) + ((e + f*x)^4*(a + b*\text{ArcTanh}[c + d*x]))/(4*f) + (b*(d*e + f - c*f)^4*\text{Log}[1 - c - d*x])/(8*d^4*f) - (b*(d*e - f - c*f)^4*\text{Log}[1 + c + d*x])/(8*d^4*f)$

Rule 31

$\text{Int}[(a + (b*x)^m)^{-1}, x_Symbol] \rightarrow \text{Simp}[\text{Log}[\text{RemoveContent}[a + b*x, x]]/b, x] /; \text{FreeQ}\{a, b, x\}$

Rule 633

$\text{Int}[(d + (e*x)^m)/(a + (c*x)^2), x_Symbol] \rightarrow \text{With}\{q = \text{Rt}[-(a*c), 2]\}, \text{Dist}[e/2 + (c*d)/(2*q), \text{Int}[1/(-q + c*x), x], x] + \text{Dist}[e/2 - (c*d)/(2*q), \text{Int}[1/(q + c*x), x], x] /; \text{FreeQ}\{a, c, d, e, x\} \&\& \text{NiceSqrtQ}[-(a*c)]$

Rule 702

$\text{Int}[(d + (e*x)^m)^m/(a + (c*x)^2), x_Symbol] \rightarrow \text{Int}[\text{PolynomialDivide}[(d + e*x)^m, a + c*x^2, x], x] /; \text{FreeQ}\{a, c, d, e, x\} \&\& \text{NeQ}[$

$c*d^2 + a*e^2, 0]$ && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])

Rule 5926

Int[((a_.) + ArcTanh[(c_.)*(x_.)]*(b_.))*((d_.) + (e_.)*(x_.))^(q_.), x_Symbol] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTanh[c*x]))/(e*(q + 1)), x] - Dist[(b*c)/(e*(q + 1)), Int[(d + e*x)^(q + 1)/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[q, -1]

Rule 6111

Int[((a_.) + ArcTanh[(c_.) + (d_.)*(x_.)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_.))^(m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*ArcTanh[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]

Rubi steps

$$\begin{aligned}
 \int (e + fx)^3 (a + b \tanh^{-1}(c + dx)) dx &= \frac{\text{Subst}\left(\int \left(\frac{de - cf}{d} + \frac{fx}{d}\right)^3 (a + b \tanh^{-1}(x)) dx, x, c + dx\right)}{d} \\
 &= \frac{(e + fx)^4 (a + b \tanh^{-1}(c + dx))}{4f} - \frac{b \text{Subst}\left(\int \frac{\left(\frac{de - cf}{d} + \frac{fx}{d}\right)^4}{1 - x^2} dx, x, c + dx\right)}{4f} \\
 &= \frac{(e + fx)^4 (a + b \tanh^{-1}(c + dx))}{4f} - \frac{b \text{Subst}\left(\int \left(-\frac{f^2(6d^2e^2 - 12cdef + (1 + 6c^2)f^2)}{d^4}\right) dx, x, c + dx\right)}{4f} \\
 &= \frac{bf(6d^2e^2 - 12cdef + (1 + 6c^2)f^2)x}{4d^3} + \frac{bf^2(de - cf)(c + dx)^2}{2d^4} + \frac{bf^3(c + dx)^3}{12d^4} \\
 &= \frac{bf(6d^2e^2 - 12cdef + (1 + 6c^2)f^2)x}{4d^3} + \frac{bf^2(de - cf)(c + dx)^2}{2d^4} + \frac{bf^3(c + dx)^3}{12d^4} \\
 &= \frac{bf(6d^2e^2 - 12cdef + (1 + 6c^2)f^2)x}{4d^3} + \frac{bf^2(de - cf)(c + dx)^2}{2d^4} + \frac{bf^3(c + dx)^3}{12d^4}
 \end{aligned}$$

Mathematica [A] time = 0.28, size = 270, normalized size = 1.61

$$\frac{6dx(4ad^3e^3 + bf((3c^2 + 1)f^2 - 8cdef + 6d^2e^2)) + 6d^2fx^2(6ad^2e^2 + bf(2de - cf)) + 2d^3f^2x^3(12ade + bf) + \dots}{\dots}$$

Antiderivative was successfully verified.

[In] Integrate[(e + f*x)^3*(a + b*ArcTanh[c + d*x]),x]

[Out] (6*d*(4*a*d^3*e^3 + b*f*(6*d^2*e^2 - 8*c*d*e*f + (1 + 3*c^2)*f^2))*x + 6*d^2*f*(6*a*d^2*e^2 + b*f*(2*d*e - c*f))*x^2 + 2*d^3*f^2*(12*a*d*e + b*f)*x^3 + 6*a*d^4*f^3*x^4 + 6*b*d^4*x*(4*e^3 + 6*e^2*f*x + 4*e*f^2*x^2 + f^3*x^3)*ArcTanh[c + d*x] - 3*b*(-1 + c)*(4*d^3*e^3 - 6*(-1 + c)*d^2*e^2*f + 4*(-1 + c)^2*d*e*f^2 - (-1 + c)^3*f^3)*Log[1 - c - d*x] - 3*b*(1 + c)*(-4*d^3*e^3 + 6*(1 + c)*d^2*e^2*f - 4*(1 + c)^2*d*e*f^2 + (1 + c)^3*f^3)*Log[1 + c + d*x])/ (24*d^4)

fricas [B] time = 0.51, size = 386, normalized size = 2.30

$$6ad^4f^3x^4 + 2(12ad^4ef^2 + bd^3f^3)x^3 + 6(6ad^4e^2f + 2bd^3ef^2 - bcd^2f^3)x^2 + 6(4ad^4e^3 + 6bd^3e^2f - 8bcd^2ef^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x+e)^3*(a+b*arctanh(d*x+c)),x, algorithm="fricas")

[Out] 1/24*(6*a*d^4*f^3*x^4 + 2*(12*a*d^4*e*f^2 + b*d^3*f^3)*x^3 + 6*(6*a*d^4*e^2*f + 2*b*d^3*e*f^2 - b*c*d^2*f^3)*x^2 + 6*(4*a*d^4*e^3 + 6*b*d^3*e^2*f - 8*b*c*d^2*e*f^2 + (3*b*c^2 + b)*d*f^3)*x + 3*(4*(b*c + b)*d^3*e^3 - 6*(b*c^2 + 2*b*c + b)*d^2*e^2*f + 4*(b*c^3 + 3*b*c^2 + 3*b*c + b)*d*e*f^2 - (b*c^4 + 4*b*c^3 + 6*b*c^2 + 4*b*c + b)*f^3)*log(d*x + c + 1) - 3*(4*(b*c - b)*d^3*e^3 - 6*(b*c^2 - 2*b*c + b)*d^2*e^2*f + 4*(b*c^3 - 3*b*c^2 + 3*b*c - b)*d*e*f^2 - (b*c^4 - 4*b*c^3 + 6*b*c^2 - 4*b*c + b)*f^3)*log(d*x + c - 1) + 3*(b*d^4*f^3*x^4 + 4*b*d^4*e*f^2*x^3 + 6*b*d^4*e^2*f*x^2 + 4*b*d^4*e^3*x)*log(-(d*x + c + 1)/(d*x + c - 1)))/d^4

giac [B] time = 0.63, size = 3990, normalized size = 23.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x+e)^3*(a+b*arctanh(d*x+c)),x, algorithm="giac")

[Out] 1/6*(3*(d*x + c + 1)^4*b*c^3*f^3*log(-(d*x + c + 1)/(d*x + c - 1) + 1)/(d*x + c - 1)^4 - 12*(d*x + c + 1)^3*b*c^3*f^3*log(-(d*x + c + 1)/(d*x + c - 1)

$$\begin{aligned}
& + 1)/(d*x + c - 1)^3 + 18*(d*x + c + 1)^2*b*c^3*f^3*\log(-(d*x + c + 1)/(d*x \\
& x + c - 1) + 1)/(d*x + c - 1)^2 - 12*(d*x + c + 1)*b*c^3*f^3*\log(-(d*x + c \\
& + 1)/(d*x + c - 1) + 1)/(d*x + c - 1) + 3*b*c^3*f^3*\log(-(d*x + c + 1)/(d*x \\
& + c - 1) + 1) - 9*(d*x + c + 1)^4*b*c^2*d*f^2*e*\log(-(d*x + c + 1)/(d*x + \\
& c - 1) + 1)/(d*x + c - 1)^4 + 36*(d*x + c + 1)^3*b*c^2*d*f^2*e*\log(-(d*x + \\
& c + 1)/(d*x + c - 1) + 1)/(d*x + c - 1)^3 - 54*(d*x + c + 1)^2*b*c^2*d*f^2* \\
& e*\log(-(d*x + c + 1)/(d*x + c - 1) + 1)/(d*x + c - 1)^2 + 36*(d*x + c + 1)* \\
& b*c^2*d*f^2*e*\log(-(d*x + c + 1)/(d*x + c - 1) + 1)/(d*x + c - 1) - 9*b*c^2 \\
& *d*f^2*e*\log(-(d*x + c + 1)/(d*x + c - 1) + 1) - 3*(d*x + c + 1)^4*b*c^3*f^ \\
& 3*\log(-(d*x + c + 1)/(d*x + c - 1))/(d*x + c - 1)^4 + 9*(d*x + c + 1)^3*b*c \\
& ^3*f^3*\log(-(d*x + c + 1)/(d*x + c - 1))/(d*x + c - 1)^3 - 9*(d*x + c + 1)^ \\
& 2*b*c^3*f^3*\log(-(d*x + c + 1)/(d*x + c - 1))/(d*x + c - 1)^2 + 3*(d*x + c \\
& + 1)*b*c^3*f^3*\log(-(d*x + c + 1)/(d*x + c - 1))/(d*x + c - 1) + 9*(d*x + c \\
& + 1)^4*b*c^2*d*f^2*e*\log(-(d*x + c + 1)/(d*x + c - 1))/(d*x + c - 1)^4 - 2 \\
& 7*(d*x + c + 1)^3*b*c^2*d*f^2*e*\log(-(d*x + c + 1)/(d*x + c - 1))/(d*x + c \\
& - 1)^3 + 27*(d*x + c + 1)^2*b*c^2*d*f^2*e*\log(-(d*x + c + 1)/(d*x + c - 1)) \\
& /(d*x + c - 1)^2 - 9*(d*x + c + 1)*b*c^2*d*f^2*e*\log(-(d*x + c + 1)/(d*x + \\
& c - 1))/(d*x + c - 1) - 6*(d*x + c + 1)^3*a*c^3*f^3/(d*x + c - 1)^3 + 18*(d \\
& *x + c + 1)^2*a*c^3*f^3/(d*x + c - 1)^2 - 18*(d*x + c + 1)*a*c^3*f^3/(d*x + \\
& c - 1) + 6*a*c^3*f^3 + 18*(d*x + c + 1)^3*a*c^2*d*f^2*e/(d*x + c - 1)^3 - \\
& 54*(d*x + c + 1)^2*a*c^2*d*f^2*e/(d*x + c - 1)^2 + 54*(d*x + c + 1)*a*c^2*d \\
& *f^2*e/(d*x + c - 1) - 18*a*c^2*d*f^2*e + 9*(d*x + c + 1)^4*b*c*d^2*f*e^2*\log \\
& \log(-(d*x + c + 1)/(d*x + c - 1) + 1)/(d*x + c - 1)^4 - 36*(d*x + c + 1)^3*b \\
& *c*d^2*f*e^2*\log(-(d*x + c + 1)/(d*x + c - 1) + 1)/(d*x + c - 1)^3 + 54*(d \\
& x + c + 1)^2*b*c*d^2*f*e^2*\log(-(d*x + c + 1)/(d*x + c - 1) + 1)/(d*x + c - \\
& 1)^2 - 36*(d*x + c + 1)*b*c*d^2*f*e^2*\log(-(d*x + c + 1)/(d*x + c - 1) + 1 \\
&)/(d*x + c - 1) + 9*b*c*d^2*f*e^2*\log(-(d*x + c + 1)/(d*x + c - 1) + 1) + 9 \\
& *(d*x + c + 1)^3*b*c^2*f^3*\log(-(d*x + c + 1)/(d*x + c - 1))/(d*x + c - 1)^ \\
& 3 - 18*(d*x + c + 1)^2*b*c^2*f^3*\log(-(d*x + c + 1)/(d*x + c - 1))/(d*x + c \\
& - 1)^2 + 9*(d*x + c + 1)*b*c^2*f^3*\log(-(d*x + c + 1)/(d*x + c - 1))/(d*x \\
& + c - 1) - 9*(d*x + c + 1)^4*b*c*d^2*f*e^2*\log(-(d*x + c + 1)/(d*x + c - 1) \\
&)/(d*x + c - 1)^4 + 27*(d*x + c + 1)^3*b*c*d^2*f*e^2*\log(-(d*x + c + 1)/(d* \\
& x + c - 1))/(d*x + c - 1)^3 - 27*(d*x + c + 1)^2*b*c*d^2*f*e^2*\log(-(d*x + \\
& c + 1)/(d*x + c - 1))/(d*x + c - 1)^2 + 9*(d*x + c + 1)*b*c*d^2*f*e^2*\log(- \\
& (d*x + c + 1)/(d*x + c - 1))/(d*x + c - 1) - 18*(d*x + c + 1)^3*b*c*d*f^2*e \\
& *\log(-(d*x + c + 1)/(d*x + c - 1))/(d*x + c - 1)^3 + 36*(d*x + c + 1)^2*b*c \\
& *d*f^2*e*\log(-(d*x + c + 1)/(d*x + c - 1))/(d*x + c - 1)^2 - 18*(d*x + c + \\
& 1)*b*c*d*f^2*e*\log(-(d*x + c + 1)/(d*x + c - 1))/(d*x + c - 1) + 18*(d*x + \\
& c + 1)^3*a*c^2*f^3/(d*x + c - 1)^3 - 36*(d*x + c + 1)^2*a*c^2*f^3/(d*x + c \\
& - 1)^2 + 18*(d*x + c + 1)*a*c^2*f^3/(d*x + c - 1) + 9*(d*x + c + 1)^3*b*c^2 \\
& *f^3/(d*x + c - 1)^3 - 27*(d*x + c + 1)^2*b*c^2*f^3/(d*x + c - 1)^2 + 27*(d \\
& *x + c + 1)*b*c^2*f^3/(d*x + c - 1) - 9*b*c^2*f^3 - 18*(d*x + c + 1)^3*a*c* \\
& d^2*f*e^2/(d*x + c - 1)^3 + 54*(d*x + c + 1)^2*a*c*d^2*f*e^2/(d*x + c - 1)^ \\
& 2 - 54*(d*x + c + 1)*a*c*d^2*f*e^2/(d*x + c - 1) + 18*a*c*d^2*f*e^2 - 36*(d \\
& *x + c + 1)^3*a*c*d*f^2*e/(d*x + c - 1)^3 + 72*(d*x + c + 1)^2*a*c*d*f^2*e/
\end{aligned}$$

$$\begin{aligned}
& (d*x + c - 1)^2 - 36*(d*x + c + 1)*a*c*d*f^2*e/(d*x + c - 1) - 18*(d*x + c \\
& + 1)^3*b*c*d*f^2*e/(d*x + c - 1)^3 + 54*(d*x + c + 1)^2*b*c*d*f^2*e/(d*x + \\
& c - 1)^2 - 54*(d*x + c + 1)*b*c*d*f^2*e/(d*x + c - 1) + 18*b*c*d*f^2*e + 3* \\
& (d*x + c + 1)^4*b*c*f^3*log(-(d*x + c + 1)/(d*x + c - 1) + 1)/(d*x + c - 1) \\
& ^4 - 12*(d*x + c + 1)^3*b*c*f^3*log(-(d*x + c + 1)/(d*x + c - 1) + 1)/(d*x \\
& + c - 1)^3 + 18*(d*x + c + 1)^2*b*c*f^3*log(-(d*x + c + 1)/(d*x + c - 1) + \\
& 1)/(d*x + c - 1)^2 - 12*(d*x + c + 1)*b*c*f^3*log(-(d*x + c + 1)/(d*x + c - \\
& 1) + 1)/(d*x + c - 1) + 3*b*c*f^3*log(-(d*x + c + 1)/(d*x + c - 1) + 1) - \\
& 3*(d*x + c + 1)^4*b*d^3*e^3*log(-(d*x + c + 1)/(d*x + c - 1) + 1)/(d*x + c \\
& - 1)^4 + 12*(d*x + c + 1)^3*b*d^3*e^3*log(-(d*x + c + 1)/(d*x + c - 1) + 1) \\
& /(d*x + c - 1)^3 - 18*(d*x + c + 1)^2*b*d^3*e^3*log(-(d*x + c + 1)/(d*x + c \\
& - 1) + 1)/(d*x + c - 1)^2 + 12*(d*x + c + 1)*b*d^3*e^3*log(-(d*x + c + 1)/ \\
& (d*x + c - 1) + 1)/(d*x + c - 1) - 3*b*d^3*e^3*log(-(d*x + c + 1)/(d*x + c \\
& - 1) + 1) - 3*(d*x + c + 1)^4*b*d*f^2*e*log(-(d*x + c + 1)/(d*x + c - 1) + \\
& 1)/(d*x + c - 1)^4 + 12*(d*x + c + 1)^3*b*d*f^2*e*log(-(d*x + c + 1)/(d*x + \\
& c - 1) + 1)/(d*x + c - 1)^3 - 18*(d*x + c + 1)^2*b*d*f^2*e*log(-(d*x + c + \\
& 1)/(d*x + c - 1) + 1)/(d*x + c - 1)^2 + 12*(d*x + c + 1)*b*d*f^2*e*log(-(d \\
& *x + c + 1)/(d*x + c - 1) + 1)/(d*x + c - 1) - 3*b*d*f^2*e*log(-(d*x + c + \\
& 1)/(d*x + c - 1) + 1) - 3*(d*x + c + 1)^4*b*c*f^3*log(-(d*x + c + 1)/(d*x + \\
& c - 1))/(d*x + c - 1)^4 + 3*(d*x + c + 1)^3*b*c*f^3*log(-(d*x + c + 1)/(d* \\
& x + c - 1))/(d*x + c - 1)^3 - 9*(d*x + c + 1)^2*b*c*f^3*log(-(d*x + c + 1)/ \\
& (d*x + c - 1))/(d*x + c - 1)^2 + 9*(d*x + c + 1)*b*c*f^3*log(-(d*x + c + 1) \\
& /(d*x + c - 1))/(d*x + c - 1) + 3*(d*x + c + 1)^4*b*d^3*e^3*log(-(d*x + c + \\
& 1)/(d*x + c - 1))/(d*x + c - 1)^4 - 9*(d*x + c + 1)^3*b*d^3*e^3*log(-(d*x \\
& + c + 1)/(d*x + c - 1))/(d*x + c - 1)^3 + 9*(d*x + c + 1)^2*b*d^3*e^3*log(- \\
& (d*x + c + 1)/(d*x + c - 1))/(d*x + c - 1)^2 - 3*(d*x + c + 1)*b*d^3*e^3*lo \\
& g(-(d*x + c + 1)/(d*x + c - 1))/(d*x + c - 1) + 9*(d*x + c + 1)^3*b*d^2*f*e \\
& ^2*log(-(d*x + c + 1)/(d*x + c - 1))/(d*x + c - 1)^3 - 18*(d*x + c + 1)^2*b \\
& *d^2*f*e^2*log(-(d*x + c + 1)/(d*x + c - 1))/(d*x + c - 1)^2 + 9*(d*x + c + \\
& 1)*b*d^2*f*e^2*log(-(d*x + c + 1)/(d*x + c - 1))/(d*x + c - 1) + 3*(d*x + \\
& c + 1)^4*b*d*f^2*e*log(-(d*x + c + 1)/(d*x + c - 1))/(d*x + c - 1)^4 - 3*(d \\
& *x + c + 1)^3*b*d*f^2*e*log(-(d*x + c + 1)/(d*x + c - 1))/(d*x + c - 1)^3 + \\
& 9*(d*x + c + 1)^2*b*d*f^2*e*log(-(d*x + c + 1)/(d*x + c - 1))/(d*x + c - 1 \\
&)^2 - 9*(d*x + c + 1)*b*d*f^2*e*log(-(d*x + c + 1)/(d*x + c - 1))/(d*x + c \\
& - 1) - 18*(d*x + c + 1)^3*a*c*f^3/(d*x + c - 1)^3 + 18*(d*x + c + 1)^2*a*c* \\
& f^3/(d*x + c - 1)^2 - 6*(d*x + c + 1)*a*c*f^3/(d*x + c - 1) + 6*a*c*f^3 - 6 \\
& *(d*x + c + 1)^3*b*c*f^3/(d*x + c - 1)^3 + 12*(d*x + c + 1)^2*b*c*f^3/(d*x \\
& + c - 1)^2 - 6*(d*x + c + 1)*b*c*f^3/(d*x + c - 1) + 6*(d*x + c + 1)^3*a*d \\
& ^3*e^3/(d*x + c - 1)^3 - 18*(d*x + c + 1)^2*a*d^3*e^3/(d*x + c - 1)^2 + 18*(\\
& d*x + c + 1)*a*d^3*e^3/(d*x + c - 1) - 6*a*d^3*e^3 + 18*(d*x + c + 1)^3*a*d \\
& ^2*f*e^2/(d*x + c - 1)^3 - 36*(d*x + c + 1)^2*a*d^2*f*e^2/(d*x + c - 1)^2 + \\
& 18*(d*x + c + 1)*a*d^2*f*e^2/(d*x + c - 1) + 9*(d*x + c + 1)^3*b*d^2*f*e^2 \\
& /(d*x + c - 1)^3 - 27*(d*x + c + 1)^2*b*d^2*f*e^2/(d*x + c - 1)^2 + 27*(d*x \\
& + c + 1)*b*d^2*f*e^2/(d*x + c - 1) - 9*b*d^2*f*e^2 + 18*(d*x + c + 1)^3*a* \\
& d*f^2*e/(d*x + c - 1)^3 - 18*(d*x + c + 1)^2*a*d*f^2*e/(d*x + c - 1)^2 + 6*
\end{aligned}$$

$$\begin{aligned} & (d*x + c + 1)*a*d*f^2*e/(d*x + c - 1) - 6*a*d*f^2*e + 6*(d*x + c + 1)^3*b*d \\ & *f^2*e/(d*x + c - 1)^3 - 12*(d*x + c + 1)^2*b*d*f^2*e/(d*x + c - 1)^2 + 6*(\\ & d*x + c + 1)*b*d*f^2*e/(d*x + c - 1) + 3*(d*x + c + 1)^3*b*f^3*\log(-(d*x + \\ & c + 1)/(d*x + c - 1))/(d*x + c - 1)^3 + 3*(d*x + c + 1)*b*f^3*\log(-(d*x + \\ & c + 1)/(d*x + c - 1))/(d*x + c - 1) + 6*(d*x + c + 1)^3*a*f^3/(d*x + c - 1)^ \\ & 3 + 6*(d*x + c + 1)*a*f^3/(d*x + c - 1) + 3*(d*x + c + 1)^3*b*f^3/(d*x + c \\ & - 1)^3 - 6*(d*x + c + 1)^2*b*f^3/(d*x + c - 1)^2 + 5*(d*x + c + 1)*b*f^3/(d \\ & *x + c - 1) - 2*b*f^3*((c + 1)*d - (c - 1)*d)/((d*x + c + 1)^4*d^5/(d*x + \\ & c - 1)^4 - 4*(d*x + c + 1)^3*d^5/(d*x + c - 1)^3 + 6*(d*x + c + 1)^2*d^5/(d \\ & *x + c - 1)^2 - 4*(d*x + c + 1)*d^5/(d*x + c - 1) + d^5) \end{aligned}$$

maple [B] time = 0.04, size = 786, normalized size = 4.68

$$\frac{b \ln(dx + c - 1)e^4}{8f} - \frac{b \ln(dx + c + 1)e^4}{8f} + \frac{b f^3 \ln(dx + c - 1)}{8d^4} - \frac{b f^3 \ln(dx + c + 1)}{8d^4} + \frac{b f^3 x^3}{12d} + \frac{b f^3 x}{4d^3} + a f^2 x^3 e + \frac{3afx}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f*x+e)^3*(a+b*arctanh(d*x+c)),x)

[Out] $\frac{1}{8}b/f*\ln(d*x+c-1)*e^4 - \frac{1}{8}b/f*\ln(d*x+c+1)*e^4 + \frac{1}{8}d^4*b*f^3*\ln(d*x+c-1) - \frac{1}{8}d^4*b*f^3*\ln(d*x+c+1) + \frac{1}{12}d*b*f^3*x^3 + \frac{1}{4}b/d^3*f^3*x+a*f^2*x^3*e + \frac{3}{2}a*f*x^2*e^2 + \frac{1}{2}d*e^3*b*\ln(d*x+c+1) + \frac{1}{2}d*e^3*b*\ln(d*x+c-1) + \frac{1}{4}b*f^3*arctanh(d*x+c)*x^4 + arctanh(d*x+c)*x*b*e^3 + \frac{1}{4}b/f*arctanh(d*x+c)*e^4 + \frac{13}{12}d^4*b*f^3*c^3 + \frac{1}{4}a/f*e^4 + \frac{1}{4}d^4*b*f^3*c + \frac{1}{4}a*f^3*x^4 + a*x*e^3 - \frac{2}{d^2}b*f^2*c*e*x - \frac{3}{2}d^2*b*f*\ln(d*x+c-1)*c*e^2 + \frac{3}{2}d^3*b*f^2*\ln(d*x+c+1)*c^2*e - \frac{3}{4}d^2*b*f*\ln(d*x+c+1)*c^2*e^2 - \frac{3}{2}d^3*b*f^2*\ln(d*x+c-1)*c*e + \frac{3}{2}d^3*b*f^2*\ln(d*x+c+1)*c*e + \frac{1}{2}d^3*b*f^2*\ln(d*x+c+1)*c^3*e + \frac{3}{4}d^2*b*f*\ln(d*x+c-1)*c^2*e^2 + \frac{3}{2}d^3*b*f^2*\ln(d*x+c-1)*c^2*e - \frac{3}{2}d^2*b*f*\ln(d*x+c+1)*c*e^2 - \frac{1}{2}d^3*b*f^2*\ln(d*x+c-1)*c^3*e + \frac{3}{4}b/d^3*f^3*c^2*x + \frac{3}{2}b/d*f*e^2*x - \frac{1}{2}d^4*b*f^3*\ln(d*x+c+1)*c - \frac{1}{2}d^4*b*f^3*\ln(d*x+c-1)*c^3 - \frac{1}{2}d^4*b*f^3*\ln(d*x+c-1)*c - \frac{1}{2}d^4*b*f^3*\ln(d*x+c+1)*c^3 - \frac{1}{4}d^2*b*f^3*x^2*c + \frac{1}{8}d^4*b*f^3*\ln(d*x+c-1)*c^4 + \frac{3}{2}b*f*arctanh(d*x+c)*e^2*x^2 + b*f^2*arctanh(d*x+c)*e*x^3 - \frac{3}{4}d^4*b*f^3*\ln(d*x+c+1)*c^2 - \frac{1}{8}d^4*b*f^3*\ln(d*x+c+1)*c^4 + \frac{3}{4}d^2*b*f*\ln(d*x+c-1)*e^2 - \frac{3}{4}d^2*b*f*\ln(d*x+c+1)*e^2 + \frac{1}{2}d^3*b*f^2*\ln(d*x+c-1)*e + \frac{3}{4}d^4*b*f^3*\ln(d*x+c-1)*c^2 + \frac{1}{2}d^3*b*f^2*\ln(d*x+c+1)*e + \frac{1}{2}d*b*\ln(d*x+c+1)*c*e^3 - \frac{5}{2}d^3*b*f^2*c^2*e - \frac{1}{2}d*b*\ln(d*x+c-1)*c*e^3 + \frac{1}{2}d*b*f^2*e*x^2 + \frac{3}{2}d^2*b*f*c*e^2$

maxima [B] time = 0.36, size = 333, normalized size = 1.98

$$\frac{1}{4}af^3x^4 + aef^2x^3 + \frac{3}{2}ae^2fx^2 + \frac{3}{4}\left(2x^2 \operatorname{artanh}(dx + c) + d\left(\frac{2x}{d^2} - \frac{(c^2 + 2c + 1)\log(dx + c + 1)}{d^3} + \frac{(c^2 - 2c + 1)\log(dx + c - 1)}{d^3}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x+e)^3*(a+b*arctanh(d*x+c)),x, algorithm="maxima")

```
[Out] 1/4*a*f^3*x^4 + a*e*f^2*x^3 + 3/2*a*e^2*f*x^2 + 3/4*(2*x^2*arctanh(d*x + c)
+ d*(2*x/d^2 - (c^2 + 2*c + 1)*log(d*x + c + 1)/d^3 + (c^2 - 2*c + 1)*log(
d*x + c - 1)/d^3))*b*e^2*f + 1/2*(2*x^3*arctanh(d*x + c) + d*((d*x^2 - 4*c*
x)/d^3 + (c^3 + 3*c^2 + 3*c + 1)*log(d*x + c + 1)/d^4 - (c^3 - 3*c^2 + 3*c
- 1)*log(d*x + c - 1)/d^4))*b*e*f^2 + 1/24*(6*x^4*arctanh(d*x + c) + d*(2*(
d^2*x^3 - 3*c*d*x^2 + 3*(3*c^2 + 1)*x)/d^4 - 3*(c^4 + 4*c^3 + 6*c^2 + 4*c +
1)*log(d*x + c + 1)/d^5 + 3*(c^4 - 4*c^3 + 6*c^2 - 4*c + 1)*log(d*x + c -
1)/d^5))*b*f^3 + a*e^3*x + 1/2*(2*(d*x + c)*arctanh(d*x + c) + log(-(d*x +
c)^2 + 1))*b*e^3/d
```

mupad [B] time = 1.72, size = 737, normalized size = 4.39

$$\ln(c + dx + 1) \left(\frac{be^3x}{2} + \frac{3be^2fx^2}{4} + \frac{bef^2x^3}{2} + \frac{bf^3x^4}{8} \right) - \ln(1 - dx - c) \left(\frac{be^3x}{2} + \frac{3be^2fx^2}{4} + \frac{bef^2x^3}{2} + \frac{bf^3x^4}{8} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)^3*(a + b*atanh(c + d*x)),x)
```

```
[Out] log(c + d*x + 1)*((b*f^3*x^4)/8 + (b*e^3*x)/2 + (3*b*e^2*f*x^2)/4 + (b*e*f^
2*x^3)/2) - log(1 - d*x - c)*((b*f^3*x^4)/8 + (b*e^3*x)/2 + (3*b*e^2*f*x^2)
/4 + (b*e*f^2*x^3)/2) + x*((e*(6*a*c^2*f^2 - 6*a*f^2 + 2*a*d^2*e^2 + 3*b*d*
e*f + 12*a*c*d*e*f))/(2*d^2) - ((4*c^2 - 4)*((f^2*(b*f + 8*a*c*f + 12*a*d*e
)))/(4*d) - (2*a*c*f^3)/d))/(4*d^2) + (2*c*((2*c*((f^2*(b*f + 8*a*c*f + 12*a
*d*e)))/(4*d) - (2*a*c*f^3)/d))/d - (4*a*c^2*f^3 - 4*a*f^3 + 4*b*d*e*f^2 + 1
2*a*d^2*e^2*f + 24*a*c*d*e*f^2)/(4*d^2) + (a*f^3*(4*c^2 - 4))/(4*d^2))/d)
- x^2*((c*((f^2*(b*f + 8*a*c*f + 12*a*d*e)))/(4*d) - (2*a*c*f^3)/d))/d - (4*
a*c^2*f^3 - 4*a*f^3 + 4*b*d*e*f^2 + 12*a*d^2*e^2*f + 24*a*c*d*e*f^2)/(8*d^2
) + (a*f^3*(4*c^2 - 4))/(8*d^2) + x^3*((f^2*(b*f + 8*a*c*f + 12*a*d*e))/(1
2*d) - (2*a*c*f^3)/(3*d)) + (a*f^3*x^4)/4 + (log(c + d*x - 1)*(b*f^3 + 6*b*
c^2*f^3 - 4*b*c^3*f^3 + 4*b*d^3*e^3 + b*c^4*f^3 - 4*b*c*f^3 + 4*b*d*e*f^2 -
4*b*c*d^3*e^3 + 6*b*d^2*e^2*f - 12*b*c*d^2*e^2*f + 12*b*c^2*d*e*f^2 - 4*b*
c^3*d*e*f^2 + 6*b*c^2*d^2*e^2*f - 12*b*c*d*e*f^2))/(8*d^4) - (log(c + d*x +
1)*(b*f^3 + 6*b*c^2*f^3 + 4*b*c^3*f^3 - 4*b*d^3*e^3 + b*c^4*f^3 + 4*b*c*f^
3 - 4*b*d*e*f^2 - 4*b*c*d^3*e^3 + 6*b*d^2*e^2*f + 12*b*c*d^2*e^2*f - 12*b*c
^2*d*e*f^2 - 4*b*c^3*d*e*f^2 + 6*b*c^2*d^2*e^2*f - 12*b*c*d*e*f^2))/(8*d^4)
```

sympy [A] time = 7.64, size = 644, normalized size = 3.83

$$\left\{ \begin{array}{l} ae^3x + \frac{3ae^2fx^2}{2} + ae^2fx^3 + \frac{af^3x^4}{4} - \frac{bc^4f^3 \operatorname{atanh}(c+dx)}{4d^4} + \frac{bc^3ef^2 \operatorname{atanh}(c+dx)}{d^3} - \frac{bc^3f^3 \log\left(\frac{c}{d} + x + \frac{1}{d}\right)}{d^4} + \frac{bc^3f^3 \operatorname{atanh}(c+dx)}{d^4} - \frac{3bc^2e^2f}{d^4} \\ (a + b \operatorname{atanh}(c)) \left(e^3x + \frac{3e^2fx^2}{2} + e^2fx^3 + \frac{f^3x^4}{4} \right) \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x+e)**3*(a+b*atanh(d*x+c)),x)

[Out] Piecewise((a*e**3*x + 3*a*e**2*f*x**2/2 + a*e*f**2*x**3 + a*f**3*x**4/4 - b*c**4*f**3*atanh(c + d*x)/(4*d**4) + b*c**3*e*f**2*atanh(c + d*x)/d**3 - b*c**3*f**3*log(c/d + x + 1/d)/d**4 + b*c**3*f**3*atanh(c + d*x)/d**4 - 3*b*c**2*e**2*f*atanh(c + d*x)/(2*d**2) + 3*b*c**2*e*f**2*log(c/d + x + 1/d)/d**3 - 3*b*c**2*e*f**2*atanh(c + d*x)/d**3 + 3*b*c**2*f**3*x/(4*d**3) - 3*b*c**2*f**3*atanh(c + d*x)/(2*d**4) + b*c*e**3*atanh(c + d*x)/d - 3*b*c*e**2*f*log(c/d + x + 1/d)/d**2 + 3*b*c*e**2*f*atanh(c + d*x)/d**2 - 2*b*c*e*f**2*x/d**2 - b*c*f**3*x**2/(4*d**2) + 3*b*c*e*f**2*atanh(c + d*x)/d**3 - b*c*f**3*log(c/d + x + 1/d)/d**4 + b*c*f**3*atanh(c + d*x)/d**4 + b*e**3*x*atanh(c + d*x) + 3*b*e**2*f*x**2*atanh(c + d*x)/2 + b*e*f**2*x**3*atanh(c + d*x) + b*f**3*x**4*atanh(c + d*x)/4 + b*e**3*log(c/d + x + 1/d)/d - b*e**3*atanh(c + d*x)/d + 3*b*e**2*f*x/(2*d) + b*e*f**2*x**2/(2*d) + b*f**3*x**3/(12*d) - 3*b*e**2*f*atanh(c + d*x)/(2*d**2) + b*e*f**2*log(c/d + x + 1/d)/d**3 - b*e*f**2*atanh(c + d*x)/d**3 + b*f**3*x/(4*d**3) - b*f**3*atanh(c + d*x)/(4*d**4), Ne(d, 0)), ((a + b*atanh(c))*(e**3*x + 3*e**2*f*x**2/2 + e*f**2*x**3 + f**3*x**4/4), True))

3.32 $\int (e + fx)^2 (a + b \tanh^{-1}(c + dx)) dx$

Optimal. Leaf size=120

$$\frac{(e + fx)^3 (a + b \tanh^{-1}(c + dx))}{3f} + \frac{b(-cf + de + f)^3 \log(-c - dx + 1)}{6d^3 f} - \frac{b(de - (c + 1)f)^3 \log(c + dx + 1)}{6d^3 f} + \frac{bf^2(c + dx)^2}{6d^3 f}$$

[Out] b*f*(-c*f+d*e)*x/d^2+1/6*b*f^2*(d*x+c)^2/d^3+1/3*(f*x+e)^3*(a+b*arctanh(d*x+c))/f+1/6*b*(-c*f+d*e+f)^3*ln(-d*x-c+1)/d^3/f-1/6*b*(d*e-(1+c)*f)^3*ln(d*x+c+1)/d^3/f

Rubi [A] time = 0.20, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {6111, 5926, 702, 633, 31}

$$\frac{(e + fx)^3 (a + b \tanh^{-1}(c + dx))}{3f} + \frac{bfx(de - cf)}{d^2} + \frac{b(-cf + de + f)^3 \log(-c - dx + 1)}{6d^3 f} - \frac{b(de - (c + 1)f)^3 \log(c + dx + 1)}{6d^3 f}$$

Antiderivative was successfully verified.

[In] Int[(e + f*x)^2*(a + b*ArcTanh[c + d*x]),x]

[Out] (b*f*(d*e - c*f)*x)/d^2 + (b*f^2*(c + d*x)^2)/(6*d^3) + ((e + f*x)^3*(a + b*ArcTanh[c + d*x]))/(3*f) + (b*(d*e + f - c*f)^3*Log[1 - c - d*x])/(6*d^3*f) - (b*(d*e - (1 + c)*f)^3*Log[1 + c + d*x])/(6*d^3*f)

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 633

Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] := With[{q = Rt[-(a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x), x], x] + Dist[e/2 - (c*d)/(2*q), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[-(a*c)]

Rule 702

Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] := Int[PolynomialDivide[(d + e*x)^m, a + c*x^2, x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])

Rule 5926

```
Int[((a_.) + ArcTanh[(c_.)*(x_.)]*(b_.))*((d_.) + (e_.)*(x_.))^(q_.), x_Symbol]
:> Simp[((d + e*x)^(q + 1)*(a + b*ArcTanh[c*x]))/(e*(q + 1)), x] - Dist[(b*c)/(e*(q + 1)), Int[(d + e*x)^(q + 1)/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[q, -1]
```

Rule 6111

```
Int[((a_.) + ArcTanh[(c_.) + (d_.)*(x_.)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_.))^(m_.), x_Symbol]
:> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*ArcTanh[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned} \int (e + fx)^2 (a + b \tanh^{-1}(c + dx)) dx &= \frac{\text{Subst}\left(\int \left(\frac{de-cf}{d} + \frac{fx}{d}\right)^2 (a + b \tanh^{-1}(x)) dx, x, c + dx\right)}{d} \\ &= \frac{(e + fx)^3 (a + b \tanh^{-1}(c + dx))}{3f} - \frac{b \text{Subst}\left(\int \frac{\left(\frac{de-cf}{d} + \frac{fx}{d}\right)^3}{1-x^2} dx, x, c + dx\right)}{3f} \\ &= \frac{(e + fx)^3 (a + b \tanh^{-1}(c + dx))}{3f} - \frac{b \text{Subst}\left(\int \left(-\frac{3f^2(de-cf)}{d^3} - \frac{f^3x}{d^3} + \frac{(de-cf)}{d^3}\right) dx, x, c + dx\right)}{3f} \\ &= \frac{bf(de - cf)x}{d^2} + \frac{bf^2(c + dx)^2}{6d^3} + \frac{(e + fx)^3 (a + b \tanh^{-1}(c + dx))}{3f} - \frac{b \text{Subst}\left(\int \frac{1}{1-x^2} dx, x, c + dx\right)}{3f} \\ &= \frac{bf(de - cf)x}{d^2} + \frac{bf^2(c + dx)^2}{6d^3} + \frac{(e + fx)^3 (a + b \tanh^{-1}(c + dx))}{3f} - \frac{b(d^2 - (c + dx)^2)}{6d^3} \\ &= \frac{bf(de - cf)x}{d^2} + \frac{bf^2(c + dx)^2}{6d^3} + \frac{(e + fx)^3 (a + b \tanh^{-1}(c + dx))}{3f} + \frac{b(de - c^2 - 2cdx - dx^2)}{6d^3} \end{aligned}$$

Mathematica [A] time = 0.17, size = 174, normalized size = 1.45

$$\frac{2dx(3ad^2e^2 + bf(3de - 2cf)) + d^2fx^2(6ade + bf) + 2ad^3f^2x^3 + 2bd^3x(3e^2 + 3efx + f^2x^2) \tanh^{-1}(c + dx) - b(d^2 - (c + dx)^2)}{6d^3}$$

Antiderivative was successfully verified.

[In] Integrate[(e + f*x)^2*(a + b*ArcTanh[c + d*x]),x]

[Out] (2*d*(3*a*d^2*e^2 + b*f*(3*d*e - 2*c*f))*x + d^2*f*(6*a*d*e + b*f)*x^2 + 2*a*d^3*f^2*x^3 + 2*b*d^3*x*(3*e^2 + 3*e*f*x + f^2*x^2)*ArcTanh[c + d*x] - b*(-1 + c)*(3*d^2*e^2 - 3*(-1 + c)*d*e*f + (-1 + c)^2*f^2)*Log[1 - c - d*x] + b*(1 + c)*(3*d^2*e^2 - 3*(1 + c)*d*e*f + (1 + c)^2*f^2)*Log[1 + c + d*x])/(6*d^3)

fricas [B] time = 0.48, size = 242, normalized size = 2.02

$$\frac{2ad^3f^2x^3 + (6ad^3ef + bd^2f^2)x^2 + 2(3ad^3e^2 + 3bd^2ef - 2bcd^2f^2)x + (3(bc + b)d^2e^2 - 3(bc^2 + 2bc + b)def + \dots)}{6d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x+e)^2*(a+b*arctanh(d*x+c)),x, algorithm="fricas")

[Out] 1/6*(2*a*d^3*f^2*x^3 + (6*a*d^3*e*f + b*d^2*f^2)*x^2 + 2*(3*a*d^3*e^2 + 3*b*d^2*e*f - 2*b*c*d*f^2)*x + (3*(b*c + b)*d^2*e^2 - 3*(b*c^2 + 2*b*c + b)*d*e*f + (b*c^3 + 3*b*c^2 + 3*b*c + b)*f^2)*log(d*x + c + 1) - (3*(b*c - b)*d^2*e^2 - 3*(b*c^2 - 2*b*c + b)*d*e*f + (b*c^3 - 3*b*c^2 + 3*b*c - b)*f^2)*log(d*x + c - 1) + (b*d^3*f^2*x^3 + 3*b*d^3*e*f*x^2 + 3*b*d^3*e^2*x)*log(-(d*x + c + 1)/(d*x + c - 1))/d^3

giac [B] time = 0.30, size = 1717, normalized size = 14.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x+e)^2*(a+b*arctanh(d*x+c)),x, algorithm="giac")

[Out] -1/6*(3*(d*x + c + 1)^3*b*c^2*f^2*log(-(d*x + c + 1)/(d*x + c - 1) + 1)/(d*x + c - 1)^3 - 9*(d*x + c + 1)^2*b*c^2*f^2*log(-(d*x + c + 1)/(d*x + c - 1) + 1)/(d*x + c - 1)^2 + 9*(d*x + c + 1)*b*c^2*f^2*log(-(d*x + c + 1)/(d*x + c - 1) + 1)/(d*x + c - 1) - 3*b*c^2*f^2*log(-(d*x + c + 1)/(d*x + c - 1) + 1) - 6*(d*x + c + 1)^3*b*c*d*f*e*log(-(d*x + c + 1)/(d*x + c - 1) + 1)/(d*x + c - 1)^3 + 18*(d*x + c + 1)^2*b*c*d*f*e*log(-(d*x + c + 1)/(d*x + c - 1) + 1)/(d*x + c - 1)^2 - 18*(d*x + c + 1)*b*c*d*f*e*log(-(d*x + c + 1)/(d*x + c - 1) + 1)/(d*x + c - 1) + 6*b*c*d*f*e*log(-(d*x + c + 1)/(d*x + c - 1) + 1) - 3*(d*x + c + 1)^3*b*c^2*f^2*log(-(d*x + c + 1)/(d*x + c - 1))/(d*x + c - 1)^3 + 6*(d*x + c + 1)^2*b*c^2*f^2*log(-(d*x + c + 1)/(d*x + c - 1))/(d*x + c - 1)^2 - 3*(d*x + c + 1)*b*c^2*f^2*log(-(d*x + c + 1)/(d*x + c - 1))/(d*x + c - 1) + 6*(d*x + c + 1)^3*b*c*d*f*e*log(-(d*x + c + 1)/(d*x + c - 1))/(d*x + c - 1)^3 - 12*(d*x + c + 1)^2*b*c*d*f*e*log(-(d*x + c + 1)/(d*x + c - 1))/(d*x + c - 1)^2 + 6*(d*x + c + 1)*b*c*d*f*e*log(-(d*x + c + 1)/(d*x + c - 1))/(d*x + c - 1)^2 - 6*(d*x + c + 1)*b*c*d*f*e*log(-(d*x + c + 1)/(d*x + c - 1))/(d*x + c - 1) + 3*(d*x + c + 1)^3*b*c^2*f^2*log(-(d*x + c + 1)/(d*x + c - 1))/(d*x + c - 1)^3 - 9*(d*x + c + 1)^2*b*c^2*f^2*log(-(d*x + c + 1)/(d*x + c - 1))/(d*x + c - 1)^2 + 9*(d*x + c + 1)*b*c^2*f^2*log(-(d*x + c + 1)/(d*x + c - 1))/(d*x + c - 1) - 3*b*c^2*f^2*log(-(d*x + c + 1)/(d*x + c - 1)) - 6*(d*x + c + 1)^3*b*c*d*f*e*log(-(d*x + c + 1)/(d*x + c - 1)) + 18*(d*x + c + 1)^2*b*c*d*f*e*log(-(d*x + c + 1)/(d*x + c - 1)) - 18*(d*x + c + 1)*b*c*d*f*e*log(-(d*x + c + 1)/(d*x + c - 1)) + 6*b*c*d*f*e*log(-(d*x + c + 1)/(d*x + c - 1)) - 3*(d*x + c + 1)^3*b*c^2*f^2*log(-(d*x + c + 1)/(d*x + c - 1)) + 6*(d*x + c + 1)^2*b*c^2*f^2*log(-(d*x + c + 1)/(d*x + c - 1)) - 3*(d*x + c + 1)*b*c^2*f^2*log(-(d*x + c + 1)/(d*x + c - 1)) + 6*(d*x + c + 1)^3*b*c*d*f*e*log(-(d*x + c + 1)/(d*x + c - 1)) - 12*(d*x + c + 1)^2*b*c*d*f*e*log(-(d*x + c + 1)/(d*x + c - 1)) + 6*(d*x + c + 1)*b*c*d*f*e*log(-(d*x + c + 1)/(d*x + c - 1)) - 6*(d*x + c + 1)*b*c*d*f*e*log(-(d*x + c + 1)/(d*x + c - 1)) + 3*(d*x + c + 1)^3*b*c^2*f^2*log(-(d*x + c + 1)/(d*x + c - 1)) - 9*(d*x + c + 1)^2*b*c^2*f^2*log(-(d*x + c + 1)/(d*x + c - 1)) + 9*(d*x + c + 1)*b*c^2*f^2*log(-(d*x + c + 1)/(d*x + c - 1)) - 3*b*c^2*f^2*log(-(d*x + c + 1)/(d*x + c - 1)))/d^3

```

(d*x + c - 1))/(d*x + c - 1) - 6*(d*x + c + 1)^2*a*c^2*f^2/(d*x + c - 1)^2
+ 12*(d*x + c + 1)*a*c^2*f^2/(d*x + c - 1) - 6*a*c^2*f^2 + 12*(d*x + c + 1)
^2*a*c*d*f*e/(d*x + c - 1)^2 - 24*(d*x + c + 1)*a*c*d*f*e/(d*x + c - 1) +
12*a*c*d*f*e + 3*(d*x + c + 1)^3*b*d^2*e^2*log(-(d*x + c + 1)/(d*x + c - 1)
+ 1)/(d*x + c - 1)^3 - 9*(d*x + c + 1)^2*b*d^2*e^2*log(-(d*x + c + 1)/(d*x
+ c - 1) + 1)/(d*x + c - 1)^2 + 9*(d*x + c + 1)*b*d^2*e^2*log(-(d*x + c + 1)
)/(d*x + c - 1) + 1)/(d*x + c - 1) - 3*b*d^2*e^2*log(-(d*x + c + 1)/(d*x +
c - 1) + 1) + 6*(d*x + c + 1)^2*b*c*f^2*log(-(d*x + c + 1)/(d*x + c - 1))/(
d*x + c - 1)^2 - 6*(d*x + c + 1)*b*c*f^2*log(-(d*x + c + 1)/(d*x + c - 1))/
(d*x + c - 1) - 3*(d*x + c + 1)^3*b*d^2*e^2*log(-(d*x + c + 1)/(d*x + c - 1
))/(d*x + c - 1)^3 + 6*(d*x + c + 1)^2*b*d^2*e^2*log(-(d*x + c + 1)/(d*x +
c - 1))/(d*x + c - 1)^2 - 3*(d*x + c + 1)*b*d^2*e^2*log(-(d*x + c + 1)/(d*x
+ c - 1))/(d*x + c - 1) - 6*(d*x + c + 1)^2*b*d*f*e*log(-(d*x + c + 1)/(d*
x + c - 1))/(d*x + c - 1)^2 + 6*(d*x + c + 1)*b*d*f*e*log(-(d*x + c + 1)/(d
*x + c - 1))/(d*x + c - 1) + 12*(d*x + c + 1)^2*a*c*f^2/(d*x + c - 1)^2 - 1
2*(d*x + c + 1)*a*c*f^2/(d*x + c - 1) + 6*(d*x + c + 1)^2*b*c*f^2/(d*x + c
- 1)^2 - 12*(d*x + c + 1)*b*c*f^2/(d*x + c - 1) + 6*b*c*f^2 - 6*(d*x + c +
1)^2*a*d^2*e^2/(d*x + c - 1)^2 + 12*(d*x + c + 1)*a*d^2*e^2/(d*x + c - 1) -
6*a*d^2*e^2 - 12*(d*x + c + 1)^2*a*d*f*e/(d*x + c - 1)^2 + 12*(d*x + c + 1
)*a*d*f*e/(d*x + c - 1) - 6*(d*x + c + 1)^2*b*d*f*e/(d*x + c - 1)^2 + 12*(d
*x + c + 1)*b*d*f*e/(d*x + c - 1) - 6*b*d*f*e + (d*x + c + 1)^3*b*f^2*log(-
(d*x + c + 1)/(d*x + c - 1) + 1)/(d*x + c - 1)^3 - 3*(d*x + c + 1)^2*b*f^2*
log(-(d*x + c + 1)/(d*x + c - 1) + 1)/(d*x + c - 1)^2 + 3*(d*x + c + 1)*b*f
^2*log(-(d*x + c + 1)/(d*x + c - 1) + 1)/(d*x + c - 1) - b*f^2*log(-(d*x +
c + 1)/(d*x + c - 1) + 1) - (d*x + c + 1)^3*b*f^2*log(-(d*x + c + 1)/(d*x +
c - 1))/(d*x + c - 1)^3 - 3*(d*x + c + 1)*b*f^2*log(-(d*x + c + 1)/(d*x +
c - 1))/(d*x + c - 1) - 6*(d*x + c + 1)^2*a*f^2/(d*x + c - 1)^2 - 2*a*f^2 -
2*(d*x + c + 1)^2*b*f^2/(d*x + c - 1)^2 + 2*(d*x + c + 1)*b*f^2/(d*x + c
- 1))*((c + 1)*d - (c - 1)*d)/((d*x + c + 1)^3*d^4/(d*x + c - 1)^3 - 3*(d*x
+ c + 1)^2*d^4/(d*x + c - 1)^2 + 3*(d*x + c + 1)*d^4/(d*x + c - 1) - d^4)

```

maple [B] time = 0.04, size = 477, normalized size = 3.98

$$\frac{b \operatorname{arctanh}(dx+c)e^3}{3f} + \frac{bf^2x^2}{6d} + afx^2e + \frac{bf^2 \operatorname{arctanh}(dx+c)x^3}{3} + \operatorname{arctanh}(dx+c)xb e^2 + \frac{b \ln(dx+c-1)e^3}{6f} + \frac{e^2b}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f*x+e)^2*(a+b*arctanh(d*x+c)),x)

[Out] 1/6/d*b*f^2*x^2+a*f*x^2*e+1/6*b/f*ln(d*x+c-1)*e^3+1/3*b*f^2*arctanh(d*x+c)*x^3+arctanh(d*x+c)*x*b*e^2+1/2/d*e^2*b*ln(d*x+c+1)+1/2/d*e^2*b*ln(d*x+c-1)+1/6/d^3*b*f^2*ln(d*x+c-1)+1/6/d^3*b*f^2*ln(d*x+c+1)-1/6*b/f*ln(d*x+c+1)*e^3+1/3*b/f*arctanh(d*x+c)*e^3+1/3*a/f*e^3-5/6/d^3*b*f^2*c^2+1/3*a*f^2*x^3+a*x*e^2-2/3*b/d^2*f^2*c*x-1/2/d^2*b*f*ln(d*x+c+1)*c^2*e+1/2/d^2*b*f*ln(d*x+c-1)*c^2*e-1/d^2*b*f*ln(d*x+c-1)*c*e-1/d^2*b*f*ln(d*x+c+1)*c*e+1/2/d^3*b*f^2*

$n(d*x+c+1)*c+1/2/d*b*\ln(d*x+c+1)*c*e^{-1/2}/d*b*\ln(d*x+c-1)*c*e^{1/2}/d^3*b*f^2*\ln(d*x+c+1)*c^2+1/6/d^3*b*f^2*\ln(d*x+c+1)*c^3-1/2/d^3*b*f^2*\ln(d*x+c-1)*c-1/2/d^2*b*f*\ln(d*x+c+1)*e+1/d^2*b*f*c*e+b/d*f*e*x+b*f*\operatorname{arctanh}(d*x+c)*e*x^2+1/2/d^3*b*f^2*\ln(d*x+c-1)*c^2-1/6/d^3*b*f^2*\ln(d*x+c-1)*c^3+1/2/d^2*b*f*\ln(d*x+c-1)*e$

maxima [A] time = 0.32, size = 207, normalized size = 1.72

$$\frac{1}{3}af^2x^3+aeafx^2+\frac{1}{2}\left(2x^2\operatorname{artanh}(dx+c)+d\left(\frac{2x}{d^2}-\frac{(c^2+2c+1)\log(dx+c+1)}{d^3}+\frac{(c^2-2c+1)\log(dx+c-1)}{d^3}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x+e)^2*(a+b*arctanh(d*x+c)),x, algorithm="maxima")

[Out] $\frac{1}{3}af^2x^3+aeafx^2+\frac{1}{2}(2x^2\operatorname{artanh}(dx+c)+d(2x/d^2-(c^2+2c+1)\log(dx+c+1)/d^3+(c^2-2c+1)\log(dx+c-1)/d^3))*e*f+1/6(2x^3\operatorname{arctanh}(d*x+c)+d((d*x^2-4*c*x)/d^3+(c^3+3*c^2+3*c+1)\log(d*x+c+1)/d^4-(c^3-3*c^2+3*c-1)\log(d*x+c-1)/d^4))*b*f^2+a*e^2*x+1/2(2*(d*x+c)*\operatorname{arctanh}(d*x+c)+\log(-(d*x+c)^2+1))*b*e^2/d$

mupad [B] time = 1.39, size = 381, normalized size = 3.18

$$x^2\left(\frac{f(bf+6acf+6ade)}{6d}-\frac{acf^2}{d}\right)-\ln(1-dx-c)\left(\frac{be^2x}{2}+\frac{befx^2}{2}+\frac{bf^2x^3}{6}\right)-x\left(\frac{2c\left(\frac{f(bf+6acf+6ade)}{3d}\right)}{d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e+f*x)^2*(a+b*atanh(c+d*x)),x)

[Out] $x^2((f*(b*f+6*a*c*f+6*a*d*e))/(6*d)-(a*c*f^2)/d)-\log(1-d*x-c)*((b*f^2*x^3)/6+(b*e^2*x)/2+(b*e*f*x^2)/2)-x((2*c*((f*(b*f+6*a*c*f+6*a*d*e))/(3*d)-(2*a*c*f^2)/d))/d-(3*a*c^2*f^2-3*a*f^2+3*a*d^2*e^2+3*b*d*e*f+12*a*c*d*e*f)/(3*d^2)+(a*f^2*(3*c^2-3))/(3*d^2))+\log(c+d*x+1)*((b*f^2*x^3)/6+(b*e^2*x)/2+(b*e*f*x^2)/2)+(a*f^2*x^3)/3+(\log(c+d*x-1))*((b*f^2)/6+d*((b*e*f)/2+(b*c^2*e*f)/2-b*c*e*f)+d^2*((b*e^2)/2-(b*c*e^2)/2)+(b*c^2*f^2)/2-(b*c^3*f^2)/6-(b*c*f^2)/2))/d^3+(\log(c+d*x+1))*((b*f^2)/6-d*((b*e*f)/2+(b*c^2*e*f)/2+b*c*e*f)+d^2*((b*e^2)/2+(b*c*e^2)/2)+(b*c^2*f^2)/2+(b*c^3*f^2)/6+(b*c*f^2)/2))/d^3$

sympy [A] time = 4.57, size = 369, normalized size = 3.08

$$\left\{ \begin{array}{l} ae^2x + aefx^2 + \frac{af^2x^3}{3} + \frac{bc^3f^2 \operatorname{atanh}(c+dx)}{3d^3} - \frac{bc^2ef \operatorname{atanh}(c+dx)}{d^2} + \frac{bc^2f^2 \log\left(\frac{c}{d} + x + \frac{1}{d}\right)}{d^3} - \frac{bc^2f^2 \operatorname{atanh}(c+dx)}{d^3} + \frac{bce^2 \operatorname{atanh}(c+dx)}{d} - \\ (a + b \operatorname{atanh}(c)) \left(e^2x + efx^2 + \frac{f^2x^3}{3} \right) \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x+e)**2*(a+b*atanh(d*x+c)),x)

[Out] Piecewise((a*e**2*x + a*e*f*x**2 + a*f**2*x**3/3 + b*c**3*f**2*atanh(c + d*x)/(3*d**3) - b*c**2*e*f*atanh(c + d*x)/d**2 + b*c**2*f**2*log(c/d + x + 1/d)/d**3 - b*c**2*f**2*atanh(c + d*x)/d**3 + b*c*e**2*atanh(c + d*x)/d - 2*b*c*e*f*log(c/d + x + 1/d)/d**2 + 2*b*c*e*f*atanh(c + d*x)/d**2 - 2*b*c*f**2*x/(3*d**2) + b*c*f**2*atanh(c + d*x)/d**3 + b*e**2*x*atanh(c + d*x) + b*e*f*x**2*atanh(c + d*x) + b*f**2*x**3*atanh(c + d*x)/3 + b*e**2*log(c/d + x + 1/d)/d - b*e**2*atanh(c + d*x)/d + b*e*f*x/d + b*f**2*x**2/(6*d) - b*e*f*atanh(c + d*x)/d**2 + b*f**2*log(c/d + x + 1/d)/(3*d**3) - b*f**2*atanh(c + d*x)/(3*d**3), Ne(d, 0)), ((a + b*atanh(c))*(e**2*x + e*f*x**2 + f**2*x**3/3), True))

3.33 $\int (e + fx) (a + b \tanh^{-1}(c + dx)) dx$

Optimal. Leaf size=97

$$\frac{(e + fx)^2 (a + b \tanh^{-1}(c + dx))}{2f} + \frac{b(-cf + de + f)^2 \log(-c - dx + 1)}{4d^2 f} - \frac{b(de - (c + 1)f)^2 \log(c + dx + 1)}{4d^2 f} + \frac{bf x}{2d}$$

[Out] $1/2*b*f*x/d+1/2*(f*x+e)^2*(a+b*\operatorname{arctanh}(d*x+c))/f+1/4*b*(-c*f+d*e+f)^2*\ln(-d*x-c+1)/d^2/f-1/4*b*(d*e-(1+c)*f)^2*\ln(d*x+c+1)/d^2/f$

Rubi [A] time = 0.17, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {6111, 5926, 702, 633, 31}

$$\frac{(e + fx)^2 (a + b \tanh^{-1}(c + dx))}{2f} + \frac{b(-cf + de + f)^2 \log(-c - dx + 1)}{4d^2 f} - \frac{b(de - (c + 1)f)^2 \log(c + dx + 1)}{4d^2 f} + \frac{bf x}{2d}$$

Antiderivative was successfully verified.

[In] `Int[(e + f*x)*(a + b*ArcTanh[c + d*x]),x]`

[Out] $(b*f*x)/(2*d) + ((e + f*x)^2*(a + b*ArcTanh[c + d*x]))/(2*f) + (b*(d*e + f - c*f)^2*\operatorname{Log}[1 - c - d*x])/(4*d^2*f) - (b*(d*e - (1 + c)*f)^2*\operatorname{Log}[1 + c + d*x])/(4*d^2*f)$

Rule 31

`Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]`

Rule 633

`Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] := With[{q = Rt[-(a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x), x], x] + Dist[e/2 - (c*d)/(2*q), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[-(a*c)]`

Rule 702

`Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] := Int[PolynomialDivide[(d + e*x)^m, a + c*x^2, x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])`

Rule 5926

```
Int[((a_.) + ArcTanh[(c_.)*(x_.)]*(b_.))*((d_.) + (e_.)*(x_.))^(q_.), x_Symbol]
:> Simp[((d + e*x)^(q + 1)*(a + b*ArcTanh[c*x]))/(e*(q + 1)), x] - Dist[(b*c)/(e*(q + 1)), Int[(d + e*x)^(q + 1)/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[q, -1]
```

Rule 6111

```
Int[((a_.) + ArcTanh[(c_.) + (d_.)*(x_.)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_.))^(m_.), x_Symbol]
:> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*ArcTanh[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int (e + fx) (a + b \tanh^{-1}(c + dx)) dx &= \frac{\text{Subst}\left(\int \left(\frac{de-cf}{d} + \frac{fx}{d}\right) (a + b \tanh^{-1}(x)) dx, x, c + dx\right)}{d} \\
&= \frac{(e + fx)^2 (a + b \tanh^{-1}(c + dx))}{2f} - \frac{b \text{Subst}\left(\int \frac{\left(\frac{de-cf}{d} + \frac{fx}{d}\right)^2}{1-x^2} dx, x, c + dx\right)}{2f} \\
&= \frac{(e + fx)^2 (a + b \tanh^{-1}(c + dx))}{2f} - \frac{b \text{Subst}\left(\int \left(-\frac{f^2}{d^2} + \frac{d^2e^2 - 2cdef + (1+c^2)f^2 + 2d^2ef - 2cde^2}{d^2(1-x^2)} dx, x, c + dx\right)}{2f} \\
&= \frac{bfx}{2d} + \frac{(e + fx)^2 (a + b \tanh^{-1}(c + dx))}{2f} - \frac{b \text{Subst}\left(\int \frac{d^2e^2 - 2cdef + (1+c^2)f^2 + 2d^2ef - 2cde^2}{1-x^2} dx, x, c + dx\right)}{2d^2f} \\
&= \frac{bfx}{2d} + \frac{(e + fx)^2 (a + b \tanh^{-1}(c + dx))}{2f} - \frac{(b(de + f - cf)^2) \text{Subst}\left(\int \frac{1}{1-x} dx, x, c + dx\right)}{4d^2f} \\
&= \frac{bfx}{2d} + \frac{(e + fx)^2 (a + b \tanh^{-1}(c + dx))}{2f} + \frac{b(de + f - cf)^2 \log(1 - c - dx)}{4d^2f}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 138, normalized size = 1.42

$$aex + \frac{1}{2}afx^2 + \frac{b(c^2 - 2c + 1)f \log(-c - dx + 1)}{4d^2} + \frac{b(-c^2 - 2c - 1)f \log(c + dx + 1)}{4d^2} + \frac{be((c + 1) \log(c + dx + 1) - \log(-c - dx + 1))}{2d}$$

Antiderivative was successfully verified.

[In] Integrate[(e + f*x)*(a + b*ArcTanh[c + d*x]),x]

[Out] $a*e*x + (b*f*x)/(2*d) + (a*f*x^2)/2 + b*e*x*ArcTanh[c + d*x] + (b*f*x^2*ArcTanh[c + d*x])/2 + (b*(1 - 2*c + c^2)*f*Log[1 - c - d*x])/(4*d^2) + (b*(-1 - 2*c - c^2)*f*Log[1 + c + d*x])/(4*d^2) + (b*e*(-((-1 + c)*Log[1 - c - d*x]) + (1 + c)*Log[1 + c + d*x]))/(2*d)$

fricas [A] time = 0.52, size = 134, normalized size = 1.38

$$\frac{2ad^2fx^2 + 2(2ad^2e + bdf)x + (2(bc + b)de - (bc^2 + 2bc + b)f)\log(dx + c + 1) - (2(bc - b)de - (bc^2 - 2bc + b)f)\log(dx + c - 1)}{4d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x+e)*(a+b*arctanh(d*x+c)),x, algorithm="fricas")

[Out] $1/4*(2*a*d^2*f*x^2 + 2*(2*a*d^2*e + b*d*f)*x + (2*(b*c + b)*d*e - (b*c^2 + 2*b*c + b)*f)*\log(d*x + c + 1) - (2*(b*c - b)*d*e - (b*c^2 - 2*b*c + b)*f)*\log(d*x + c - 1) + (b*d^2*f*x^2 + 2*b*d^2*e*x)*\log(-(d*x + c + 1)/(d*x + c - 1)))/d^2$

giac [B] time = 0.21, size = 536, normalized size = 5.53

$$\left(\frac{(dx+c+1)^2bcf \log\left(\frac{dx+c+1}{dx+c-1} + 1\right)}{(dx+c-1)^2} - \frac{2(dx+c+1)bcf \log\left(\frac{dx+c+1}{dx+c-1} + 1\right)}{dx+c-1} + bcf \log\left(-\frac{dx+c+1}{dx+c-1} + 1\right) - \frac{(dx+c+1)^2bde \log\left(\frac{dx+c+1}{dx+c-1} + 1\right)}{(dx+c-1)^2} + \frac{2(dx+c+1)bde \log\left(\frac{dx+c+1}{dx+c-1} + 1\right)}{dx+c-1} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x+e)*(a+b*arctanh(d*x+c)),x, algorithm="giac")

[Out] $1/2*((d*x + c + 1)^2*b*c*f*\log(-(d*x + c + 1)/(d*x + c - 1) + 1)/(d*x + c - 1)^2 - 2*(d*x + c + 1)*b*c*f*\log(-(d*x + c + 1)/(d*x + c - 1) + 1)/(d*x + c - 1) + b*c*f*\log(-(d*x + c + 1)/(d*x + c - 1) + 1) - (d*x + c + 1)^2*b*d*e*\log(-(d*x + c + 1)/(d*x + c - 1) + 1)/(d*x + c - 1)^2 + 2*(d*x + c + 1)*b*d*e*\log(-(d*x + c + 1)/(d*x + c - 1) + 1)/(d*x + c - 1) - b*d*e*\log(-(d*x + c + 1)/(d*x + c - 1) + 1) - (d*x + c + 1)^2*b*c*f*\log(-(d*x + c + 1)/(d*x + c - 1))/(d*x + c - 1)^2 + (d*x + c + 1)*b*c*f*\log(-(d*x + c + 1)/(d*x + c - 1))/(d*x + c - 1) + (d*x + c + 1)^2*b*d*e*\log(-(d*x + c + 1)/(d*x + c - 1))/(d*x + c - 1)^2 - (d*x + c + 1)*b*d*e*\log(-(d*x + c + 1)/(d*x + c - 1))/(d*x + c - 1) - 2*(d*x + c + 1)*a*c*f/(d*x + c - 1) + 2*a*c*f + 2*(d*x + c + 1)*a*d*e/(d*x + c - 1) - 2*a*d*e + (d*x + c + 1)*b*f*\log(-(d*x + c + 1)/(d*x + c - 1))/(d*x + c - 1) + 2*(d*x + c + 1)*a*f/(d*x + c - 1) + (d*x + c + 1)*b*f/(d*x + c - 1) - b*f)*((c + 1)*d - (c - 1)*d)/((d*x + c + 1)^2*d^3/(d*x + c - 1)^2 - 2*(d*x + c + 1)*d^3/(d*x + c - 1) + d^3)$

maple [B] time = 0.04, size = 184, normalized size = 1.90

$$\frac{ax^2f}{2} - \frac{ac^2f}{2d^2} + axe + \frac{ace}{d} + \frac{b \operatorname{arctanh}(dx+c)fx^2}{2} - \frac{b \operatorname{arctanh}(dx+c)fc^2}{2d^2} + \operatorname{arctanh}(dx+c)xbe + \frac{\operatorname{arctanh}(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f*x+e)*(a+b*arctanh(d*x+c)),x)

[Out] 1/2*a*x^2*f-1/2/d^2*a*c^2*f+a*x*e+1/d*a*c*e+1/2*b*arctanh(d*x+c)*f*x^2-1/2/d^2*b*arctanh(d*x+c)*f*c^2+arctanh(d*x+c)*x*b*e+1/d*arctanh(d*x+c)*b*c*e+1/2*b*f*x/d+1/2/d^2*b*c*f-1/2/d^2*b*ln(d*x+c-1)*c*f+1/2/d*b*ln(d*x+c-1)*e+1/4/d^2*b*ln(d*x+c-1)*f-1/2/d^2*b*ln(d*x+c+1)*c*f+1/2/d*b*ln(d*x+c+1)*e-1/4/d^2*b*ln(d*x+c+1)*f

maxima [A] time = 0.32, size = 109, normalized size = 1.12

$$\frac{1}{2}afx^2 + \frac{1}{4} \left(2x^2 \operatorname{artanh}(dx+c) + d \left(\frac{2x}{d^2} - \frac{(c^2+2c+1) \log(dx+c+1)}{d^3} + \frac{(c^2-2c+1) \log(dx+c-1)}{d^3} \right) \right) bfa + aex + 1$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x+e)*(a+b*arctanh(d*x+c)),x, algorithm="maxima")

[Out] 1/2*a*f*x^2 + 1/4*(2*x^2*arctanh(d*x+c) + d*(2*x/d^2 - (c^2+2*c+1)*log(d*x+c+1)/d^3 + (c^2-2*c+1)*log(d*x+c-1)/d^3))*b*f + a*e*x + 1/2*(2*(d*x+c)*arctanh(d*x+c) + log(-(d*x+c)^2+1))*b*e/d

mupad [B] time = 1.34, size = 136, normalized size = 1.40

$$aex + \frac{afx^2}{2} + \frac{be \ln(c^2+2cdx+d^2x^2-1)}{2d} - \frac{bf \operatorname{atanh}(c+dx)}{2d^2} + \frac{bf x^2 \operatorname{atanh}(c+dx)}{2} + \frac{bf x}{2d} + bex \operatorname{atanh}(c+dx)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e+f*x)*(a+b*atanh(c+d*x)),x)

[Out] a*e*x + (a*f*x^2)/2 + (b*e*log(c^2+d^2*x^2+2*c*d*x-1))/(2*d) - (b*f*atanh(c+d*x))/(2*d^2) + (b*f*x^2*atanh(c+d*x))/2 + (b*f*x)/(2*d) + b*e*x*atanh(c+d*x) - (b*c^2*f*atanh(c+d*x))/(2*d^2) - (b*c*f*log(c^2+d^2*x^2+2*c*d*x-1))/(2*d^2) + (b*c*e*atanh(c+d*x))/d

sympy [A] time = 2.35, size = 173, normalized size = 1.78

$$\left\{ \begin{array}{l} aex + \frac{afx^2}{2} - \frac{bc^2f \operatorname{atanh}(c+dx)}{2d^2} + \frac{bce \operatorname{atanh}(c+dx)}{d} - \frac{bcf \log\left(\frac{c}{d}+x+\frac{1}{d}\right)}{d^2} + \frac{bcf \operatorname{atanh}(c+dx)}{d^2} + bex \operatorname{atanh}(c+dx) + \frac{bf x^2 \operatorname{atanh}(c+dx)}{2} \\ (a+b \operatorname{atanh}(c)) \left(ex + \frac{fx^2}{2} \right) \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(a+b*atanh(d*x+c)),x)
```

```
[Out] Piecewise((a*e*x + a*f*x**2/2 - b*c**2*f*atanh(c + d*x)/(2*d**2) + b*c*e*at
anh(c + d*x)/d - b*c*f*log(c/d + x + 1/d)/d**2 + b*c*f*atanh(c + d*x)/d**2
+ b*e*x*atanh(c + d*x) + b*f*x**2*atanh(c + d*x)/2 + b*e*log(c/d + x + 1/d)
/d - b*e*atanh(c + d*x)/d + b*f*x/(2*d) - b*f*atanh(c + d*x)/(2*d**2), Ne(d
, 0)), ((a + b*atanh(c))*(e*x + f*x**2/2), True))
```

3.34 $\int (a + b \tanh^{-1}(c + dx)) dx$

Optimal. Leaf size=40

$$ax + \frac{b \log(1 - (c + dx)^2)}{2d} + \frac{b(c + dx) \tanh^{-1}(c + dx)}{d}$$

[Out] a*x+b*(d*x+c)*arctanh(d*x+c)/d+1/2*b*ln(1-(d*x+c)^2)/d

Rubi [A] time = 0.02, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {6103, 5910, 260}

$$ax + \frac{b \log(1 - (c + dx)^2)}{2d} + \frac{b(c + dx) \tanh^{-1}(c + dx)}{d}$$

Antiderivative was successfully verified.

[In] Int[a + b*ArcTanh[c + d*x], x]

[Out] a*x + (b*(c + d*x)*ArcTanh[c + d*x])/d + (b*Log[1 - (c + d*x)^2])/(2*d)

Rule 260

Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] :> Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 5910

Int[((a_.) + ArcTanh[(c_.)*(x_)])*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*ArcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]

Rule 6103

Int[((a_.) + ArcTanh[(c_) + (d_.)*(x_)])*(b_.))^(p_.), x_Symbol] :> Dist[1/d, Subst[Int[(a + b*ArcTanh[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[p, 0]

Rubi steps

$$\begin{aligned}
\int (a + b \tanh^{-1}(c + dx)) dx &= ax + b \int \tanh^{-1}(c + dx) dx \\
&= ax + \frac{b \operatorname{Subst}\left(\int \tanh^{-1}(x) dx, x, c + dx\right)}{d} \\
&= ax + \frac{b(c + dx) \tanh^{-1}(c + dx)}{d} - \frac{b \operatorname{Subst}\left(\int \frac{x}{1-x^2} dx, x, c + dx\right)}{d} \\
&= ax + \frac{b(c + dx) \tanh^{-1}(c + dx)}{d} + \frac{b \log(1 - (c + dx)^2)}{2d}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 48, normalized size = 1.20

$$ax + \frac{b((c + 1) \log(c + dx + 1) - (c - 1) \log(-c - dx + 1))}{2d} + bx \tanh^{-1}(c + dx)$$

Antiderivative was successfully verified.

[In] Integrate[a + b*ArcTanh[c + d*x], x]

[Out] a*x + b*x*ArcTanh[c + d*x] + (b*(-((-1 + c)*Log[1 - c - d*x]) + (1 + c)*Log[1 + c + d*x]))/(2*d)

fricas [A] time = 0.46, size = 61, normalized size = 1.52

$$\frac{bdx \log\left(-\frac{dx+c+1}{dx+c-1}\right) + 2adx + (bc + b) \log(dx + c + 1) - (bc - b) \log(dx + c - 1)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a+b*arctanh(d*x+c), x, algorithm="fricas")

[Out] 1/2*(b*d*x*log(-(d*x + c + 1)/(d*x + c - 1)) + 2*a*d*x + (b*c + b)*log(d*x + c + 1) - (b*c - b)*log(d*x + c - 1))/d

giac [B] time = 0.52, size = 200, normalized size = 5.00

$$\frac{1}{2} ((c + 1)d - (c - 1)d)b \left(\frac{\log\left(\left|\frac{-dx-c-1}{dx+c-1}\right|\right)}{d^2} - \frac{\log\left(\left|\frac{-dx+c+1}{dx+c-1} + 1\right|\right)}{d^2} + \frac{\log\left(\frac{c - \frac{((dx+c+1)(c-1) - c-1)d}{dx+c-1} + 1}{c - \frac{((dx+c+1)(c-1) - c-1)d}{dx+c-1} - 1}\right)}{d^2 \left(\frac{dx+c+1}{dx+c-1} - 1\right)} \right) + ax$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a+b*arctanh(d*x+c),x, algorithm="giac")

[Out] $1/2*((c + 1)*d - (c - 1)*d)*b*(\log(\text{abs}(-d*x - c - 1)/\text{abs}(d*x + c - 1))/d^2 - \log(\text{abs}(-(d*x + c + 1)/(d*x + c - 1) + 1))/d^2 + \log(-(c - ((d*x + c + 1)*(c - 1)/(d*x + c - 1) - c - 1)*d/((d*x + c + 1)*d/(d*x + c - 1) - d) + 1)/(c - ((d*x + c + 1)*(c - 1)/(d*x + c - 1) - c - 1)*d/((d*x + c + 1)*d/(d*x + c - 1) - d) - 1))/d^2*((d*x + c + 1)/(d*x + c - 1) - 1)) + a*x$

maple [A] time = 0.03, size = 44, normalized size = 1.10

$$ax + b \operatorname{arctanh}(dx + c)x + \frac{b \operatorname{arctanh}(dx + c)c}{d} + \frac{b \ln(1 - (dx + c)^2)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(a+b*arctanh(d*x+c),x)

[Out] $a*x + b*\operatorname{arctanh}(d*x+c)*x + b/d*\operatorname{arctanh}(d*x+c)*c + 1/2*b*\ln(1-(d*x+c)^2)/d$

maxima [A] time = 0.32, size = 36, normalized size = 0.90

$$ax + \frac{(2(dx + c) \operatorname{artanh}(dx + c) + \log(-(dx + c)^2 + 1))b}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a+b*arctanh(d*x+c),x, algorithm="maxima")

[Out] $a*x + 1/2*(2*(d*x + c)*\operatorname{arctanh}(d*x + c) + \log(-(d*x + c)^2 + 1))*b/d$

mupad [B] time = 1.44, size = 48, normalized size = 1.20

$$ax + \frac{\frac{b \ln(c^2 + 2cdx + d^2x^2 - 1)}{2} + bc \operatorname{atanh}(c + dx)}{d} + bx \operatorname{atanh}(c + dx)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(a + b*atanh(c + d*x),x)

[Out] $a*x + ((b*\log(c^2 + d^2*x^2 + 2*c*d*x - 1))/2 + b*c*\operatorname{atanh}(c + d*x))/d + b*x*\operatorname{atanh}(c + d*x)$

sympy [A] time = 0.60, size = 46, normalized size = 1.15

$$ax + b \left\{ \begin{array}{ll} \frac{c \operatorname{atanh}(c+dx)}{d} + x \operatorname{atanh}(c + dx) + \frac{\log(c+dx+1)}{d} - \frac{\operatorname{atanh}(c+dx)}{d} & \text{for } d \neq 0 \\ x \operatorname{atanh}(c) & \text{otherwise} \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*atanh(d*x+c),x)
```

```
[Out] a*x + b*Piecewise((c*atanh(c + d*x)/d + x*atanh(c + d*x) + log(c + d*x + 1)  
/d - atanh(c + d*x)/d, Ne(d, 0)), (x*atanh(c), True))
```

$$3.35 \quad \int \frac{a+b \tanh^{-1}(c+dx)}{e+fx} dx$$

Optimal. Leaf size=130

$$\frac{(a + b \tanh^{-1}(c + dx)) \log\left(\frac{2d(e+fx)}{(c+dx+1)(-cf+de+f)}\right)}{f} - \frac{\log\left(\frac{2}{c+dx+1}\right) (a + b \tanh^{-1}(c + dx))}{f} - \frac{b \operatorname{Li}_2\left(1 - \frac{2d(e+fx)}{(de-cf+f)(c+dx+1)}\right)}{2f}$$

[Out] $-(a+b*\operatorname{arctanh}(d*x+c))*\ln(2/(d*x+c+1))/f+(a+b*\operatorname{arctanh}(d*x+c))*\ln(2*d*(f*x+e)/(-c*f+d*e+f)/(d*x+c+1))/f+1/2*b*\operatorname{polylog}(2,1-2/(d*x+c+1))/f-1/2*b*\operatorname{polylog}(2,1-2*d*(f*x+e)/(-c*f+d*e+f)/(d*x+c+1))/f$

Rubi [A] time = 0.14, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {6111, 5920, 2402, 2315, 2447}

$$-\frac{b \operatorname{PolyLog}\left(2, 1 - \frac{2d(e+fx)}{(c+dx+1)(-cf+de+f)}\right)}{2f} + \frac{b \operatorname{PolyLog}\left(2, 1 - \frac{2}{c+dx+1}\right)}{2f} + \frac{(a + b \tanh^{-1}(c + dx)) \log\left(\frac{2d(e+fx)}{(c+dx+1)(-cf+de+f)}\right)}{f}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(a + b*\operatorname{ArcTanh}[c + d*x])/(e + f*x), x]$

[Out] $-\left(\left((a + b*\operatorname{ArcTanh}[c + d*x])*\operatorname{Log}[2/(1 + c + d*x)]\right)/f\right) + \left(\left((a + b*\operatorname{ArcTanh}[c + d*x])*\operatorname{Log}[(2*d*(e + f*x))/((d*e + f - c*f)*(1 + c + d*x))]\right)/f\right) + (b*\operatorname{PolyLog}[2, 1 - 2/(1 + c + d*x)]/(2*f) - (b*\operatorname{PolyLog}[2, 1 - (2*d*(e + f*x))/((d*e + f - c*f)*(1 + c + d*x))])/(2*f)$

Rule 2315

$\operatorname{Int}[\operatorname{Log}[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] \rightarrow -\operatorname{Simp}[\operatorname{PolyLog}[2, 1 - c*x]/e, x] /; \operatorname{FreeQ}\{c, d, e\}, x \ \&\& \operatorname{EqQ}[e + c*d, 0]$

Rule 2402

$\operatorname{Int}[\operatorname{Log}[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] \rightarrow -\operatorname{Dist}[e/g, \operatorname{Subst}[\operatorname{Int}[\operatorname{Log}[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; \operatorname{FreeQ}\{c, d, e, f, g\}, x \ \&\& \operatorname{EqQ}[c, 2*d] \ \&\& \operatorname{EqQ}[e^2*f + d^2*g, 0]$

Rule 2447

$\operatorname{Int}[\operatorname{Log}[u_]*(Pq_)^{(m_.)}, x_Symbol] \rightarrow \operatorname{With}\{C = \operatorname{FullSimplify}[(Pq^m*(1 - u))/D[u, x]]\}, \operatorname{Simp}[C*\operatorname{PolyLog}[2, 1 - u], x] /; \operatorname{FreeQ}[C, x] /; \operatorname{IntegerQ}[m] \ \&\& \operatorname{PolyQ}[Pq, x] \ \&\& \operatorname{RationalFunctionQ}[u, x] \ \&\& \operatorname{LeQ}[\operatorname{RationalFunctionExponents}[u,$

x][[2]], Expon[Pq, x]]

Rule 5920

Int[((a_.) + ArcTanh[(c_.)*(x_.)]*(b_.))/((d_.) + (e_.)*(x_.)), x_Symbol] := -
Simp[((a + b*ArcTanh[c*x])*Log[2/(1 + c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x))
/((c*d + e)*(1 + c*x))]/(1 - c^2*x^2), x], x] + Simp[((a + b*ArcTanh[c*x])*
Log[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))]/e, x]) /; FreeQ[{a, b, c, d, e}
, x] && NeQ[c^2*d^2 - e^2, 0]

Rule 6111

Int[((a_.) + ArcTanh[(c_.) + (d_.)*(x_.)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_.))^(
m_.), x_Symbol] := Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcTanh[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGt
Q[p, 0]

Rubi steps

$$\int \frac{a + b \tanh^{-1}(c + dx)}{e + fx} dx = \frac{\text{Subst}\left(\int \frac{a + b \tanh^{-1}(x)}{\frac{de - cf}{d} + \frac{fx}{d}} dx, x, c + dx\right)}{d}$$

$$= -\frac{(a + b \tanh^{-1}(c + dx)) \log\left(\frac{2}{1 + c + dx}\right)}{f} + \frac{(a + b \tanh^{-1}(c + dx)) \log\left(\frac{2d(e + fx)}{(de + f - cf)(1 + c + dx)}\right)}{f}$$

$$= -\frac{(a + b \tanh^{-1}(c + dx)) \log\left(\frac{2}{1 + c + dx}\right)}{f} + \frac{(a + b \tanh^{-1}(c + dx)) \log\left(\frac{2d(e + fx)}{(de + f - cf)(1 + c + dx)}\right)}{f}$$

$$= -\frac{(a + b \tanh^{-1}(c + dx)) \log\left(\frac{2}{1 + c + dx}\right)}{f} + \frac{(a + b \tanh^{-1}(c + dx)) \log\left(\frac{2d(e + fx)}{(de + f - cf)(1 + c + dx)}\right)}{f}$$

Mathematica [A] time = 0.08, size = 126, normalized size = 0.97

$$\frac{2a \log(e + fx) - b \text{Li}_2\left(\frac{f(c + dx - 1)}{(c - 1)f - de}\right) + b \text{Li}_2\left(\frac{f(c + dx + 1)}{-de + cf + f}\right) - b \log(-c - dx + 1) \log\left(\frac{d(e + fx)}{-cf + de + f}\right) + b \log(c + dx + 1) \log\left(\frac{d(e + fx)}{-cf + de + f}\right)}{2f}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b*ArcTanh[c + d*x])/(e + f*x), x]

[Out] (2*a*Log[e + f*x] - b*Log[1 - c - d*x]*Log[(d*(e + f*x))/(d*e + f - c*f)] + b*Log[1 + c + d*x]*Log[(d*(e + f*x))/(d*e - (1 + c)*f)] - b*PolyLog[2, (f*(-1 + c + d*x))/(-(d*e) + (-1 + c)*f)] + b*PolyLog[2, (f*(1 + c + d*x))/(-(d*e) + f + c*f)])/(2*f)

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b \operatorname{artanh}(dx + c) + a}{fx + e}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arctanh(d*x+c))/(f*x+e), x, algorithm="fricas")

[Out] integral((b*arctanh(d*x + c) + a)/(f*x + e), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b \operatorname{artanh}(dx + c) + a}{fx + e} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arctanh(d*x+c))/(f*x+e), x, algorithm="giac")

[Out] integrate((b*arctanh(d*x + c) + a)/(f*x + e), x)

maple [A] time = 0.12, size = 202, normalized size = 1.55

$$\frac{a \ln((dx + c)f - cf + de)}{f} + \frac{b \ln((dx + c)f - cf + de) \operatorname{arctanh}(dx + c)}{f} + \frac{b \ln((dx + c)f - cf + de) \ln\left(\frac{(dx+c)f-f}{cf-de-f}\right)}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*arctanh(d*x+c))/(f*x+e), x)

[Out] a*ln((d*x+c)*f-c*f+d*e)/f+b*ln((d*x+c)*f-c*f+d*e)/f*arctanh(d*x+c)+1/2*b/f*ln((d*x+c)*f-c*f+d*e)*ln(((d*x+c)*f-f)/(c*f-d*e-f))+1/2*b/f*dilog(((d*x+c)*f-f)/(c*f-d*e-f))-1/2*b/f*ln((d*x+c)*f-c*f+d*e)*ln(((d*x+c)*f+f)/(c*f-d*e+f))-1/2*b/f*dilog(((d*x+c)*f+f)/(c*f-d*e+f))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2} b \int \frac{\log(dx + c + 1) - \log(-dx - c + 1)}{fx + e} dx + \frac{a \log(fx + e)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arctanh(d*x+c))/(f*x+e),x, algorithm="maxima")

[Out] 1/2*b*integrate((log(d*x + c + 1) - log(-d*x - c + 1))/(f*x + e), x) + a*log(f*x + e)/f

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a + b \operatorname{atanh}(c + dx)}{e + fx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*atanh(c + d*x))/(e + f*x),x)

[Out] int((a + b*atanh(c + d*x))/(e + f*x), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{atanh}(c + dx)}{e + fx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*atanh(d*x+c))/(f*x+e),x)

[Out] Integral((a + b*atanh(c + d*x))/(e + f*x), x)

$$3.36 \quad \int \frac{a+b \tanh^{-1}(c+dx)}{(e+fx)^2} dx$$

Optimal. Leaf size=115

$$-\frac{a+b \tanh^{-1}(c+dx)}{f(e+fx)} - \frac{bd \log(-c-dx+1)}{2f(-cf+de+f)} + \frac{bd \log(c+dx+1)}{2f(-cf+de-f)} - \frac{bd \log(e+fx)}{(-cf+de+f)(de-(c+1)f)}$$

[Out] $(-a-b*\arctanh(d*x+c))/f/(f*x+e)-1/2*b*d*\ln(-d*x-c+1)/f/(-c*f+d*e+f)+1/2*b*d*\ln(d*x+c+1)/f/(-c*f+d*e-f)-b*d*\ln(f*x+e)/(-c*f+d*e-f)/(-c*f+d*e+f)$

Rubi [A] time = 0.17, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {6109, 1982, 705, 31, 632}

$$-\frac{a+b \tanh^{-1}(c+dx)}{f(e+fx)} - \frac{bd \log(-c-dx+1)}{2f(-cf+de+f)} + \frac{bd \log(c+dx+1)}{2f(-cf+de-f)} - \frac{bd \log(e+fx)}{(-cf+de+f)(de-(c+1)f)}$$

Antiderivative was successfully verified.

[In] Int[(a + b*ArcTanh[c + d*x])/(e + f*x)^2, x]

[Out] $-((a + b*\text{ArcTanh}[c + d*x])/(f*(e + f*x))) - (b*d*\text{Log}[1 - c - d*x])/(2*f*(d*e + f - c*f)) + (b*d*\text{Log}[1 + c + d*x])/(2*f*(d*e - f - c*f)) - (b*d*\text{Log}[e + f*x])/((d*e + f - c*f)*(d*e - (1 + c)*f))$

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 632

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && NiceSqrtQ[b^2 - 4*a*c]

Rule 705

Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol] := Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0]

2, 0] && NeQ[2*c*d - b*e, 0]

Rule 1982

Int[(u_)^(m_)*(v_)^(p_), x_Symbol] := Int[ExpandToSum[u, x]^m*ExpandToSum[v, x]^p, x] /; FreeQ[{m, p}, x] && LinearQ[u, x] && QuadraticQ[v, x] && ! (LinearMatchQ[u, x] && QuadraticMatchQ[v, x])

Rule 6109

Int[((a_) + ArcTanh[(c_) + (d_)*(x_)])*(b_)^(p_)*((e_) + (f_)*(x_))^(m_), x_Symbol] := Simp[((e + f*x)^(m + 1)*(a + b*ArcTanh[c + d*x])^p)/(f*(m + 1)), x] - Dist[(b*d*p)/(f*(m + 1)), Int[((e + f*x)^(m + 1)*(a + b*ArcTanh[c + d*x])^(p - 1))/(1 - (c + d*x)^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && ILtQ[m, -1]

Rubi steps

$$\begin{aligned}
 \int \frac{a + b \tanh^{-1}(c + dx)}{(e + fx)^2} dx &= -\frac{a + b \tanh^{-1}(c + dx)}{f(e + fx)} + \frac{(bd) \int \frac{1}{(e+fx)(1-(c+dx)^2)} dx}{f} \\
 &= -\frac{a + b \tanh^{-1}(c + dx)}{f(e + fx)} + \frac{(bd) \int \frac{1}{(e+fx)(1-c^2-2cdx-d^2x^2)} dx}{f} \\
 &= -\frac{a + b \tanh^{-1}(c + dx)}{f(e + fx)} + \frac{(bd) \int \frac{-d^2e+2cdf+d^2fx}{1-c^2-2cdx-d^2x^2} dx}{f(-d^2e^2 + 2cdef + (1 - c^2) f^2)} + \frac{(bdf) \int \frac{1}{e+fx} dx}{-d^2e^2 + 2cdef + (1 - c^2) f^2} \\
 &= -\frac{a + b \tanh^{-1}(c + dx)}{f(e + fx)} - \frac{bd \log(e + fx)}{(de - f - cf)(de + f - cf)} - \frac{(bd^3) \int \frac{1}{-d-cd-d^2x} dx}{2f(de - f - cf)} + \frac{(bdf) \int \frac{1}{e+fx} dx}{2f(de - f - cf)} \\
 &= -\frac{a + b \tanh^{-1}(c + dx)}{f(e + fx)} - \frac{bd \log(1 - c - dx)}{2f(de + f - cf)} + \frac{bd \log(1 + c + dx)}{2f(de - f - cf)} - \frac{bd \log(e + fx)}{(de - f - cf)}
 \end{aligned}$$

Mathematica [A] time = 0.18, size = 125, normalized size = 1.09

$$\frac{1}{2} \left(-\frac{2a}{f(e + fx)} - \frac{2bd \log(e + fx)}{(c^2 - 1) f^2 - 2cdef + d^2e^2} + \frac{bd \log(-c - dx + 1)}{f((c - 1)f - de)} - \frac{bd \log(c + dx + 1)}{f(cf - de + f)} - \frac{2b \tanh^{-1}(c + dx)}{f(e + fx)} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*ArcTanh[c + d*x])/(e + f*x)^2,x]

[Out] $((-2*a)/(f*(e + f*x)) - (2*b*ArcTanh[c + d*x])/(f*(e + f*x)) + (b*d*Log[1 - c - d*x])/(f*(-(d*e) + (-1 + c)*f)) - (b*d*Log[1 + c + d*x])/(f*(-(d*e) + f + c*f)) - (2*b*d*Log[e + f*x])/(d^2*e^2 - 2*c*d*e*f + (-1 + c^2)*f^2))/2$

fricas [B] time = 0.61, size = 263, normalized size = 2.29

$$\frac{2ad^2e^2 - 4acdef + 2(ac^2 - a)f^2 - (bd^2e^2 - (bc - b)def + (bd^2ef - (bc - b)df^2)x) \log(dx + c + 1) + (bd^2e^2 - (bc - b)def + (bd^2ef - (bc - b)df^2)x)}{2(d^2e^3f - 2cde^2f^2 + (c^2 - 1)f^3 + (d^2e^2f^2 - 2cde^2f^2 + (c^2 - 1)f^4)x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arctanh(d*x+c))/(f*x+e)^2,x, algorithm="fricas")

[Out] $-1/2*(2*a*d^2*e^2 - 4*a*c*d*e*f + 2*(a*c^2 - a)*f^2 - (b*d^2*e^2 - (b*c - b)*d*e*f + (b*d^2*e*f - (b*c - b)*d*f^2)*x)*\log(d*x + c + 1) + (b*d^2*e^2 - (b*c + b)*d*e*f + (b*d^2*e*f - (b*c + b)*d*f^2)*x)*\log(d*x + c - 1) + 2*(b*d*f^2*x + b*d*e*f)*\log(f*x + e) + (b*d^2*e^2 - 2*b*c*d*e*f + (b*c^2 - b)*f^2)*\log(-(d*x + c + 1)/(d*x + c - 1))/(d^2*e^3*f - 2*c*d*e^2*f^2 + (c^2 - 1)*e*f^3 + (d^2*e^2*f^2 - 2*c*d*e*f^3 + (c^2 - 1)*f^4)*x)$

giac [B] time = 0.27, size = 896, normalized size = 7.79

$$\frac{\left(\frac{(dx+c+1)bcf \log\left(\frac{(dx+c+1)cf}{dx+c-1} - cf - \frac{(dx+c+1)de}{dx+c-1} + de - \frac{(dx+c+1)f}{dx+c-1} - f\right)}{dx+c-1} - bcf \log\left(\frac{(dx+c+1)cf}{dx+c-1} - cf - \frac{(dx+c+1)de}{dx+c-1} + de - \frac{(dx+c+1)f}{dx+c-1} - f\right) \right)}{2\left(\frac{d^2e^3f - 2cde^2f^2 + (c^2 - 1)f^3 + (d^2e^2f^2 - 2cde^2f^2 + (c^2 - 1)f^4)x}{d^2e^3f - 2cde^2f^2 + (c^2 - 1)f^3 + (d^2e^2f^2 - 2cde^2f^2 + (c^2 - 1)f^4)x}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arctanh(d*x+c))/(f*x+e)^2,x, algorithm="giac")

[Out] $-1/2*((d*x + c + 1)*b*c*f*\log((d*x + c + 1)*c*f/(d*x + c - 1) - c*f - (d*x + c + 1)*d*e/(d*x + c - 1) + d*e - (d*x + c + 1)*f/(d*x + c - 1) - f)/(d*x + c - 1) - b*c*f*\log((d*x + c + 1)*c*f/(d*x + c - 1) - c*f - (d*x + c + 1)*d*e/(d*x + c - 1) + d*e - (d*x + c + 1)*f/(d*x + c - 1) - f) - (d*x + c + 1)*b*d*e*\log((d*x + c + 1)*c*f/(d*x + c - 1) - c*f - (d*x + c + 1)*d*e/(d*x + c - 1) + d*e - (d*x + c + 1)*f/(d*x + c - 1) - f)/(d*x + c - 1) + b*d*e*\log((d*x + c + 1)*c*f/(d*x + c - 1) - c*f - (d*x + c + 1)*d*e/(d*x + c - 1) + d*e - (d*x + c + 1)*f/(d*x + c - 1) - f) - (d*x + c + 1)*b*c*f*\log(-(d*x + c + 1)/(d*x + c - 1))/(d*x + c - 1) + (d*x + c + 1)*b*d*e*\log(-(d*x + c + 1)/(d*x + c - 1))/(d*x + c - 1) - 2*a*c*f + 2*a*d*e - (d*x + c + 1)*b*f*\log((d*x + c + 1)*c*f/(d*x + c - 1) - c*f - (d*x + c + 1)*d*e/(d*x + c - 1) + d*e - (d*x + c + 1)*f/(d*x + c - 1) - f)/(d*x + c - 1) - b*f*\log((d*x + c + 1)*c*f/(d*x + c - 1) - c*f - (d*x + c + 1)*d*e/(d*x + c - 1) + d*e - (d*x + c + 1)*f/(d*x + c - 1) - f) + (d*x + c + 1)*b*f*\log(-(d*x + c + 1)/(d*x + c - 1))/(d*x + c - 1)$

+ c - 1))/(d*x + c - 1) - 2*a*f)*((c + 1)*d - (c - 1)*d)/((d*x + c + 1)*c^3 *f^3/(d*x + c - 1) - c^3*f^3 - 3*(d*x + c + 1)*c^2*d*f^2*e/(d*x + c - 1) + 3*c^2*d*f^2*e - (d*x + c + 1)*c^2*f^3/(d*x + c - 1) - c^2*f^3 + 3*(d*x + c + 1)*c*d^2*f*e^2/(d*x + c - 1) - 3*c*d^2*f*e^2 + 2*(d*x + c + 1)*c*d*f^2*e/(d*x + c - 1) + 2*c*d*f^2*e - (d*x + c + 1)*c*f^3/(d*x + c - 1) + c*f^3 - (d*x + c + 1)*d^3*e^3/(d*x + c - 1) + d^3*e^3 - (d*x + c + 1)*d^2*f*e^2/(d*x + c - 1) - d^2*f*e^2 + (d*x + c + 1)*d*f^2*e/(d*x + c - 1) - d*f^2*e + (d*x + c + 1)*f^3/(d*x + c - 1) + f^3)

maple [A] time = 0.04, size = 141, normalized size = 1.23

$$\frac{da}{(dfx + de)f} - \frac{db \operatorname{arctanh}(dx + c)}{(dfx + de)f} - \frac{db \ln((dx + c)f - cf + de)}{(cf - de - f)(cf - de + f)} + \frac{db \ln(dx + c - 1)}{f(2cf - 2de - 2f)} - \frac{db \ln(dx + c + 1)}{f(2cf - 2de + 2f)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*arctanh(d*x+c))/(f*x+e)^2,x)

[Out] -d*a/(d*f*x+d*e)/f-d*b/(d*f*x+d*e)/f*arctanh(d*x+c)-d*b/(c*f-d*e-f)/(c*f-d*e+f)*ln((d*x+c)*f-c*f+d*e)+d*b/f/(2*c*f-2*d*e-2*f)*ln(d*x+c-1)-d*b/f/(2*c*f-2*d*e+2*f)*ln(d*x+c+1)

maxima [A] time = 0.32, size = 121, normalized size = 1.05

$$\frac{1}{2} \left(d \left(\frac{\log(dx + c + 1)}{def - (c + 1)f^2} - \frac{\log(dx + c - 1)}{def - (c - 1)f^2} - \frac{2 \log(fx + e)}{d^2e^2 - 2cdef + (c^2 - 1)f^2} \right) - \frac{2 \operatorname{artanh}(dx + c)}{f^2x + ef} \right) b - \frac{a}{f^2x + ef}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arctanh(d*x+c))/(f*x+e)^2,x, algorithm="maxima")

[Out] 1/2*(d*(log(d*x + c + 1)/(d*e*f - (c + 1)*f^2) - log(d*x + c - 1)/(d*e*f - (c - 1)*f^2) - 2*log(f*x + e)/(d^2*e^2 - 2*c*d*e*f + (c^2 - 1)*f^2)) - 2*arctanh(d*x + c)/(f^2*x + e*f))*b - a/(f^2*x + e*f)

mupad [B] time = 1.64, size = 170, normalized size = 1.48

$$\ln(e + fx) \left(\frac{b(c-1)}{2e(de-f(c-1))} - \frac{b(c+1)}{2e(de-f(c+1))} \right) - \frac{a}{xf^2 + ef} + \frac{b \ln(1 - dx - c)}{f(2e + 2fx)} - \frac{b \ln(c + dx + 1)}{2f(e + fx)} - \frac{ba}{2f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*atanh(c + d*x))/(e + f*x)^2,x)

[Out] log(e + f*x)*((b*(c - 1))/(2*e*(d*e - f*(c - 1))) - (b*(c + 1))/(2*e*(d*e - f*(c + 1)))) - a/(e*f + f^2*x) + (b*log(1 - d*x - c))/(f*(2*e + 2*f*x)) -

$$(b \cdot \log(c + d \cdot x + 1)) / (2 \cdot f \cdot (e + f \cdot x)) - (b \cdot d \cdot \log(c + d \cdot x - 1)) / (2 \cdot f^2 - 2 \cdot c \cdot f^2 + 2 \cdot d \cdot e \cdot f) - (b \cdot d \cdot \log(c + d \cdot x + 1)) / (2 \cdot c \cdot f^2 + 2 \cdot f^2 - 2 \cdot d \cdot e \cdot f)$$

sympy [A] time = 10.01, size = 1658, normalized size = 14.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*atanh(d*x+c))/(f*x+e)**2,x)

[Out] Piecewise(((a*x + b*c*atanh(c + d*x)/d + b*x*atanh(c + d*x) + b*log(c/d + x + 1/d)/d - b*atanh(c + d*x)/d)/e**2, Eq(f, 0)), (-2*a*f/(2*e*f**2 + 2*f**3*x) + b*d*e*atanh(d*e/f + d*x - 1)/(2*e*f**2 + 2*f**3*x) + b*d*f*x*atanh(d*e/f + d*x - 1)/(2*e*f**2 + 2*f**3*x) - 2*b*f*atanh(d*e/f + d*x - 1)/(2*e*f**2 + 2*f**3*x) - b*f/(2*e*f**2 + 2*f**3*x), Eq(c, (d*e - f)/f)), (-2*a*f/(2*e*f**2 + 2*f**3*x) - b*d*e*atanh(d*e/f + d*x + 1)/(2*e*f**2 + 2*f**3*x) - b*d*f*x*atanh(d*e/f + d*x + 1)/(2*e*f**2 + 2*f**3*x) - 2*b*f*atanh(d*e/f + d*x + 1)/(2*e*f**2 + 2*f**3*x) + b*f/(2*e*f**2 + 2*f**3*x), Eq(c, (d*e + f)/f)), (zoo*(a*x + b*c*atanh(c + d*x)/d + b*x*atanh(c + d*x) + b*log(c/d + x + 1/d)/d - b*atanh(c + d*x)/d), Eq(e, -f*x)), (-(a + b*atanh(c))/(e*f + f**2*x), Eq(d, 0)), (-a*c**2*f**2/(c**2*e*f**3 + c**2*f**4*x - 2*c*d*e**2*f**2 - 2*c*d*e*f**3*x + d**2*e**3*f + d**2*e**2*f**2*x - e*f**3 - f**4*x) + 2*a*c*d*e*f/(c**2*e*f**3 + c**2*f**4*x - 2*c*d*e**2*f**2 - 2*c*d*e*f**3*x + d**2*e**3*f + d**2*e**2*f**2*x - e*f**3 - f**4*x) - a*d**2*e**2/(c**2*e*f**3 + c**2*f**4*x - 2*c*d*e**2*f**2 - 2*c*d*e*f**3*x + d**2*e**3*f + d**2*e**2*f**2*x - e*f**3 - f**4*x) + a*f**2/(c**2*e*f**3 + c**2*f**4*x - 2*c*d*e**2*f**2 - 2*c*d*e*f**3*x + d**2*e**3*f + d**2*e**2*f**2*x - e*f**3 - f**4*x) - b*c**2*f**2*atanh(c + d*x)/(c**2*e*f**3 + c**2*f**4*x - 2*c*d*e**2*f**2 - 2*c*d*e*f**3*x + d**2*e**3*f + d**2*e**2*f**2*x - e*f**3 - f**4*x) + b*c*d*e*f*atanh(c + d*x)/(c**2*e*f**3 + c**2*f**4*x - 2*c*d*e**2*f**2 - 2*c*d*e*f**3*x + d**2*e**3*f + d**2*e**2*f**2*x - e*f**3 - f**4*x) - b*c*d*f**2*x*atanh(c + d*x)/(c**2*e*f**3 + c**2*f**4*x - 2*c*d*e**2*f**2 - 2*c*d*e*f**3*x + d**2*e**3*f + d**2*e**2*f**2*x - e*f**3 - f**4*x) + b*d**2*e*f*x*atanh(c + d*x)/(c**2*e*f**3 + c**2*f**4*x - 2*c*d*e**2*f**2 - 2*c*d*e*f**3*x + d**2*e**3*f + d**2*e**2*f**2*x - e*f**3 - f**4*x) - b*d*e*f*log(e/f + x)/(c**2*e*f**3 + c**2*f**4*x - 2*c*d*e**2*f**2 - 2*c*d*e*f**3*x + d**2*e**3*f + d**2*e**2*f**2*x - e*f**3 - f**4*x) + b*d*e*f*log(c/d + x + 1/d)/(c**2*e*f**3 + c**2*f**4*x - 2*c*d*e**2*f**2 - 2*c*d*e*f**3*x + d**2*e**3*f + d**2*e**2*f**2*x - e*f**3 - f**4*x) - b*d*e*f*atanh(c + d*x)/(c**2*e*f**3 + c**2*f**4*x - 2*c*d*e**2*f**2 - 2*c*d*e*f**3*x + d**2*e**3*f + d**2*e**2*f**2*x - e*f**3 - f**4*x) - b*d*f**2*x*log(e/f + x)/(c**2*e*f**3 + c**2*f**4*x - 2*c*d*e**2*f**2 - 2*c*d*e*f**3*x + d**2*e**3*f + d**2*e**2*f**2*x - e*f**3 - f**4*x) + b*d*f**2*x*log(c/d + x + 1/d)/(c**2*e*f**3 + c**2*f**4*x - 2*c*d*e**2*f**2 - 2*c*d*e*f**3*x + d**2*e**3*f + d**2*e**2*f**2*x - e*f**3 - f**4*x) - b*d*f**2*x*atanh(c + d*x)/(c**2*e*f**3 + c**2*f**4*x - 2*c*d*e**2*f**2


```
- 2*c*d*e*f**3*x + d**2*e**3*f + d**2*e**2*f**2*x - e*f**3 - f**4*x) + b*f*  
*2*atanh(c + d*x)/(c**2*e*f**3 + c**2*f**4*x - 2*c*d*e**2*f**2 - 2*c*d*e*f*  
*3*x + d**2*e**3*f + d**2*e**2*f**2*x - e*f**3 - f**4*x), True))
```

$$3.37 \quad \int \frac{a+b \tanh^{-1}(c+dx)}{(e+fx)^3} dx$$

Optimal. Leaf size=167

$$\frac{a+b \tanh^{-1}(c+dx)}{2f(e+fx)^2} - \frac{bd^2 \log(-c-dx+1)}{4f(-cf+de+f)^2} + \frac{bd^2 \log(c+dx+1)}{4f(-cf+de-f)^2} - \frac{bd^2(de-cf) \log(e+fx)}{(-cf+de+f)^2(de-(c+1)f)^2} + \frac{1}{2(e+fx)}$$

[Out] $1/2*b*d/(-c*f+d*e-f)/(-c*f+d*e+f)/(f*x+e)+1/2*(-a-b*arctanh(d*x+c))/f/(f*x+e)^2-1/4*b*d^2*\ln(-d*x-c+1)/f/(-c*f+d*e+f)^2+1/4*b*d^2*\ln(d*x+c+1)/f/(-c*f+d*e-f)^2-b*d^2*(-c*f+d*e)*\ln(f*x+e)/(-c*f+d*e+f)^2/(d*e-(1+c)*f)^2$

Rubi [A] time = 0.25, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {6109, 1982, 709, 800}

$$\frac{a+b \tanh^{-1}(c+dx)}{2f(e+fx)^2} - \frac{bd^2 \log(-c-dx+1)}{4f(-cf+de+f)^2} + \frac{bd^2 \log(c+dx+1)}{4f(-cf+de-f)^2} - \frac{bd^2(de-cf) \log(e+fx)}{(-cf+de+f)^2(de-(c+1)f)^2} + \frac{1}{2(e+fx)}$$

Antiderivative was successfully verified.

[In] Int[(a + b*ArcTanh[c + d*x])/(e + f*x)^3,x]

[Out] $(b*d)/(2*(d*e + f - c*f)*(d*e - (1 + c)*f)*(e + f*x)) - (a + b*ArcTanh[c + d*x])/(2*f*(e + f*x)^2) - (b*d^2*Log[1 - c - d*x])/(4*f*(d*e + f - c*f)^2) + (b*d^2*Log[1 + c + d*x])/(4*f*(d*e - f - c*f)^2) - (b*d^2*(d*e - c*f)*Log[e + f*x])/((d*e + f - c*f)^2*(d*e - (1 + c)*f)^2)$

Rule 709

Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(e*(d + e*x)^(m + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(c*d^2 - b*d*e + a*e^2), Int[((d + e*x)^(m + 1)*Simp[c*d - b*e - c*e*x, x])/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m, -1]

Rule 800

Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]

Rule 1982

```
Int[(u_)^(m_)*(v_)^(p_), x_Symbol] := Int[ExpandToSum[u, x]^m*ExpandToSum
[v, x]^p, x] /; FreeQ[{m, p}, x] && LinearQ[u, x] && QuadraticQ[v, x] && !
(LinearMatchQ[u, x] && QuadraticMatchQ[v, x])
```

Rule 6109

```
Int[((a_.) + ArcTanh[(c_) + (d_.)*(x_)])*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_), x_Symbol] := Simp[((e + f*x)^(m + 1)*(a + b*ArcTanh[c + d*x])^p)/(f*(m
+ 1)), x] - Dist[(b*d*p)/(f*(m + 1)), Int[((e + f*x)^(m + 1)*(a + b*ArcTan
h[c + d*x])^(p - 1))/(1 - (c + d*x)^2), x], x] /; FreeQ[{a, b, c, d, e, f},
x] && IGtQ[p, 0] && ILtQ[m, -1]
```

Rubi steps

$$\begin{aligned} \int \frac{a + b \tanh^{-1}(c + dx)}{(e + fx)^3} dx &= -\frac{a + b \tanh^{-1}(c + dx)}{2f(e + fx)^2} + \frac{(bd) \int \frac{1}{(e+fx)^2(1-(c+dx)^2)} dx}{2f} \\ &= -\frac{a + b \tanh^{-1}(c + dx)}{2f(e + fx)^2} + \frac{(bd) \int \frac{1}{(e+fx)^2(1-c^2-2cdx-d^2x^2)} dx}{2f} \\ &= \frac{bd}{2(de + f - cf)(de - (1 + c)f)(e + fx)} - \frac{a + b \tanh^{-1}(c + dx)}{2f(e + fx)^2} + \frac{(bd) \int \frac{-d(de-2cdx-d^2x^2)}{(e+fx)(1-c-dx)^2} dx}{2f(-d^2e^2 + 2cde)} \\ &= \frac{bd}{2(de + f - cf)(de - (1 + c)f)(e + fx)} - \frac{a + b \tanh^{-1}(c + dx)}{2f(e + fx)^2} + \frac{(bd) \int \left(\frac{d^2(-de+(1-c)d^2x^2)}{2(de+f-cf)} \right) dx}{2f(-d^2e^2 + 2cde)} \\ &= \frac{bd}{2(de + f - cf)(de - (1 + c)f)(e + fx)} - \frac{a + b \tanh^{-1}(c + dx)}{2f(e + fx)^2} - \frac{bd^2 \log(1 - c - dx)}{4f(de + f - cf)^2} \end{aligned}$$

Mathematica [A] time = 0.32, size = 174, normalized size = 1.04

$$\frac{1}{4} \left(-\frac{2a}{f(e + fx)^2} + \frac{2bd}{(e + fx)((c^2 - 1)f^2 - 2cdef + d^2e^2)} - \frac{4bd^2(de - cf) \log(e + fx)}{((c^2 - 1)f^2 - 2cdef + d^2e^2)^2} - \frac{bd^2 \log(-c - dx + 1)}{f(-cf + de + f)^2} \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c + d*x])/(e + f*x)^3, x]
```

```
[Out] ((-2*a)/(f*(e + f*x)^2) + (2*b*d)/((d^2*e^2 - 2*c*d*e*f + (-1 + c^2)*f^2)*(
e + f*x)) - (2*b*ArcTanh[c + d*x])/(f*(e + f*x)^2) - (b*d^2*Log[1 - c - d*x
```

)]/(f*(d*e + f - c*f)^2) + (b*d^2*Log[1 + c + d*x])/(f*(-(d*e) + f + c*f)^2) - (4*b*d^2*(d*e - c*f)*Log[e + f*x])/(d^2*e^2 - 2*c*d*e*f + (-1 + c^2)*f^2)^2)/4

fricas [B] time = 1.60, size = 834, normalized size = 4.99

$$2ad^4e^4 - 2(4ac + b)d^3e^3f + 4(3ac^2 + bc - a)d^2e^2f^2 - 2(4ac^3 + bc^2 - 4ac - b)def^3 + 2(ac^4 - 2ac^2 + a)f^4 -$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arctanh(d*x+c))/(f*x+e)^3,x, algorithm="fricas")

[Out]
$$\begin{aligned} & -1/4*(2*a*d^4*e^4 - 2*(4*a*c + b)*d^3*e^3*f + 4*(3*a*c^2 + b*c - a)*d^2*e^2*f^2 - 2*(4*a*c^3 + b*c^2 - 4*a*c - b)*d*e*f^3 + 2*(a*c^4 - 2*a*c^2 + a)*f^4 - 2*(b*d^3*e^2*f^2 - 2*b*c*d^2*e*f^3 + (b*c^2 - b)*d*f^4)*x - (b*d^4*e^4 - 2*(b*c - b)*d^3*e^3*f + (b*c^2 - 2*b*c + b)*d^2*e^2*f^2 + (b*d^4*e^2*f^2 - 2*(b*c - b)*d^3*e*f^3 + (b*c^2 - 2*b*c + b)*d^2*f^4)*x^2 + 2*(b*d^4*e^3*f - 2*(b*c - b)*d^3*e^2*f^2 + (b*c^2 - 2*b*c + b)*d^2*e*f^3)*x*\log(d*x + c + 1) + (b*d^4*e^4 - 2*(b*c + b)*d^3*e^3*f + (b*c^2 + 2*b*c + b)*d^2*e^2*f^2 + (b*d^4*e^2*f^2 - 2*(b*c + b)*d^3*e*f^3 + (b*c^2 + 2*b*c + b)*d^2*f^4)*x^2 + 2*(b*d^4*e^3*f - 2*(b*c + b)*d^3*e^2*f^2 + (b*c^2 + 2*b*c + b)*d^2*e*f^3)*x*\log(d*x + c - 1) + 4*(b*d^3*e^3*f - b*c*d^2*e^2*f^2 + (b*d^3*e*f^3 - b*c*d^2*f^4)*x^2 + 2*(b*d^3*e^2*f^2 - b*c*d^2*e*f^3)*x)*\log(f*x + e) + (b*d^4*e^4 - 4*b*c*d^3*e^3*f + 2*(3*b*c^2 - b)*d^2*e^2*f^2 - 4*(b*c^3 - b*c)*d*e*f^3 + (b*c^4 - 2*b*c^2 + b)*f^4)*\log(-(d*x + c + 1)/(d*x + c - 1)))/(d^4*e^6*f - 4*c*d^3*e^5*f^2 + 2*(3*c^2 - 1)*d^2*e^4*f^3 - 4*(c^3 - c)*d*e^3*f^4 + (c^4 - 2*c^2 + 1)*e^2*f^5 + (d^4*e^4*f^3 - 4*c*d^3*e^3*f^4 + 2*(3*c^2 - 1)*d^2*e^2*f^5 - 4*(c^3 - c)*d*e*f^6 + (c^4 - 2*c^2 + 1)*f^7)*x^2 + 2*(d^4*e^5*f^2 - 4*c*d^3*e^4*f^3 + 2*(3*c^2 - 1)*d^2*e^3*f^4 - 4*(c^3 - c)*d*e^2*f^5 + (c^4 - 2*c^2 + 1)*e*f^6)*x \end{aligned}$$

giac [B] time = 0.41, size = 4720, normalized size = 28.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arctanh(d*x+c))/(f*x+e)^3,x, algorithm="giac")

[Out]
$$\begin{aligned} & 1/2*((d*x + c + 1)^2*b*c^3*d*f^3*\log((d*x + c + 1)*c*f/(d*x + c - 1) - c*f - (d*x + c + 1)*d*e/(d*x + c - 1) + d*e - (d*x + c + 1)*f/(d*x + c - 1) - f)/(d*x + c - 1)^2 - 2*(d*x + c + 1)*b*c^3*d*f^3*\log((d*x + c + 1)*c*f/(d*x + c - 1) - c*f - (d*x + c + 1)*d*e/(d*x + c - 1) + d*e - (d*x + c + 1)*f/(d*x + c - 1) - f)/(d*x + c - 1) + b*c^3*d*f^3*\log((d*x + c + 1)*c*f/(d*x + c \end{aligned}$$

$$\begin{aligned}
& - 1) - c*f - (d*x + c + 1)*d*e/(d*x + c - 1) + d*e - (d*x + c + 1)*f/(d*x \\
& + c - 1) - f) - 3*(d*x + c + 1)^2*b*c^2*d^2*f^2*e*log((d*x + c + 1)*c*f/(d* \\
& x + c - 1) - c*f - (d*x + c + 1)*d*e/(d*x + c - 1) + d*e - (d*x + c + 1)*f/ \\
& (d*x + c - 1) - f)/(d*x + c - 1)^2 + 6*(d*x + c + 1)*b*c^2*d^2*f^2*e*log((d \\
& *x + c + 1)*c*f/(d*x + c - 1) - c*f - (d*x + c + 1)*d*e/(d*x + c - 1) + d*e \\
& - (d*x + c + 1)*f/(d*x + c - 1) - f)/(d*x + c - 1) - 3*b*c^2*d^2*f^2*e*log \\
& ((d*x + c + 1)*c*f/(d*x + c - 1) - c*f - (d*x + c + 1)*d*e/(d*x + c - 1) + \\
& d*e - (d*x + c + 1)*f/(d*x + c - 1) - f) - (d*x + c + 1)^2*b*c^3*d*f^3*log(\\
& -(d*x + c + 1)/(d*x + c - 1))/(d*x + c - 1)^2 + (d*x + c + 1)*b*c^3*d*f^3*1 \\
& og(-(d*x + c + 1)/(d*x + c - 1))/(d*x + c - 1) + 3*(d*x + c + 1)^2*b*c^2*d^ \\
& 2*f^2*e*log(-(d*x + c + 1)/(d*x + c - 1))/(d*x + c - 1)^2 - 3*(d*x + c + 1) \\
& *b*c^2*d^2*f^2*e*log(-(d*x + c + 1)/(d*x + c - 1))/(d*x + c - 1) - 2*(d*x + \\
& c + 1)*a*c^3*d*f^3/(d*x + c - 1) + 2*a*c^3*d*f^3 + 6*(d*x + c + 1)*a*c^2*d \\
& ^2*f^2*e/(d*x + c - 1) - 6*a*c^2*d^2*f^2*e - 2*(d*x + c + 1)^2*b*c^2*d*f^3* \\
& log((d*x + c + 1)*c*f/(d*x + c - 1) - c*f - (d*x + c + 1)*d*e/(d*x + c - 1) \\
& + d*e - (d*x + c + 1)*f/(d*x + c - 1) - f)/(d*x + c - 1)^2 + 2*b*c^2*d*f^3 \\
& *log((d*x + c + 1)*c*f/(d*x + c - 1) - c*f - (d*x + c + 1)*d*e/(d*x + c - 1 \\
&) + d*e - (d*x + c + 1)*f/(d*x + c - 1) - f) + 3*(d*x + c + 1)^2*b*c*d^3*f* \\
& e^2*log((d*x + c + 1)*c*f/(d*x + c - 1) - c*f - (d*x + c + 1)*d*e/(d*x + c \\
& - 1) + d*e - (d*x + c + 1)*f/(d*x + c - 1) - f)/(d*x + c - 1)^2 - 6*(d*x + \\
& c + 1)*b*c*d^3*f*e^2*log((d*x + c + 1)*c*f/(d*x + c - 1) - c*f - (d*x + c + \\
& 1)*d*e/(d*x + c - 1) + d*e - (d*x + c + 1)*f/(d*x + c - 1) - f)/(d*x + c - \\
& 1) + 3*b*c*d^3*f*e^2*log((d*x + c + 1)*c*f/(d*x + c - 1) - c*f - (d*x + c \\
& + 1)*d*e/(d*x + c - 1) + d*e - (d*x + c + 1)*f/(d*x + c - 1) - f) + 4*(d*x \\
& + c + 1)^2*b*c*d^2*f^2*e*log((d*x + c + 1)*c*f/(d*x + c - 1) - c*f - (d*x + \\
& c + 1)*d*e/(d*x + c - 1) + d*e - (d*x + c + 1)*f/(d*x + c - 1) - f)/(d*x + \\
& c - 1)^2 - 4*b*c*d^2*f^2*e*log((d*x + c + 1)*c*f/(d*x + c - 1) - c*f - (d* \\
& x + c + 1)*d*e/(d*x + c - 1) + d*e - (d*x + c + 1)*f/(d*x + c - 1) - f) + 2 \\
& *(d*x + c + 1)^2*b*c^2*d*f^3*log(-(d*x + c + 1)/(d*x + c - 1))/(d*x + c - 1 \\
&)^2 - (d*x + c + 1)*b*c^2*d*f^3*log(-(d*x + c + 1)/(d*x + c - 1))/(d*x + c \\
& - 1) - 3*(d*x + c + 1)^2*b*c*d^3*f*e^2*log(-(d*x + c + 1)/(d*x + c - 1))/(d \\
& *x + c - 1)^2 + 3*(d*x + c + 1)*b*c*d^3*f*e^2*log(-(d*x + c + 1)/(d*x + c - \\
& 1))/(d*x + c - 1) - 4*(d*x + c + 1)^2*b*c*d^2*f^2*e*log(-(d*x + c + 1)/(d* \\
& x + c - 1))/(d*x + c - 1)^2 + 2*(d*x + c + 1)*b*c*d^2*f^2*e*log(-(d*x + c + \\
& 1)/(d*x + c - 1))/(d*x + c - 1) - 2*(d*x + c + 1)*a*c^2*d*f^3/(d*x + c - 1 \\
&) + 4*a*c^2*d*f^3 - (d*x + c + 1)*b*c^2*d*f^3/(d*x + c - 1) + b*c^2*d*f^3 - \\
& 6*(d*x + c + 1)*a*c*d^3*f*e^2/(d*x + c - 1) + 6*a*c*d^3*f*e^2 + 4*(d*x + c \\
& + 1)*a*c*d^2*f^2*e/(d*x + c - 1) - 8*a*c*d^2*f^2*e + 2*(d*x + c + 1)*b*c*d \\
& ^2*f^2*e/(d*x + c - 1) - 2*b*c*d^2*f^2*e + (d*x + c + 1)^2*b*c*d*f^3*log((d \\
& *x + c + 1)*c*f/(d*x + c - 1) - c*f - (d*x + c + 1)*d*e/(d*x + c - 1) + d*e \\
& - (d*x + c + 1)*f/(d*x + c - 1) - f)/(d*x + c - 1)^2 + 2*(d*x + c + 1)*b*c \\
& *d*f^3*log((d*x + c + 1)*c*f/(d*x + c - 1) - c*f - (d*x + c + 1)*d*e/(d*x + \\
& c - 1) + d*e - (d*x + c + 1)*f/(d*x + c - 1) - f)/(d*x + c - 1) + b*c*d*f^ \\
& 3*log((d*x + c + 1)*c*f/(d*x + c - 1) - c*f - (d*x + c + 1)*d*e/(d*x + c - \\
& 1) + d*e - (d*x + c + 1)*f/(d*x + c - 1) - f) - (d*x + c + 1)^2*b*d^4*e^3*1
\end{aligned}$$

$$\begin{aligned}
& \log((d*x + c + 1)*c*f/(d*x + c - 1) - c*f - (d*x + c + 1)*d*e/(d*x + c - 1) \\
& + d*e - (d*x + c + 1)*f/(d*x + c - 1) - f)/(d*x + c - 1)^2 + 2*(d*x + c + 1) \\
&)*b*d^4*e^3*\log((d*x + c + 1)*c*f/(d*x + c - 1) - c*f - (d*x + c + 1)*d*e/(d*x + c - 1) \\
& + d*e - (d*x + c + 1)*f/(d*x + c - 1) - f)/(d*x + c - 1) - b*d^4*e^3*\log((d*x + c + 1)*c*f/(d*x + c - 1) - c*f - (d*x + c + 1)*d*e/(d*x + c - 1) \\
& + d*e - (d*x + c + 1)*f/(d*x + c - 1) - f) - 2*(d*x + c + 1)^2*b*d^3*f*e^2*\log((d*x + c + 1)*c*f/(d*x + c - 1) - c*f - (d*x + c + 1)*d*e/(d*x + c - 1) \\
& + d*e - (d*x + c + 1)*f/(d*x + c - 1) - f)/(d*x + c - 1)^2 + 2*b*d^3*f*e^2*\log((d*x + c + 1)*c*f/(d*x + c - 1) - c*f - (d*x + c + 1)*d*e/(d*x + c - 1) \\
& + d*e - (d*x + c + 1)*f/(d*x + c - 1) - f) - (d*x + c + 1)^2*b*d^2*f^2*e*\log((d*x + c + 1)*c*f/(d*x + c - 1) - c*f - (d*x + c + 1)*d*e/(d*x + c - 1) \\
& + d*e - (d*x + c + 1)*f/(d*x + c - 1) - f)/(d*x + c - 1)^2 - 2*(d*x + c + 1)*b*d^2*f^2*e*\log((d*x + c + 1)*c*f/(d*x + c - 1) - c*f - (d*x + c + 1)*d*e/(d*x + c - 1) \\
& + d*e - (d*x + c + 1)*f/(d*x + c - 1) - f)/(d*x + c - 1) - b*d^2*f^2*e*\log((d*x + c + 1)*c*f/(d*x + c - 1) - c*f - (d*x + c + 1)*d*e/(d*x + c - 1) \\
& + d*e - (d*x + c + 1)*f/(d*x + c - 1) - f) - (d*x + c + 1)^2*b*c*d*f^3*\log(-(d*x + c + 1)/(d*x + c - 1))/(d*x + c - 1)^2 - (d*x + c + 1)*b*c*d*f^3*\log(-(d*x + c + 1)/(d*x + c - 1))/(d*x + c - 1) \\
& + (d*x + c + 1)^2*b*d^4*e^3*\log(-(d*x + c + 1)/(d*x + c - 1))/(d*x + c - 1)^2 - (d*x + c + 1)*b*d^4*e^3*\log(-(d*x + c + 1)/(d*x + c - 1))/(d*x + c - 1) + 2*(d*x + c + 1)^2*b*d^3*f*e^2*\log(-(d*x + c + 1)/(d*x + c - 1))/(d*x + c - 1)^2 \\
& - (d*x + c + 1)*b*d^3*f*e^2*\log(-(d*x + c + 1)/(d*x + c - 1))/(d*x + c - 1) + (d*x + c + 1)^2*b*d^2*f^2*e*\log(-(d*x + c + 1)/(d*x + c - 1))/(d*x + c - 1)^2 + (d*x + c + 1)*b*d^2*f^2*e*\log(-(d*x + c + 1)/(d*x + c - 1))/(d*x + c - 1) \\
& + 2*(d*x + c + 1)*a*c*d*f^3/(d*x + c - 1) + 2*a*c*d*f^3 + 2*b*c*d*f^3 + 2*(d*x + c + 1)*a*d^4*e^3/(d*x + c - 1) - 2*a*d^4*e^3 - 2*(d*x + c + 1)*a*d^3*f*e^2/(d*x + c - 1) + 4*a*d^3*f*e^2 - (d*x + c + 1)*b*d^3*f*e^2/(d*x + c - 1) \\
& + b*d^3*f*e^2 - 2*(d*x + c + 1)*a*d^2*f^2*e/(d*x + c - 1) - 2*a*d^2*f^2*e - 2*b*d^2*f^2*e + (d*x + c + 1)*b*d*f^3*\log(-(d*x + c + 1)/(d*x + c - 1))/(d*x + c - 1) + 2*(d*x + c + 1)*a*d*f^3/(d*x + c - 1) + (d*x + c + 1)*b*d*f^3/(d*x + c - 1) \\
& + b*d*f^3*((c + 1)*d - (c - 1)*d)/((d*x + c + 1)^2*c^6*f^6/(d*x + c - 1)^2 - 2*(d*x + c + 1)*c^6*f^6/(d*x + c - 1) + c^6*f^6 - 6*(d*x + c + 1)^2*c^5*d*f^5*e/(d*x + c - 1)^2 + 12*(d*x + c + 1)*c^5*d*f^5*e/(d*x + c - 1) - 6*c^5*d*f^5*e - 2*(d*x + c + 1)^2*c^5*f^6/(d*x + c - 1)^2 + 2*c^5*f^6 + 15*(d*x + c + 1)^2*c^4*d^2*f^4*e^2/(d*x + c - 1)^2 - 30*(d*x + c + 1)*c^4*d^2*f^4*e^2/(d*x + c - 1) + 15*c^4*d^2*f^4*e^2 + 10*(d*x + c + 1)^2*c^4*d*f^5*e/(d*x + c - 1)^2 - 10*c^4*d*f^5*e - (d*x + c + 1)^2*c^4*f^6/(d*x + c - 1)^2 + 6*(d*x + c + 1)*c^4*f^6/(d*x + c - 1) - c^4*f^6 - 20*(d*x + c + 1)^2*c^3*d^3*f^3*e^3/(d*x + c - 1)^2 + 40*(d*x + c + 1)*c^3*d^3*f^3*e^3/(d*x + c - 1) - 20*c^3*d^3*f^3*e^3 - 20*(d*x + c + 1)^2*c^3*d^2*f^4*e^2/(d*x + c - 1)^2 + 20*c^3*d^2*f^4*e^2 + 4*(d*x + c + 1)^2*c^3*d*f^5*e/(d*x + c - 1)^2 - 24*(d*x + c + 1)*c^3*d*f^5*e/(d*x + c - 1) + 4*c^3*d*f^5*e + 4*(d*x + c + 1)^2*c^3*f^6/(d*x + c - 1)^2 - 4*c^3*f^6 + 15*(d*x + c + 1)^2*c^2*d^4*f^2*e^4/(d*x + c - 1)^2 - 30*(d*x + c + 1)*c^2*d^4*f^2*e^4/(d*x + c - 1) + 15*c^2*d^4*f^2*e^4 + 20*(d*x + c + 1)^2*c^2*d^3*f^3*e^3/(d*x + c - 1)
\end{aligned}$$

$$\begin{aligned}
& (c-1)^2 - 20c^2d^3f^3e^3 - 6(dx+c+1)^2c^2d^2f^4e^2/(dx+c-1)^2 + 36(dx+c+1)c^2d^2f^4e^2/(dx+c-1) - 6c^2d^2f^4e^2 \\
& - 12(dx+c+1)^2c^2d^2f^5e/(dx+c-1)^2 + 12c^2d^2f^5e - (dx+c+1)^2c^2f^6/(dx+c-1)^2 - 6(dx+c+1)c^2f^6/(dx+c-1) \\
& - c^2f^6 - 6(dx+c+1)^2c^2d^5f^5e^5/(dx+c-1)^2 + 12(dx+c+1)c^2d^5f^5e^5/(dx+c-1) - 6c^2d^5f^5e^5 - 10(dx+c+1)^2c^2d^4f^2e^4 \\
& / (dx+c-1)^2 + 10c^2d^4f^2e^4 + 4(dx+c+1)^2c^2d^3f^3e^3/(dx+c-1)^2 - 24(dx+c+1)c^2d^3f^3e^3/(dx+c-1) + 4c^2d^3f^3e^3 \\
& + 12(dx+c+1)^2c^2d^2f^4e^2/(dx+c-1)^2 - 12c^2d^2f^4e^2 + 2(dx+c+1)^2c^2d^2f^5e/(dx+c-1)^2 + 12(dx+c+1)c^2d^2f^5e/(dx+c-1) \\
& + 2c^2d^2f^5e - 2(dx+c+1)^2c^2f^6/(dx+c-1)^2 + 2c^2f^6 + (dx+c+1)^2d^6e^6/(dx+c-1)^2 - 2(dx+c+1)d^6e^6/(dx+c-1) \\
& + d^6e^6 + 2(dx+c+1)^2d^5f^5e^5/(dx+c-1)^2 - 2d^5f^5e^5 - (dx+c+1)^2d^4f^2e^4/(dx+c-1)^2 + 6(dx+c+1)d^4f^2e^4/(dx+c-1) \\
& - d^4f^2e^4 - 4(dx+c+1)^2d^3f^3e^3/(dx+c-1)^2 + 4d^3f^3e^3 - (dx+c+1)^2d^2f^4e^2/(dx+c-1)^2 - 6(dx+c+1)d^2f^4e^2/(dx+c-1) \\
& - d^2f^4e^2 + 2(dx+c+1)^2d^2f^5e/(dx+c-1)^2 - 2d^2f^5e + (dx+c+1)^2f^6/(dx+c-1)^2 + 2(dx+c+1)f^6/(dx+c-1) + f^6
\end{aligned}$$

maple [A] time = 0.04, size = 236, normalized size = 1.41

$$\frac{d^2a}{2(dfx+de)^2f} - \frac{d^2b \operatorname{arctanh}(dx+c)}{2(dfx+de)^2f} + \frac{d^2b}{2(cf-de-f)(cf-de+f)(dfx+de)} + \frac{d^2bf \ln((dx+c)f-cf+a)}{(cf-de-f)^2(cf-de+f)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*arctanh(dx+c))/(f*x+e)^3,x)

[Out] $-1/2*d^2*a/(d*f*x+d*e)^2/f - 1/2*d^2*b/(d*f*x+d*e)^2/f*\operatorname{arctanh}(d*x+c) + 1/2*d^2*b/(c*f-d*e-f)/(c*f-d*e+f)/(d*f*x+d*e) + d^2*b*f/(c*f-d*e-f)^2/(c*f-d*e+f)^2*\ln((d*x+c)*f-c*f+d*e)*c-d^3*b/(c*f-d*e-f)^2/(c*f-d*e+f)^2*\ln((d*x+c)*f-c*f+d*e)*e - 1/4*d^2*b/f/(c*f-d*e-f)^2*\ln(d*x+c-1) + 1/4*d^2*b/f/(c*f-d*e+f)^2*\ln(d*x+c+1)$

maxima [A] time = 0.34, size = 291, normalized size = 1.74

$$\frac{1}{4} \left(d \left(\frac{d \log(dx+c+1)}{d^2e^2f - 2(c+1)def^2 + (c^2+2c+1)f^3} - \frac{d \log(dx+c-1)}{d^2e^2f - 2(c-1)def^2 + (c^2-2c+1)f^3} - \frac{d^4e^4 - 4cd^3e^3f + 2d^2c^2e^2f^2}{d^4e^4 - 4cd^3e^3f + 2d^2c^2e^2f^2} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arctanh(dx+c))/(f*x+e)^3,x, algorithm="maxima")

[Out] $1/4*(d*(d*\log(dx+c+1)/(d^2*e^2*f - 2*(c+1)*d*e*f^2 + (c^2+2*c+1)*f^3) - d*\log(dx+c-1)/(d^2*e^2*f - 2*(c-1)*d*e*f^2 + (c^2-2*c+1)*f^3))$

```
*f^3) - 4*(d^2*e - c*d*f)*log(f*x + e)/(d^4*e^4 - 4*c*d^3*e^3*f + 2*(3*c^2
- 1)*d^2*e^2*f^2 - 4*(c^3 - c)*d*e*f^3 + (c^4 - 2*c^2 + 1)*f^4) + 2/(d^2*e^
3 - 2*c*d*e^2*f + (c^2 - 1)*e*f^2 + (d^2*e^2*f - 2*c*d*e*f^2 + (c^2 - 1)*f^
3)*x)) - 2*arctanh(d*x + c)/(f^3*x^2 + 2*e*f^2*x + e^2*f)))*b - 1/2*a/(f^3*x
^2 + 2*e*f^2*x + e^2*f)
```

mupad [B] time = 3.03, size = 417, normalized size = 2.50

$$\frac{bd^2 \ln(c + dx + 1)}{4c^2 f^3 - 8cde f^2 + 8cf^3 + 4d^2 e^2 f - 8de f^2 + 4f^3} - \frac{\ln(e + fx) (bd^3 e - bcd^2)}{c^4 f^4 - 4c^3 de f^3 + 6c^2 d^2 e^2 f^2 - 2c^2 f^4 - 4cd^3 e^3 f + 4d^4 e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c + d*x))/(e + f*x)^3,x)
```

```
[Out] (b*d^2*log(c + d*x + 1))/(8*c*f^3 + 4*f^3 + 4*c^2*f^3 + 4*d^2*e^2*f - 8*d*e
*f^2 - 8*c*d*e*f^2) - (log(e + f*x)*(b*d^3*e - b*c*d^2*f))/(f^4 - 2*c^2*f^4
+ c^4*f^4 + d^4*e^4 - 2*d^2*e^2*f^2 + 4*c*d*e*f^3 + 6*c^2*d^2*e^2*f^2 - 4*
c*d^3*e^3*f - 4*c^3*d*e*f^3) - (b*log(c + d*x + 1))/(4*f*(e^2 + f^2*x^2 + 2
*e*f*x)) - (b*d^2*log(c + d*x - 1))/(4*f^3 - 8*c*f^3 + 4*c^2*f^3 + 4*d^2*e^
2*f + 8*d*e*f^2 - 8*c*d*e*f^2) - ((a*f^2 - a*c^2*f^2 - a*d^2*e^2 + b*d*e*f
+ 2*a*c*d*e*f)/(f^2 - c^2*f^2 - d^2*e^2 + 2*c*d*e*f) + (b*d*f^2*x)/(f^2 - c
^2*f^2 - d^2*e^2 + 2*c*d*e*f))/(2*e^2*f + 2*f^3*x^2 + 4*e*f^2*x) + (b*log(1
- d*x - c))/(2*f*(2*e^2 + 2*f^2*x^2 + 4*e*f*x))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(d*x+c))/(f*x+e)**3,x)
```

```
[Out] Timed out
```


3.38 $\int (e + fx)^3 \left(a + b \tanh^{-1}(c + dx) \right)^2 dx$

Optimal. Leaf size=562

$$\frac{(de - cf) \left((c^2 + 1) f^2 - 2cdef + d^2 e^2 \right) \left(a + b \tanh^{-1}(c + dx) \right)^2}{d^4} - \frac{2b(de - cf) \left((c^2 + 1) f^2 - 2cdef + d^2 e^2 \right) \log\left(\frac{-c}{-c}\right)}{d^4}$$

[Out] $b^2 f^2 (-c f + d e) x / d^3 + 1/2 a b f (6 d^2 e^2 - 12 c d e f + (6 c^2 + 1) f^2) x / d^3 + 1/12 b^2 f^3 (d x + c)^2 / d^4 - b^2 f^2 (-c f + d e) \operatorname{arctanh}(d x + c) / d^4 + 1/2 b^2 f (6 d^2 e^2 - 12 c d e f + (6 c^2 + 1) f^2) (d x + c) \operatorname{arctanh}(d x + c) / d^4 + b f^2 (-c f + d e) (d x + c)^2 (a + b \operatorname{arctanh}(d x + c)) / d^4 + 1/6 b f^3 (d x + c)^3 (a + b \operatorname{arctanh}(d x + c)) / d^4 + (-c f + d e) (d^2 e^2 - 2 c d e f + (c^2 + 1) f^2) (a + b \operatorname{arctanh}(d x + c))^2 / d^4 - 1/4 (d^4 e^4 - 4 c d^3 e^3 f + 6 (c^2 + 1) d^2 e^2 f^2 - 4 c (c^2 + 3) d e f^3 + (c^4 + 6 c^2 + 1) f^4) (a + b \operatorname{arctanh}(d x + c))^2 / d^4 + f + 1/4 (f x + e)^4 (a + b \operatorname{arctanh}(d x + c))^2 / f - 2 b (-c f + d e) (d^2 e^2 - 2 c d e f + (c^2 + 1) f^2) (a + b \operatorname{arctanh}(d x + c)) \ln(2 / (-d x - c + 1)) / d^4 + 1/12 b^2 f^3 \ln(1 - (d x + c)^2) / d^4 + 1/4 b^2 f (6 d^2 e^2 - 12 c d e f + (6 c^2 + 1) f^2) \ln(1 - (d x + c)^2) / d^4 - b^2 (-c f + d e) (d^2 e^2 - 2 c d e f + (c^2 + 1) f^2) \operatorname{polylog}(2, (-d x - c - 1) / (-d x - c + 1)) / d^4$

Rubi [A] time = 1.04, antiderivative size = 562, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 15, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {6111, 5928, 5910, 260, 5916, 321, 206, 266, 43, 6048, 5948, 5984, 5918, 2402, 2315}

$$\frac{b^2 (de - cf) \left((c^2 + 1) f^2 - 2cdef + d^2 e^2 \right) \operatorname{PolyLog}\left(2, -\frac{c + dx + 1}{-c - dx + 1}\right)}{d^4} + \frac{abfx \left((6c^2 + 1) f^2 - 12cdef + 6d^2 e^2 \right)}{2d^3} + \frac{(de - cf) \left((c^2 + 1) f^2 - 2cdef + d^2 e^2 \right) \operatorname{Log}\left(\frac{-c}{-c}\right)}{d^4}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(e + f x)^3 (a + b \operatorname{ArcTanh}[c + d x])^2, x]$

[Out] $(b^2 f^2 (d e - c f) x) / d^3 + (a b f (6 d^2 e^2 - 12 c d e f + (1 + 6 c^2) f^2) x) / (2 d^3) + (b^2 f^3 (c + d x)^2) / (12 d^4) - (b^2 f^2 (d e - c f) \operatorname{ArcTanh}[c + d x]) / d^4 + (b^2 f (6 d^2 e^2 - 12 c d e f + (1 + 6 c^2) f^2) (c + d x) \operatorname{ArcTanh}[c + d x]) / (2 d^4) + (b f^2 (d e - c f) (c + d x)^2 (a + b \operatorname{ArcTanh}[c + d x])) / d^4 + (b f^3 (c + d x)^3 (a + b \operatorname{ArcTanh}[c + d x])) / (6 d^4) + ((d e - c f) (d^2 e^2 - 2 c d e f + (1 + c^2) f^2) (a + b \operatorname{ArcTanh}[c + d x]))^2 / d^4 - ((d^4 e^4 - 4 c d^3 e^3 f + 6 (1 + c^2) d^2 e^2 f^2 - 4 c (3 + c^2) d e f^3 + (1 + 6 c^2 + c^4) f^4) (a + b \operatorname{ArcTanh}[c + d x])^2) / (4 d^4 f) + ((e + f x)^4 (a + b \operatorname{ArcTanh}[c + d x])^2) / (4 f) - (2 b (d e - c f) (d^2 e^2 - 2 c d e f + (1 + c^2) f^2) (a + b \operatorname{ArcTanh}[c + d x]) \operatorname{Log}[2 / (1 - c - d x)]) / d^4 + (b^2 f^3 \operatorname{Log}[1 - (c + d x)^2]) / (12 d^4) + (b^2 f (6 d^2 e^2 - 12 c d e f + (1 + 6 c^2) f^2) \operatorname{Log}[1 - (c + d x)^2]) / (4 d^4) - (b^2 (d e - c f) \operatorname{Log}[2 / (1 - c - d x)]) / d^4$

$(d^2e^2 - 2cd*ef + (1 + c^2)*f^2)*PolyLog[2, -((1 + c + d*x)/(1 - c - d*x))]/d^4$

Rule 43

$Int[(a_.) + (b_.)*(x_)^(m_.)*((c_.) + (d_.)*(x_)^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] \&\& NeQ[b*c - a*d, 0] \&\& IGtQ[m, 0] \&\& (!IntegerQ[n] || (EqQ[c, 0] \&\& LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])$

Rule 206

$Int[(a_) + (b_.)*(x_)^2)^{-1}, x_Symbol] := Simp[(1*ArcTanh[Rt[-b, 2]*x]/Rt[a, 2])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] \&\& NegQ[a/b] \&\& (GtQ[a, 0] || LtQ[b, 0])$

Rule 260

$Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] \&\& EqQ[m, n - 1]$

Rule 266

$Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] \&\& IntegerQ[Simplify[(m + 1)/n]]$

Rule 321

$Int[((c_.)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] := Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] \&\& IGtQ[n, 0] \&\& GtQ[m, n - 1] \&\& NeQ[m + n*p + 1, 0] \&\& IntBinomialQ[a, b, c, n, m, p, x]$

Rule 2315

$Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] := -Simp[PolyLog[2, 1 - c*x]/e, x] /; FreeQ[{c, d, e}, x] \&\& EqQ[e + c*d, 0]$

Rule 2402

$Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] := -Dist[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{c, d, e, f, g}, x] \&\& EqQ[c, 2*d] \&\& EqQ[e^2*f + d^2*g, 0]$

Rule 5910

Int[((a_.) + ArcTanh[(c_.)*(x_)])*(b_.))^(p_.), x_Symbol] := Simp[x*(a + b*ArcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]

Rule 5916

Int[((a_.) + ArcTanh[(c_.)*(x_)])*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol] := Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || IntegerQ[m]) && NeQ[m, -1]

Rule 5918

Int[((a_.) + ArcTanh[(c_.)*(x_)])*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol] := -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/((1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0]

Rule 5928

Int[((a_.) + ArcTanh[(c_.)*(x_)])*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.), x_Symbol] := Simp[((d + e*x)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(e*(q + 1)), x] - Dist[(b*c*p)/(e*(q + 1)), Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^(p - 1), (d + e*x)^(q + 1)/(1 - c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]

Rule 5948

Int[((a_.) + ArcTanh[(c_.)*(x_)])*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbol] := Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]

Rule 5984

Int[(((a_.) + ArcTanh[(c_.)*(x_)])*(b_.))^(p_.)*(x_)/((d_) + (e_.)*(x_)^2), x_Symbol] := Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]

Rule 6048

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^ (p_.)*((f_) + (g_.)*(x_))^(m_.))/
(d_) + (e_.)*(x_)^2), x_Symbol] := Int[ExpandIntegrand[(a + b*ArcTanh[c*x])
^p/(d + e*x^2), (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && I
GtQ[p, 0] && EqQ[c^2*d + e, 0] && IGtQ[m, 0]
```

Rule 6111

```
Int[(((a_.) + ArcTanh[(c_) + (d_.)*(x_)]*(b_.))^ (p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] := Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcTanh[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGt
Q[p, 0]
```

Rubi steps

$$\begin{aligned}
\int (e + fx)^3 (a + b \tanh^{-1}(c + dx))^2 dx &= \frac{\text{Subst} \left(\int \left(\frac{de - cf}{d} + \frac{fx}{d} \right)^3 (a + b \tanh^{-1}(x))^2 dx, x, c + dx \right)}{d} \\
&= \frac{(e + fx)^4 (a + b \tanh^{-1}(c + dx))^2}{4f} - \frac{b \text{Subst} \left(\int \left(-\frac{f^2(6d^2e^2 - 12cdef + (1 + 6c^2))}{d^4} \right) dx \right)}{d^4} \\
&= \frac{(e + fx)^4 (a + b \tanh^{-1}(c + dx))^2}{4f} - \frac{b \text{Subst} \left(\int \frac{(d^4e^4 - 4cd^3e^3f + 6(1 + c^2)d^2e^2)}{d^4} dx \right)}{d^4} \\
&= \frac{abf(6d^2e^2 - 12cdef + (1 + 6c^2)f^2)x}{2d^3} + \frac{bf^2(de - cf)(c + dx)^2(a + b \tanh^{-1}(c + dx))}{d^4} \\
&= \frac{b^2f^2(de - cf)x}{d^3} + \frac{abf(6d^2e^2 - 12cdef + (1 + 6c^2)f^2)x}{2d^3} + \frac{b^2f(6d^2e^2 - 12cdef + (1 + 6c^2)f^2)}{2d^3} \\
&= \frac{b^2f^2(de - cf)x}{d^3} + \frac{abf(6d^2e^2 - 12cdef + (1 + 6c^2)f^2)x}{2d^3} - \frac{b^2f^2(de - cf)}{2d^3} \\
&= \frac{b^2f^2(de - cf)x}{d^3} + \frac{abf(6d^2e^2 - 12cdef + (1 + 6c^2)f^2)x}{2d^3} + \frac{b^2f^3(c + dx)}{12d^4} \\
&= \frac{b^2f^2(de - cf)x}{d^3} + \frac{abf(6d^2e^2 - 12cdef + (1 + 6c^2)f^2)x}{2d^3} + \frac{b^2f^3(c + dx)}{12d^4} \\
&= \frac{b^2f^2(de - cf)x}{d^3} + \frac{abf(6d^2e^2 - 12cdef + (1 + 6c^2)f^2)x}{2d^3} + \frac{b^2f^3(c + dx)}{12d^4}
\end{aligned}$$

Mathematica [A] time = 7.48, size = 1082, normalized size = 1.93

$$\frac{1}{12} \left(3a^2 f^3 x^4 + 12a^2 e f^2 x^3 + 18a^2 e^2 f x^2 + 12a^2 e^3 x + ab \left(6x (4e^3 + 6f x e^2 + 4f^2 x^2 e + f^3 x^3) \tanh^{-1}(c + dx) - \frac{-2df}{\dots} \right) \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[(e + f*x)^3*(a + b*ArcTanh[c + d*x])^2,x]

[Out] (12*a^2*e^3*x + 18*a^2*e^2*f*x^2 + 12*a^2*e*f^2*x^3 + 3*a^2*f^3*x^4 + a*b*(6*x*(4*e^3 + 6*e^2*f*x + 4*e*f^2*x^2 + f^3*x^3)*ArcTanh[c + d*x] - (-2*d*f*x*(3*(1 + 3*c^2)*f^2 - 3*c*d*f*(8*e + f*x) + d^2*(18*e^2 + 6*e*f*x + f^2*x^2)) + 3*(-1 + c)*(4*d^3*e^3 - 6*(-1 + c)*d^2*e^2*f + 4*(-1 + c)^2*d*e*f^2 - (-1 + c)^3*f^3)*Log[1 - c - d*x] + 3*(1 + c)*(-4*d^3*e^3 + 6*(1 + c)*d^2*e^2*f - 4*(1 + c)^2*d*e*f^2 + (1 + c)^3*f^3)*Log[1 + c + d*x])/d^4 + (12*b^2*e^3*(ArcTanh[c + d*x]*((-1 + c + d*x)*ArcTanh[c + d*x] - 2*Log[1 + E^(-2*ArcTanh[c + d*x])])) + PolyLog[2, -E^(-2*ArcTanh[c + d*x])])/d - (18*b^2*e^2*f*((1 - 2*c + c^2 - d^2*x^2)*ArcTanh[c + d*x]^2 - 2*ArcTanh[c + d*x]*(c + d*x + 2*c*Log[1 + E^(-2*ArcTanh[c + d*x])])) + 2*Log[1/Sqrt[1 - (c + d*x)^2]] + 2*c*PolyLog[2, -E^(-2*ArcTanh[c + d*x])])/d^2 + (b^2*f^3*(-1 - 11*c^2 - 10*c*d*x + d^2*x^2 - 3*(1 - 4*c + 6*c^2 - 4*c^3 + c^4 - d^4*x^4)*ArcTanh[c + d*x]^2 + 2*ArcTanh[c + d*x]*(9*c + 13*c^3 + 3*d*x + 9*c^2*d*x - 3*c*d^2*x^2 + d^3*x^3 + 12*(c + c^3)*Log[1 + E^(-2*ArcTanh[c + d*x])])) - 8*Log[1/Sqrt[1 - (c + d*x)^2]] - 36*c^2*Log[1/Sqrt[1 - (c + d*x)^2]] - 12*(c + c^3)*PolyLog[2, -E^(-2*ArcTanh[c + d*x])])/d^4 - (3*b^2*e*f^2*(1 - (c + d*x)^2)^(3/2)*(-((c + d*x)/Sqrt[1 - (c + d*x)^2]) + (6*c*(c + d*x)*ArcTanh[c + d*x])/Sqrt[1 - (c + d*x)^2] + (3*(c + d*x)*ArcTanh[c + d*x]^2)/Sqrt[1 - (c + d*x)^2] - (3*c^2*(c + d*x)*ArcTanh[c + d*x]^2)/Sqrt[1 - (c + d*x)^2] + ArcTanh[c + d*x]^2*Cosh[3*ArcTanh[c + d*x]] + 3*c^2*ArcTanh[c + d*x]^2*Cosh[3*ArcTanh[c + d*x]] + 2*ArcTanh[c + d*x]*Cosh[3*ArcTanh[c + d*x]]*Log[1 + E^(-2*ArcTanh[c + d*x])] + 6*c^2*ArcTanh[c + d*x]*Cosh[3*ArcTanh[c + d*x]]*Log[1 + E^(-2*ArcTanh[c + d*x])] - 6*c*Cosh[3*ArcTanh[c + d*x]]*Log[1/Sqrt[1 - (c + d*x)^2]] + (3*(1 - 4*c + 3*c^2)*ArcTanh[c + d*x]^2 + 2*ArcTanh[c + d*x]*(2 + (3 + 9*c^2)*Log[1 + E^(-2*ArcTanh[c + d*x])])) - 18*c*Log[1/Sqrt[1 - (c + d*x)^2])/Sqrt[1 - (c + d*x)^2] - (4*(1 + 3*c^2)*PolyLog[2, -E^(-2*ArcTanh[c + d*x])])/(1 - (c + d*x)^2)^(3/2) - Sinh[3*ArcTanh[c + d*x]] + 6*c*ArcTanh[c + d*x]*Sinh[3*ArcTanh[c + d*x]] - ArcTanh[c + d*x]^2*Sinh[3*ArcTanh[c + d*x]] - 3*c^2*ArcTanh[c + d*x]^2*Sinh[3*ArcTanh[c + d*x]))/d^3)/12

fricas [F] time = 0.59, size = 0, normalized size = 0.00

$\text{integral}\left(a^2 f^3 x^3 + 3 a^2 e f^2 x^2 + 3 a^2 e^2 f x + a^2 e^3 + \left(b^2 f^3 x^3 + 3 b^2 e f^2 x^2 + 3 b^2 e^2 f x + b^2 e^3\right) \operatorname{artanh}(dx + c)^2 + 2\right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x+e)^3*(a+b*arctanh(d*x+c))^2,x, algorithm="fricas")`

[Out] `integral(a^2*f^3*x^3 + 3*a^2*e*f^2*x^2 + 3*a^2*e^2*f*x + a^2*e^3 + (b^2*f^3*x^3 + 3*b^2*e*f^2*x^2 + 3*b^2*e^2*f*x + b^2*e^3)*arctanh(d*x + c)^2 + 2*(a*b*f^3*x^3 + 3*a*b*e*f^2*x^2 + 3*a*b*e^2*f*x + a*b*e^3)*arctanh(d*x + c), x)`

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x+e)^3*(a+b*arctanh(d*x+c))^2,x, algorithm="giac")`

[Out] Timed out

maple [B] time = 0.09, size = 4401, normalized size = 7.83

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((f*x+e)^3*(a+b*arctanh(d*x+c))^2,x)`

[Out] `1/4/d*e^3*b^2*ln(d*x+c-1)^2+1/d^3*b^2*f^2*c*e-1/d^3*b^2*f^2*arctanh(d*x+c)*ln(d*x+c-1)*c^3*e-3/4/d^2*b^2*f*ln(d*x+c+1)*ln(-1/2*d*x-1/2*c+1/2)*c^2*e^2+3/2/d^3*b^2*f^2*ln(d*x+c+1)*ln(-1/2*d*x-1/2*c+1/2)*c^2*e-3/2/d^2*b^2*f*ln(d*x+c+1)*ln(-1/2*d*x-1/2*c+1/2)*c*e^2+3/2/d^4*b^2*f^3*ln(d*x+c+1)*c^2+1/2/d^4*b^2*f^3*ln(d*x+c+1)*c-1/8*b^2/f*ln(d*x+c+1)*ln(-1/2*d*x-1/2*c+1/2)*e^4+1/8*b^2/f*ln(-1/2*d*x-1/2*c+1/2)*ln(1/2+1/2*d*x+1/2*c)*e^4+1/4*b^2/f*arctanh(d*x+c)*ln(d*x+c-1)*e^4+1/2*a*b/f*arctanh(d*x+c)*e^4+1/6/d*a*b*f^3*x^3+b^2/d^2*e*f^2*x-5/6*b^2/d^3*c*f^3*x+1/2*a*b/d^3*f^3*x+1/d*e^3*a*b*ln(d*x+c+1)+1/d*e^3*b^2*arctanh(d*x+c)*ln(d*x+c+1)-1/2/d*e^3*b^2*ln(-1/2*d*x-1/2*c+1/2)*ln(1/2+1/2*d*x+1/2*c)+1/2/d*e^3*b^2*ln(-1/2*d*x-1/2*c+1/2)*ln(d*x+c+1)-1/4/d^4*a*b*f^3*ln(d*x+c+1)+3/2/d^2*b^2*f*ln(d*x+c+1)*e^2-1/8/d^4*b^2*f^3*ln(d*x+c+1)*ln(-1/2*d*x-1/2*c+1/2)-1/2/d^4*b^2*f^3*ln(d*x+c-1)*c+1/16/d^4*b^2*f^3*ln(d*x+c-1)^2*c^4+1/6/d*b^2*f^3*arctanh(d*x+c)*x^3+13/6/d^4*b^2*f^3*arctanh(d*x+c)*c^3-1/8*b^2/f*ln(d*x+c-1)*ln(1/2+1/2*d*x+1/2*c)*e^4+1/4*a*b/f*ln(d*x+c-1)*e^4-1/4*a*b/f*ln(d*x+c+1)*e^4+1/d*e^3*b^2*arctanh(d*x+c)*ln(d*x+c-1)+1/d*e^3*a*b*ln(d*x+c-1)-1/4/d*b^2*ln(d*x+c+1)^2*c*e^3-1/4/d*b^2*ln(d*x+c-`

$$\begin{aligned}
& 1)^2 * c * e^{3+1/2/d^3 * b^2 * f^2 * \ln(dx+c-1)} * e^{3/8/d^4 * b^2 * f^3 * \ln(dx+c-1)^2 * c^2 -} \\
& 1/2/d^3 * b^2 * f^2 * \ln(dx+c+1)} * e^{-1/d^3 * b^2 * f^2 * \operatorname{dilog}(1/2+1/2 * dx+1/2 * c)} * e^{3/2/} \\
& d^2 * b^2 * f * \ln(dx+c-1)} * e^{2+3/2/d^4 * b^2 * f^3 * \ln(dx+c-1)} * c^2 + 1/8/d^4 * b^2 * f^3 * 1 \\
& n(-1/2 * dx-1/2 * c+1/2) * \ln(1/2+1/2 * dx+1/2 * c) + 1/4/d^4 * b^2 * f^3 * \operatorname{arctanh}(dx+c) * \\
& \ln(dx+c-1) - 1/4/d^4 * b^2 * f^3 * \operatorname{arctanh}(dx+c) * \ln(dx+c+1) - 1/8/d^4 * b^2 * f^3 * \ln(d \\
& * x+c-1) * \ln(1/2+1/2 * dx+1/2 * c) + 1/2/d^3 * b^2 * f^3 * \operatorname{arctanh}(dx+c) * x+1/2/d^4 * b^2 * \\
& f^3 * \operatorname{arctanh}(dx+c) * c+3/8/d^2 * b^2 * f * \ln(dx+c-1)^2 * e^{2+1/4/d^3 * b^2 * f^2 * \ln(dx} \\
& +c-1)^2 * e^{1/4/d^4 * b^2 * f^3 * \ln(dx+c+1)^2 * c^3 - 1/4/d^4 * b^2 * f^3 * \ln(dx+c-1)^2 * c} \\
& ^3 + 3/8/d^4 * b^2 * f^3 * \ln(dx+c+1)^2 * c^2 + 1/4/d^4 * b^2 * f^3 * \ln(dx+c+1)^2 * c + 1/d^4 * \\
& b^2 * f^3 * \operatorname{dilog}(1/2+1/2 * dx+1/2 * c) * c - 1/4 * b^2 / f * \operatorname{arctanh}(dx+c) * \ln(dx+c+1) * e^4 \\
& + 1/4/d^4 * a * b * f^3 * \ln(dx+c-1) + b^2 * f^2 * \operatorname{arctanh}(dx+c)^2 * e * x^3 + 3/2 * b^2 * f * \operatorname{arcta} \\
& \operatorname{nh}(dx+c)^2 * e^2 * x^2 + 2 * \operatorname{arctanh}(dx+c) * x * a * b * e^{3+1/16/d^4 * b^2 * f^3 * \ln(dx+c+1)} \\
& ^2 * c^4 - 1/4/d^4 * b^2 * f^3 * \ln(dx+c-1)^2 * c + 1/d^4 * b^2 * f^3 * \operatorname{dilog}(1/2+1/2 * dx+1/2 * \\
& c) * c^3 + 3/8/d^2 * b^2 * f * \ln(dx+c+1)^2 * e^{-2-1/4/d^3 * b^2 * f^2 * \ln(dx+c+1)^2 * e^{-4 * a *} \\
& b/d^2 * f^2 * c * e * x + 3/2/d^3 * b^2 * f^2 * \ln(dx+c+1) * \ln(-1/2 * dx-1/2 * c+1/2) * c * e + a^2 * \\
& x * e^{3+1/4 * a^2 * f^3 * x^4 + 13/6/d^4 * a * b * f^3 * c^3 + 1/2/d^4 * a * b * f^3 * c - 1/2/d * e^3 * b^2 *} \\
& \ln(dx+c-1) * \ln(1/2+1/2 * dx+1/2 * c) - 3/4/d^2 * b^2 * f * \ln(dx+c-1) * \ln(1/2+1/2 * dx+ \\
& 1/2 * c) * e^{-2-1/2/d^3 * b^2 * f^2 * \ln(dx+c-1) * \ln(1/2+1/2 * dx+1/2 * c) * e^{-3/4/d^2 * b^2 *} \\
& f * \ln(dx+c+1) * \ln(-1/2 * dx-1/2 * c+1/2) * e^{-2+1/2/d^3 * b^2 * f^2 * \ln(dx+c+1) * \ln(-1/} \\
& 2 * dx-1/2 * c+1/2) * e^{-1/2/d^2 * b^2 * f^3 * \operatorname{arctanh}(dx+c) * x^2 * c + 3/2/d^3 * b^2 * f^3 * \operatorname{arc} \\
& \operatorname{tanh}(dx+c) * c^2 * x + 1/d * b^2 * f^2 * \operatorname{arctanh}(dx+c) * e * x^2 - 5/d^3 * b^2 * f^2 * \operatorname{arctanh}(d * \\
& x+c) * e * c^2 + 3/2 * a * b/d^3 * f^3 * c^2 * x + 3 * a * b/d * f * e^2 * x - 1/d^4 * b^2 * f^3 * \operatorname{arctanh}(dx+ \\
& c) * \ln(dx+c-1) * c + 3/4/d^2 * b^2 * f * \ln(-1/2 * dx-1/2 * c+1/2) * \ln(1/2+1/2 * dx+1/2 * c) \\
& * e^{-2-1/8/d^4 * b^2 * f^3 * \ln(dx+c+1) * \ln(-1/2 * dx-1/2 * c+1/2) * c^4 - 3/2/d^4 * a * b * f^3} \\
& * \ln(dx+c+1) * c^2 - 1/d^4 * a * b * f^3 * \ln(dx+c-1) * c - 1/d^4 * a * b * f^3 * \ln(dx+c+1) * c - 1/ \\
& 2/d^4 * b^2 * f^3 * \ln(dx+c+1) * \ln(-1/2 * dx-1/2 * c+1/2) * c^3 - 3/4/d^4 * b^2 * f^3 * \ln(dx \\
& +c+1) * \ln(-1/2 * dx-1/2 * c+1/2) * c^2 + 1/2/d^4 * b^2 * f^3 * \ln(dx+c-1) * \ln(1/2+1/2 * dx \\
& +1/2 * c) * c - 1/2/d^4 * b^2 * f^3 * \ln(dx+c+1) * \ln(-1/2 * dx-1/2 * c+1/2) * c + 1/8/d^4 * b^2 * \\
& f^3 * \ln(-1/2 * dx-1/2 * c+1/2) * \ln(1/2+1/2 * dx+1/2 * c) * c^4 + 1/2/d^4 * b^2 * f^3 * \ln(-1/} \\
& 2 * dx-1/2 * c+1/2) * \ln(1/2+1/2 * dx+1/2 * c) * c^3 + 3/4/d^4 * b^2 * f^3 * \ln(-1/2 * dx-1/2 * \\
& c+1/2) * \ln(1/2+1/2 * dx+1/2 * c) * c^2 + 1/2/d^4 * b^2 * f^3 * \ln(-1/2 * dx-1/2 * c+1/2) * \ln(\\
& 1/2+1/2 * dx+1/2 * c) * c - 1/d^4 * b^2 * f^3 * \operatorname{arctanh}(dx+c) * \ln(dx+c+1) * c^3 - 3/2/d^4 * b \\
& ^2 * f^3 * \operatorname{arctanh}(dx+c) * \ln(dx+c+1) * c^2 - 1/d^4 * b^2 * f^3 * \operatorname{arctanh}(dx+c) * \ln(dx+c \\
& +1) * c + 1/4/d^4 * b^2 * f^3 * \operatorname{arctanh}(dx+c) * \ln(dx+c-1) * c^4 + 3/d^3 * a * b * f^2 * \ln(dx+c \\
& +1) * c * e + 3/d^3 * a * b * f^2 * \ln(dx+c+1) * c^2 * e - 11/12/d^4 * b^2 * f^3 * c^2 + 1/4 * a^2 / f * e^4 \\
& - 3/d^2 * a * b * f * \ln(dx+c+1) * c * e^2 + 3/d^3 * b^2 * f^2 * \operatorname{arctanh}(dx+c) * \ln(dx+c+1) * c * e \\
& + 1/3/d^4 * b^2 * f^3 * \ln(dx+c-1) - 1/4/d * e^3 * b^2 * \ln(dx+c+1)^2 + a^2 * f^2 * x^3 * e + 3/2 * \\
& a^2 * f * x^2 * e^2 + 1/12/d^2 * b^2 * f^3 * x^2 + 1/16 * b^2 / f * \ln(dx+c-1)^2 * e^4 + 1/4 * b^2 / f * a \\
& \operatorname{rctanh}(dx+c)^2 * e^4 + 1/16 * b^2 / f * \ln(dx+c+1)^2 * e^4 + 1/4 * b^2 * f^3 * \operatorname{arctanh}(dx+c) \\
& ^2 * x^4 + \operatorname{arctanh}(dx+c)^2 * x * b^2 * e^3 - 1/d * b^2 * \operatorname{dilog}(1/2+1/2 * dx+1/2 * c) * e^3 + 1/3/ \\
& d^4 * b^2 * f^3 * \ln(dx+c+1) + 1/16/d^4 * b^2 * f^3 * \ln(dx+c-1)^2 + 1/16/d^4 * b^2 * f^3 * \ln(\\
& dx+c+1)^2 + 3/4/d^2 * b^2 * f * \ln(dx+c+1)^2 * c * e^{-2-3/d^3 * b^2 * f^2 * \ln(dx+c-1) * c * e^{-} \\
& 3/d^3 * b^2 * f^2 * \ln(dx+c+1) * c * e^{-3/2/d^2 * a * b * f * \ln(dx+c+1) * e^2 + 1/4/d^4 * a * b * f^3} \\
& * \ln(dx+c-1) * c^4 + 1/d^3 * a * b * f^2 * \ln(dx+c+1) * e + 3/2/d^2 * a * b * f * \ln(dx+c-1) * e^2 + \\
& 3/2/d^4 * a * b * f^3 * \ln(dx+c-1) * c^2 - 3/4/d^3 * b^2 * f^2 * \ln(dx+c-1)^2 * c * e + 3/4/d^3 * b
\end{aligned}$$

$$\begin{aligned} &^2*f^2*\ln(d*x+c-1)^2*c^2*e+3/d*b^2*f*arctanh(d*x+c)*e^2*x+3/d^2*b^2*f*arcta \\ &n h(d*x+c)*e^2*c-1/d*a*b*\ln(d*x+c-1)*c*e^3-3/d^3*b^2*f^2*dilog(1/2+1/2*d*x+1 \\ &/2*c)*c^2*e+3/d^2*b^2*f*dilog(1/2+1/2*d*x+1/2*c)*c*e^2-1/2/d^3*b^2*f^2*\ln(- \\ &1/2*d*x-1/2*c+1/2)*\ln(1/2+1/2*d*x+1/2*c)*e+3/2/d^4*b^2*f^3*arctanh(d*x+c)*l \\ &n(d*x+c-1)*c^2-1/d^4*a*b*f^3*\ln(d*x+c-1)*c^3-1/d^4*a*b*f^3*\ln(d*x+c+1)*c^3- \\ &3/4/d^3*b^2*f^2*\ln(d*x+c+1)^2*c*e+3/8/d^2*b^2*f*\ln(d*x+c+1)^2*c^2*e^2-1/4/d \\ &^3*b^2*f^2*\ln(d*x+c+1)^2*c^3*e+1/d^3*b^2*f^2*arctanh(d*x+c)*\ln(d*x+c-1)*e+1 \\ &/d^3*b^2*f^2*arctanh(d*x+c)*\ln(d*x+c+1)*e+3/2/d^2*b^2*f*arctanh(d*x+c)*\ln(d \\ &*x+c-1)*e^2-3/2/d^2*b^2*f*arctanh(d*x+c)*\ln(d*x+c+1)*e^2-1/8/d^4*b^2*f^3*\ln \\ &(d*x+c-1)*\ln(1/2+1/2*d*x+1/2*c)*c^4+1/2/d^4*b^2*f^3*\ln(d*x+c-1)*\ln(1/2+1/2* \\ &d*x+1/2*c)*c^3-3/4/d^4*b^2*f^3*\ln(d*x+c-1)*\ln(1/2+1/2*d*x+1/2*c)*c^2-3/4/d^ \\ &2*b^2*f*\ln(d*x+c-1)^2*c*e^2+1/d*a*b*\ln(d*x+c+1)*c*e^3+2*a*b*f^2*arctanh(d*x \\ &+c)*e*x^3+3*a*b*f*arctanh(d*x+c)*e^2*x^2-3/4/d^3*b^2*f^2*\ln(d*x+c+1)^2*c^2* \\ &e-1/2/d*b^2*\ln(-1/2*d*x-1/2*c+1/2)*\ln(1/2+1/2*d*x+1/2*c)*c*e^3+1/d*b^2*arct \\ &anh(d*x+c)*\ln(d*x+c+1)*c*e^3-1/d*b^2*arctanh(d*x+c)*\ln(d*x+c-1)*c*e^3-3/d^2 \\ &*a*b*f*\ln(d*x+c-1)*c*e^2-4/d^2*b^2*f^2*arctanh(d*x+c)*e*x*c+3/d^3*b^2*f^2*a \\ &rctanh(d*x+c)*\ln(d*x+c+1)*c^2*e-3/d^2*b^2*f*arctanh(d*x+c)*\ln(d*x+c+1)*c*e^ \\ &2+3/2/d^2*b^2*f*\ln(-1/2*d*x-1/2*c+1/2)*\ln(1/2+1/2*d*x+1/2*c)*c*e^2+1/2/d^3* \\ &b^2*f^2*\ln(d*x+c+1)*\ln(-1/2*d*x-1/2*c+1/2)*c^3*e-3/2/d^2*b^2*f*arctanh(d*x+ \\ &c)*\ln(d*x+c+1)*c^2*e^2-3/2/d^3*b^2*f^2*\ln(-1/2*d*x-1/2*c+1/2)*\ln(1/2+1/2*d* \\ &x+1/2*c)*c*e+1/d^3*b^2*f^2*arctanh(d*x+c)*\ln(d*x+c+1)*c^3*e+1/2*a*b*f^3*arc \\ &tanh(d*x+c)*x^4+1/d^3*a*b*f^2*\ln(d*x+c+1)*c^3*e+3/d^3*a*b*f^2*\ln(d*x+c-1)*c \\ &^2*e+3/2/d^2*a*b*f*\ln(d*x+c-1)*c^2*e^2-1/d^3*a*b*f^2*\ln(d*x+c-1)*c^3*e-1/2/ \\ &d^3*b^2*f^2*\ln(-1/2*d*x-1/2*c+1/2)*\ln(1/2+1/2*d*x+1/2*c)*c^3*e-3/d^3*b^2*f^ \\ &2*arctanh(d*x+c)*\ln(d*x+c-1)*c*e-3/d^2*b^2*f*arctanh(d*x+c)*\ln(d*x+c-1)*c*e \\ &^2+3/4/d^2*b^2*f*\ln(-1/2*d*x-1/2*c+1/2)*\ln(1/2+1/2*d*x+1/2*c)*c^2*e^2-3/2/d \\ &^3*b^2*f^2*\ln(-1/2*d*x-1/2*c+1/2)*\ln(1/2+1/2*d*x+1/2*c)*c^2*e-3/d^3*a*b*f^2 \\ &*\ln(d*x+c-1)*c*e-3/2/d^2*a*b*f*\ln(d*x+c+1)*c^2*e^2+3/2/d^2*b^2*f*arctanh(d* \\ &x+c)*\ln(d*x+c-1)*c^2*e^2+1/2/d^3*b^2*f^2*\ln(d*x+c-1)*\ln(1/2+1/2*d*x+1/2*c)* \\ &c^3*e-3/4/d^2*b^2*f*\ln(d*x+c-1)*\ln(1/2+1/2*d*x+1/2*c)*c^2*e^2-3/2/d^3*b^2*f \\ &^2*\ln(d*x+c-1)*\ln(1/2+1/2*d*x+1/2*c)*c^2*e+3/2/d^2*b^2*f*\ln(d*x+c-1)*\ln(1/2 \\ &+1/2*d*x+1/2*c)*c*e^2+3/2/d^3*b^2*f^2*\ln(d*x+c-1)*\ln(1/2+1/2*d*x+1/2*c)*c*e \\ &+1/2/d*b^2*\ln(d*x+c-1)*\ln(1/2+1/2*d*x+1/2*c)*c*e^3+1/2/d*b^2*\ln(d*x+c+1)*\ln \\ &(-1/2*d*x-1/2*c+1/2)*c*e^3+1/d^3*a*b*f^2*\ln(d*x+c-1)*e-1/4/d^4*a*b*f^3*\ln(d \\ &*x+c+1)*c^4+3/8/d^2*b^2*f*\ln(d*x+c-1)^2*c^2*e^2-1/4/d^3*b^2*f^2*\ln(d*x+c-1) \\ &^2*c^3*e-5/d^3*a*b*f^2*c^2*e+3/d^2*a*b*f*c*e^2+3/d^3*b^2*f^2*arctanh(d*x+c) \\ &*\ln(d*x+c-1)*c^2*e-1/2/d^2*a*b*f^3*c*x^2+1/d*a*b*f^2*e*x^2-1/4/d^4*b^2*f^3* \\ &arctanh(d*x+c)*\ln(d*x+c+1)*c^4-1/d^4*b^2*f^3*arctanh(d*x+c)*\ln(d*x+c-1)*c^3 \end{aligned}$$

maxima [B] time = 0.65, size = 1363, normalized size = 2.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x+e)^3*(a+b*arctanh(d*x+c))^2,x, algorithm="maxima")

```
[Out] 1/4*a^2*f^3*x^4 + a^2*e*f^2*x^3 + 3/2*a^2*e^2*f*x^2 + 3/2*(2*x^2*arctanh(d*x + c) + d*(2*x/d^2 - (c^2 + 2*c + 1)*log(d*x + c + 1)/d^3 + (c^2 - 2*c + 1)*log(d*x + c - 1)/d^3))*a*b*e^2*f + (2*x^3*arctanh(d*x + c) + d*((d*x^2 - 4*c*x)/d^3 + (c^3 + 3*c^2 + 3*c + 1)*log(d*x + c + 1)/d^4 - (c^3 - 3*c^2 + 3*c - 1)*log(d*x + c - 1)/d^4))*a*b*e*f^2 + 1/12*(6*x^4*arctanh(d*x + c) + d*(2*(d^2*x^3 - 3*c*d*x^2 + 3*(3*c^2 + 1)*x)/d^4 - 3*(c^4 + 4*c^3 + 6*c^2 + 4*c + 1)*log(d*x + c + 1)/d^5 + 3*(c^4 - 4*c^3 + 6*c^2 - 4*c + 1)*log(d*x + c - 1)/d^5))*a*b*f^3 + a^2*e^3*x + (2*(d*x + c)*arctanh(d*x + c) + log(-(d*x + c)^2 + 1))*a*b*e^3/d + (d^3*e^3 + 3*c^2*d*e*f^2 - c^3*f^3 + d*e*f^2 - (3*d^2*e^2*f + f^3)*c)*(log(d*x + c + 1)*log(-1/2*d*x - 1/2*c + 1/2) + dilog(1/2*d*x + 1/2*c + 1/2))*b^2/d^4 + 1/12*(13*c^3*f^3 + 18*d^2*e^2*f - 6*d*e*f^2 - 6*(5*d*e*f^2 - 3*f^3)*c^2 + 4*f^3 + 9*(2*d^2*e^2*f - 4*d*e*f^2 + f^3)*c)*b^2*log(d*x + c + 1)/d^4 - 1/12*(13*c^3*f^3 - 18*d^2*e^2*f - 6*d*e*f^2 - 6*(5*d*e*f^2 + 3*f^3)*c^2 - 4*f^3 + 9*(2*d^2*e^2*f + 4*d*e*f^2 + f^3)*c)*b^2*log(d*x + c - 1)/d^4 + 1/48*(4*b^2*d^2*f^3*x^2 + 8*(6*d^2*e*f^2 - 5*c*d*f^3)*b^2*x + 3*(b^2*d^4*f^3*x^4 + 4*b^2*d^4*e*f^2*x^3 + 6*b^2*d^4*e^2*f*x^2 + 4*b^2*d^4*e^3*x - (c^4*f^3 - 4*d^3*e^3 + 6*d^2*e^2*f - 4*(d*e*f^2 - f^3)*c^3 - 4*d*e*f^2 + 6*(d^2*e^2*f - 2*d*e*f^2 + f^3)*c^2 + f^3 - 4*(d^3*e^3 - 3*d^2*e^2*f + 3*d*e*f^2 - f^3)*c)*b^2)*log(d*x + c + 1)^2 + 3*(b^2*d^4*f^3*x^4 + 4*b^2*d^4*e*f^2*x^3 + 6*b^2*d^4*e^2*f*x^2 + 4*b^2*d^4*e^3*x - (c^4*f^3 + 4*d^3*e^3 + 6*d^2*e^2*f - 4*(d*e*f^2 + f^3)*c^3 + 4*d*e*f^2 + 6*(d^2*e^2*f + 2*d*e*f^2 + f^3)*c^2 + f^3 - 4*(d^3*e^3 + 3*d^2*e^2*f + 3*d*e*f^2 + f^3)*c)*b^2)*log(-d*x - c + 1)^2 + 4*(b^2*d^3*f^3*x^3 + 3*(2*d^3*e*f^2 - c*d^2*f^3)*b^2*x^2 + 3*(6*d^3*e^2*f - 8*c*d^2*e*f^2 + 3*c^2*d*f^3 + d*f^3)*b^2*x)*log(d*x + c + 1) - 2*(2*b^2*d^3*f^3*x^3 + 6*(2*d^3*e*f^2 - c*d^2*f^3)*b^2*x^2 + 6*(6*d^3*e^2*f - 8*c*d^2*e*f^2 + 3*c^2*d*f^3 + d*f^3)*b^2*x + 3*(b^2*d^4*f^3*x^4 + 4*b^2*d^4*e*f^2*x^3 + 6*b^2*d^4*e^2*f*x^2 + 4*b^2*d^4*e^3*x - (c^4*f^3 - 4*d^3*e^3 + 6*d^2*e^2*f - 4*(d*e*f^2 - f^3)*c^3 - 4*d*e*f^2 + 6*(d^2*e^2*f - 2*d*e*f^2 + f^3)*c^2 + f^3 - 4*(d^3*e^3 - 3*d^2*e^2*f + 3*d*e*f^2 - f^3)*c)*b^2)*log(d*x + c + 1))*log(-d*x - c + 1))/d^4
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (e + fx)^3 (a + b \operatorname{atanh}(c + dx))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e + f*x)^3*(a + b*atanh(c + d*x))^2,x)

[Out] int((e + f*x)^3*(a + b*atanh(c + d*x))^2, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \operatorname{atanh}(c + dx))^2 (e + fx)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*(a+b*atanh(d*x+c))**2,x)
```

```
[Out] Integral((a + b*atanh(c + d*x))**2*(e + f*x)**3, x)
```

$$3.39 \quad \int (e + fx)^2 \left(a + b \tanh^{-1}(c + dx) \right)^2 dx$$

Optimal. Leaf size=374

$$\frac{(de - cf) \left((c^2 + 3) f^2 - 2cdef + d^2 e^2 \right) \left(a + b \tanh^{-1}(c + dx) \right)^2}{3d^3 f} + \frac{\left((3c^2 + 1) f^2 - 6cdef + 3d^2 e^2 \right) \left(a + b \tanh^{-1}(c + dx) \right)^2}{3d^3}$$

[Out] $1/3*b^2*f^2*x/d^2+2*a*b*f*(-c*f+d*e)*x/d^2-1/3*b^2*f^2*arctanh(d*x+c)/d^3+2*b^2*f*(-c*f+d*e)*(d*x+c)*arctanh(d*x+c)/d^3+1/3*b*f^2*(d*x+c)^2*(a+b*arctanh(d*x+c))/d^3-1/3*(-c*f+d*e)*(d^2*e^2-2*c*d*e*f+(c^2+3)*f^2)*(a+b*arctanh(d*x+c))^2/d^3+1/3*(3*d^2*e^2-6*c*d*e*f+(3*c^2+1)*f^2)*(a+b*arctanh(d*x+c))^2/d^3+1/3*(f*x+e)^3*(a+b*arctanh(d*x+c))^2/f-2/3*b*(3*d^2*e^2-6*c*d*e*f+(3*c^2+1)*f^2)*(a+b*arctanh(d*x+c))*ln(2/(-d*x-c+1))/d^3+b^2*f*(-c*f+d*e)*ln(1-(d*x+c)^2)/d^3-1/3*b^2*(3*d^2*e^2-6*c*d*e*f+(3*c^2+1)*f^2)*polylog(2,(-d*x-c-1)/(-d*x-c+1))/d^3$

Rubi [A] time = 0.64, antiderivative size = 374, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 13, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.650$, Rules used = {6111, 5928, 5910, 260, 5916, 321, 206, 6048, 5948, 5984, 5918, 2402, 2315}

$$\frac{b^2 \left((3c^2 + 1) f^2 - 6cdef + 3d^2 e^2 \right) \text{PolyLog} \left(2, -\frac{c+dx+1}{-c-dx+1} \right)}{3d^3} - \frac{(de - cf) \left((c^2 + 3) f^2 - 2cdef + d^2 e^2 \right) \left(a + b \tanh^{-1}(c + dx) \right)^2}{3d^3 f}$$

Antiderivative was successfully verified.

[In] Int[(e + f*x)^2*(a + b*ArcTanh[c + d*x])^2,x]

[Out] $(b^2*f^2*x)/(3*d^2) + (2*a*b*f*(d*e - c*f)*x)/d^2 - (b^2*f^2*ArcTanh[c + d*x])/(3*d^3) + (2*b^2*f*(d*e - c*f)*(c + d*x)*ArcTanh[c + d*x])/d^3 + (b*f^2*(c + d*x)^2*(a + b*ArcTanh[c + d*x]))/(3*d^3) - ((d*e - c*f)*(d^2*e^2 - 2*c*d*e*f + (3 + c^2)*f^2)*(a + b*ArcTanh[c + d*x])^2)/(3*d^3*f) + ((3*d^2*e^2 - 6*c*d*e*f + (1 + 3*c^2)*f^2)*(a + b*ArcTanh[c + d*x])^2)/(3*d^3) + ((e + f*x)^3*(a + b*ArcTanh[c + d*x])^2)/(3*f) - (2*b*(3*d^2*e^2 - 6*c*d*e*f + (1 + 3*c^2)*f^2)*(a + b*ArcTanh[c + d*x])*Log[2/(1 - c - d*x)])/(3*d^3) + (b^2*f*(d*e - c*f)*Log[1 - (c + d*x)^2])/d^3 - (b^2*(3*d^2*e^2 - 6*c*d*e*f + (1 + 3*c^2)*f^2)*PolyLog[2, -((1 + c + d*x)/(1 - c - d*x))])/(3*d^3)$

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 260

Int[(x_)^(m_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 321

Int[((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 2315

Int[Log[(c_)*(x_)]/((d_) + (e_)*(x_)), x_Symbol] := -Simp[PolyLog[2, 1 - c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]

Rule 2402

Int[Log[(c_)]/((d_) + (e_)*(x_))]/((f_) + (g_)*(x_)^2), x_Symbol] := -Dist[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]

Rule 5910

Int[((a_) + ArcTanh[(c_)*(x_)]*(b_))^(p_), x_Symbol] := Simp[x*(a + b*ArcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]

Rule 5916

Int[((a_) + ArcTanh[(c_)*(x_)]*(b_))^(p_)*((d_)*(x_))^(m_), x_Symbol] := Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || IntegerQ[m]) && NeQ[m, -1]

Rule 5918

Int[((a_) + ArcTanh[(c_)*(x_)]*(b_))^(p_)/((d_) + (e_)*(x_)), x_Symbol] := -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/((1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0]

]

Rule 5928

```
Int[((a_.) + ArcTanh[(c_.)*(x_.)]*(b_.))^(p_.)*((d_.) + (e_.)*(x_.))^(q_.), x_Symbol]
:> Simp[((d + e*x)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(e*(q + 1)), x] -
Dist[(b*c*p)/(e*(q + 1)), Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^(p - 1),
(d + e*x)^(q + 1)/(1 - c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x]
&& IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_.)]*(b_.))^(p_.)/((d_.) + (e_.)*(x_.)^2), x_Symbol]
:> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b, c, d, e, p}, x]
&& EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 5984

```
Int[(((a_.) + ArcTanh[(c_.)*(x_.)]*(b_.))^(p_.)*(x_.))/((d_.) + (e_.)*(x_.)^2), x_Symbol]
:> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e}, x]
&& EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 6048

```
Int[(((a_.) + ArcTanh[(c_.)*(x_.)]*(b_.))^(p_.)*((f_.) + (g_.)*(x_.))^(m_.))/((d_.) + (e_.)*(x_.)^2), x_Symbol]
:> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p/(d + e*x^2), (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
&& IGtQ[p, 0] && EqQ[c^2*d + e, 0] && IGtQ[m, 0]
```

Rule 6111

```
Int[((a_.) + ArcTanh[(c_.) + (d_.)*(x_.)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_.))^(m_.), x_Symbol]
:> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*ArcTanh[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
&& IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int (e + fx)^2 (a + b \tanh^{-1}(c + dx))^2 dx &= \frac{\text{Subst} \left(\int \left(\frac{de - cf}{d} + \frac{fx}{d} \right)^2 (a + b \tanh^{-1}(x))^2 dx, x, c + dx \right)}{d} \\
&= \frac{(e + fx)^3 (a + b \tanh^{-1}(c + dx))^2}{3f} - \frac{(2b) \text{Subst} \left(\int \left(-\frac{3f^2(de - cf)(a + b \tanh^{-1}(x))}{d^3} \right) dx, x, c + dx \right)}{d^3} \\
&= \frac{(e + fx)^3 (a + b \tanh^{-1}(c + dx))^2}{3f} - \frac{(2b) \text{Subst} \left(\int \frac{((de - cf)(d^2 e^2 - 2cdef + 3f^2 x^2))}{d^3} dx, x, c + dx \right)}{d^3} \\
&= \frac{2abf(de - cf)x}{d^2} + \frac{bf^2(c + dx)^2 (a + b \tanh^{-1}(c + dx))}{3d^3} + \frac{(e + fx)^3 (a + b \tanh^{-1}(c + dx))^2}{3d^3} \\
&= \frac{b^2 f^2 x}{3d^2} + \frac{2abf(de - cf)x}{d^2} + \frac{2b^2 f(de - cf)(c + dx) \tanh^{-1}(c + dx)}{d^3} + \frac{(e + fx)^3 (a + b \tanh^{-1}(c + dx))^2}{3d^3} \\
&= \frac{b^2 f^2 x}{3d^2} + \frac{2abf(de - cf)x}{d^2} - \frac{b^2 f^2 \tanh^{-1}(c + dx)}{3d^3} + \frac{2b^2 f(de - cf)(c + dx) \tanh^{-1}(c + dx)}{d^3} + \frac{(e + fx)^3 (a + b \tanh^{-1}(c + dx))^2}{3d^3} \\
&= \frac{b^2 f^2 x}{3d^2} + \frac{2abf(de - cf)x}{d^2} - \frac{b^2 f^2 \tanh^{-1}(c + dx)}{3d^3} + \frac{2b^2 f(de - cf)(c + dx) \tanh^{-1}(c + dx)}{d^3} + \frac{(e + fx)^3 (a + b \tanh^{-1}(c + dx))^2}{3d^3} \\
&= \frac{b^2 f^2 x}{3d^2} + \frac{2abf(de - cf)x}{d^2} - \frac{b^2 f^2 \tanh^{-1}(c + dx)}{3d^3} + \frac{2b^2 f(de - cf)(c + dx) \tanh^{-1}(c + dx)}{d^3} + \frac{(e + fx)^3 (a + b \tanh^{-1}(c + dx))^2}{3d^3}
\end{aligned}$$

Mathematica [B] time = 3.81, size = 795, normalized size = 2.13

$$\frac{1}{3} a^2 f^2 x^3 + a^2 e f x^2 + a^2 e^2 x + \frac{1}{3} ab \left(2x (3e^2 + 3f x e + f^2 x^2) \tanh^{-1}(c + dx) + \frac{d f x (6de - 4cf + d f x) - (c - 1) (3d^2 e^2)}{d^3} \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[(e + f*x)^2*(a + b*ArcTanh[c + d*x])^2,x]

[Out] $a^2e^{2x} + a^2efx^2 + (a^2f^2x^3)/3 + (ab(2x(3e^2 + 3efx + f^2x^2)*\text{ArcTanh}[c + dx] + (dfx(6de - 4cf + d^2fx) - (-1 + c)(3d^2e^2 - 3(-1 + c)d*ef + (-1 + c)^2f^2)*\text{Log}[1 - c - dx] + (1 + c)(3d^2e^2 - 3(1 + c)d*ef + (1 + c)^2f^2)*\text{Log}[1 + c + dx])/d^3))/3 + (b^2e^2*(\text{ArcTanh}[c + dx]*((-1 + c + dx)*\text{ArcTanh}[c + dx] - 2*\text{Log}[1 + E^{(-2*\text{ArcTanh}[c + dx])}])) + \text{PolyLog}[2, -E^{(-2*\text{ArcTanh}[c + dx])}]))/d + (b^2ef*((-1 + 2c - c^2 + d^2x^2)*\text{ArcTanh}[c + dx]^2 + 2*\text{ArcTanh}[c + dx]*(c + dx + 2c*\text{Log}[1 + E^{(-2*\text{ArcTanh}[c + dx])}])) - 2*\text{Log}[1/\text{Sqrt}[1 - (c + dx)^2]] - 2c*\text{PolyLog}[2, -E^{(-2*\text{ArcTanh}[c + dx])}]))/d^2 - (b^2f^2*(1 - (c + dx)^2)^{(3/2)}*(-((c + dx)/\text{Sqrt}[1 - (c + dx)^2]) + (6c*(c + dx)*\text{ArcTanh}[c + dx])/ \text{Sqrt}[1 - (c + dx)^2] + (3*(c + dx)*\text{ArcTanh}[c + dx]^2)/\text{Sqrt}[1 - (c + dx)^2] - (3c^2*(c + dx)*\text{ArcTanh}[c + dx]^2)/\text{Sqrt}[1 - (c + dx)^2] + \text{ArcTanh}[c + dx]^2*\text{Cosh}[3*\text{ArcTanh}[c + dx]] + 3c^2*\text{ArcTanh}[c + dx]^2*\text{Cosh}[3*\text{ArcTanh}[c + dx]] + 2*\text{ArcTanh}[c + dx]*\text{Cosh}[3*\text{ArcTanh}[c + dx]]*\text{Log}[1 + E^{(-2*\text{ArcTanh}[c + dx])}] + 6c^2*\text{ArcTanh}[c + dx]*\text{Cosh}[3*\text{ArcTanh}[c + dx]]*\text{Log}[1 + E^{(-2*\text{ArcTanh}[c + dx])}] - 6c*\text{Cosh}[3*\text{ArcTanh}[c + dx]]*\text{Log}[1/\text{Sqrt}[1 - (c + dx)^2]] + (3*(1 - 4c + 3c^2)*\text{ArcTanh}[c + dx]^2 + 2*\text{ArcTanh}[c + dx]*(2 + (3 + 9c^2)*\text{Log}[1 + E^{(-2*\text{ArcTanh}[c + dx])}])) - 18c*\text{Log}[1/\text{Sqrt}[1 - (c + dx)^2]])/\text{Sqrt}[1 - (c + dx)^2] - (4*(1 + 3c^2)*\text{PolyLog}[2, -E^{(-2*\text{ArcTanh}[c + dx])}]))/(1 - (c + dx)^2)^{(3/2)} - \text{Sinh}[3*\text{ArcTanh}[c + dx]] + 6c*\text{ArcTanh}[c + dx]*\text{Sinh}[3*\text{ArcTanh}[c + dx]] - \text{ArcTanh}[c + dx]^2*\text{Sinh}[3*\text{ArcTanh}[c + dx]] - 3c^2*\text{ArcTanh}[c + dx]^2*\text{Sinh}[3*\text{ArcTanh}[c + dx]]))/(12*d^3)$

fricas [F] time = 0.57, size = 0, normalized size = 0.00

integral($a^2f^2x^2 + 2a^2efx + a^2e^2 + (b^2f^2x^2 + 2b^2efx + b^2e^2)$ artanh(dx + c)² + 2(abf²x² + 2abefx + abe²) a

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x+e)^2*(a+b*arctanh(d*x+c))^2,x, algorithm="fricas")

[Out] integral($a^2f^2x^2 + 2a^2efx + a^2e^2 + (b^2f^2x^2 + 2b^2efx + b^2e^2)*\text{arctanh}(dx + c)^2 + 2*(abf^2x^2 + 2abefx + abe^2)*\text{arctanh}(dx + c)$, x)

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x+e)^2*(a+b*arctanh(d*x+c))^2,x, algorithm="giac")

[Out] Timed out

maple [B] time = 0.07, size = 2694, normalized size = 7.20

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((f*x+e)^2*(a+b*\text{arctanh}(d*x+c))^2,x)$

[Out]
$$\begin{aligned} & -1/d*e^2*b^2*\text{dilog}(1/2+1/2*d*x+1/2*c)-1/4/d*e^2*b^2*\ln(d*x+c+1)^2+1/4/d*e^2 \\ & *b^2*\ln(d*x+c-1)^2+1/3*b^2*f^2*x/d^2-1/d*b^2*\text{arctanh}(d*x+c)*\ln(d*x+c-1)*c*e \\ & ^2+1/d*b^2*\text{arctanh}(d*x+c)*\ln(d*x+c+1)*c*e^2+1/d*a*b*\ln(d*x+c+1)*c*e^2-1/2/d \\ & ^2*b^2*f*\ln(d*x+c+1)*\ln(-1/2*d*x-1/2*c+1/2)*e+1/6/d^3*b^2*f^2*\ln(d*x+c+1)*\ln \\ & (-1/2*d*x-1/2*c+1/2)*c^3+1/2/d^3*b^2*f^2*\ln(d*x+c+1)*\ln(-1/2*d*x-1/2*c+1/2) \\ &)*c^2+1/2/d^3*b^2*f^2*\ln(d*x+c+1)*\ln(-1/2*d*x-1/2*c+1/2)*c-1/6/d^3*b^2*f^2* \\ & \ln(1/2+1/2*d*x+1/2*c)*\ln(-1/2*d*x-1/2*c+1/2)*c^3-1/2/d^3*b^2*f^2*\ln(1/2+1/2 \\ & *d*x+1/2*c)*\ln(-1/2*d*x-1/2*c+1/2)*c^2+1/6/d^3*b^2*f^2*\ln(d*x+c-1)*\ln(1/2+1 \\ & /2*d*x+1/2*c)*c^3-1/2/d^3*b^2*f^2*\ln(d*x+c-1)*\ln(1/2+1/2*d*x+1/2*c)*c^2+1/2 \\ & /d^3*b^2*f^2*\ln(d*x+c-1)*\ln(1/2+1/2*d*x+1/2*c)*c-1/2/d^3*b^2*f^2*\ln(1/2+1/2 \\ & *d*x+1/2*c)*\ln(-1/2*d*x-1/2*c+1/2)*c+1/d^3*b^2*f^2*\text{arctanh}(d*x+c)*\ln(d*x+c+ \\ & 1)*c^2-1/3/d^3*b^2*f^2*\text{arctanh}(d*x+c)*\ln(d*x+c-1)*c^3+1/d^3*b^2*f^2*\text{arctanh} \\ & (d*x+c)*\ln(d*x+c-1)*c^2-1/d^3*b^2*f^2*\text{arctanh}(d*x+c)*\ln(d*x+c-1)*c-1/d^2*b^ \\ & 2*f*\text{arctanh}(d*x+c)*\ln(d*x+c+1)*e+1/d^2*b^2*f*\text{arctanh}(d*x+c)*\ln(d*x+c-1)*e- \\ & 1/2/d^2*b^2*f*\ln(d*x+c-1)*\ln(1/2+1/2*d*x+1/2*c)*e-1/d*a*b*\ln(d*x+c-1)*c*e^2- \\ & 1/3/d^3*a*b*f^2*\ln(d*x+c-1)*c^3+1/2/d^2*b^2*f*\ln(1/2+1/2*d*x+1/2*c)*\ln(-1/2 \\ & *d*x-1/2*c+1/2)*e+1/3/d^3*b^2*f^2*\text{arctanh}(d*x+c)*\ln(d*x+c+1)*c^3+1/d^3*b^2* \\ & f^2*\text{arctanh}(d*x+c)*\ln(d*x+c+1)*c+1/4/d^2*b^2*f*\ln(d*x+c-1)^2*c^2*e+1/d^3*a* \\ & b*f^2*\ln(d*x+c-1)*c^2+1/3/d^3*a*b*f^2*\ln(d*x+c+1)*c^3+2/d*b^2*f*\text{arctanh}(d*x \\ & +c)*e*x+2/d^2*b^2*f*\text{arctanh}(d*x+c)*e*c+2/d^2*b^2*f*\text{dilog}(1/2+1/2*d*x+1/2*c) \\ & *c*e+1/d^3*a*b*f^2*\ln(d*x+c+1)*c^2+1/d^3*a*b*f^2*\ln(d*x+c+1)*c-1/d^3*a*b*f^ \\ & 2*\ln(d*x+c-1)*c+2*a*b*f*\text{arctanh}(d*x+c)*e*x^2-4/3*a*b/d^2*x*c*f^2+2*a*b/d*x* \\ & e*f-4/3/d^2*b^2*f^2*\text{arctanh}(d*x+c)*x*c-1/2/d^2*b^2*f*\ln(d*x+c-1)^2*c*e+1/4/ \\ & d^3*b^2*f^2*\ln(d*x+c-1)^2*c^2-1/4/d^3*b^2*f^2*\ln(d*x+c-1)^2*c+1/4/d^2*b^2*f \\ & *\ln(d*x+c-1)^2*e-1/12/d^3*b^2*f^2*\ln(d*x+c-1)^2*c^3-1/d^3*b^2*f^2*\ln(d*x+c+ \\ & 1)*c+1/4/d^2*b^2*f*\ln(d*x+c+1)^2*e+1/d^2*b^2*f*\ln(d*x+c+1)*e+b^2*f*\text{arctanh} \\ & (d*x+c)^2*e*x^2+1/3*a*b/f*\ln(d*x+c-1)*e^3-1/6/d^3*b^2*f^2*\ln(1/2+1/2*d*x+1/2 \\ & *c)*\ln(-1/2*d*x-1/2*c+1/2)+1/3/d^3*b^2*f^2*\text{arctanh}(d*x+c)*\ln(d*x+c-1)+1/3/d \\ & ^3*b^2*f^2*\text{arctanh}(d*x+c)*\ln(d*x+c+1)+1/3/d*b^2*f^2*\text{arctanh}(d*x+c)*x^2-5/3/ \\ & d^3*b^2*f^2*\text{arctanh}(d*x+c)*c^2-1/d^3*b^2*f^2*\ln(d*x+c-1)*c+1/d^2*b^2*f*\ln(d \\ & *x+c-1)*e-1/2/d^2*b^2*f*\ln(d*x+c+1)*\ln(-1/2*d*x-1/2*c+1/2)*c^2*e-1/d^2*b^2* \\ & f*\ln(d*x+c+1)*\ln(-1/2*d*x-1/2*c+1/2)*c*e+1/2/d^2*b^2*f*\ln(1/2+1/2*d*x+1/2*c \\ &)*\ln(-1/2*d*x-1/2*c+1/2)*c^2*e+1/d^2*b^2*f*\ln(d*x+c-1)*\ln(1/2+1/2*d*x+1/2*c \\ &)*c*e+1/d^2*b^2*f*\ln(1/2+1/2*d*x+1/2*c)*\ln(-1/2*d*x-1/2*c+1/2)*c*e-2/d^2*a* \\ & b*f*\ln(d*x+c-1)*c*e-1/d^2*a*b*f*\ln(d*x+c+1)*c^2*e+1/d^2*b^2*f*\text{arctanh}(d*x+c \\ &)*\ln(d*x+c-1)*c^2*e-1/4/d^3*b^2*f^2*\ln(d*x+c+1)^2*c+1/6/d^3*b^2*f^2*\ln(d*x+ \\ & c-1)-1/6/d^3*b^2*f^2*\ln(d*x+c+1)+1/12/d^3*b^2*f^2*\ln(d*x+c-1)^2-1/3/d^3*b^2 \end{aligned}$$

```

*f^2*dilog(1/2+1/2*d*x+1/2*c)-1/12/d^3*b^2*f^2*ln(d*x+c+1)^2+1/3*a^2/f*e^3+
1/3/d^3*b^2*f^2*c+1/12*b^2/f*ln(d*x+c-1)^2*e^3+1/3*b^2/f*arctanh(d*x+c)^2*e
^3+a^2*f*x^2*e+1/3*b^2*f^2*arctanh(d*x+c)^2*x^3+arctanh(d*x+c)^2*x*b^2*e^2+
1/12*b^2/f*ln(d*x+c+1)^2*e^3+1/3/d^3*a*b*f^2*ln(d*x+c+1)+1/3/d*a*b*f^2*x^2+
2/3*a*b*f^2*arctanh(d*x+c)*x^3+2*arctanh(d*x+c)*x*a*b*e^2+2/d^2*a*b*f*c*e+1
/d*e^2*b^2*arctanh(d*x+c)*ln(d*x+c-1)+1/d*e^2*b^2*arctanh(d*x+c)*ln(d*x+c+1
)-1/2/d*e^2*b^2*ln(d*x+c-1)*ln(1/2+1/2*d*x+1/2*c)+1/2/d*e^2*b^2*ln(-1/2*d*x
-1/2*c+1/2)*ln(d*x+c+1)-1/2/d*e^2*b^2*ln(-1/2*d*x-1/2*c+1/2)*ln(1/2+1/2*d*x
+1/2*c)+1/d*e^2*a*b*ln(d*x+c+1)+1/d*e^2*a*b*ln(d*x+c-1)+a^2*x*e^2+1/3*a^2*f
^2*x^3-1/3*b^2/f*arctanh(d*x+c)*ln(d*x+c+1)*e^3-1/6*b^2/f*ln(d*x+c-1)*ln(1/
2+1/2*d*x+1/2*c)*e^3-1/6*b^2/f*ln(d*x+c+1)*ln(-1/2*d*x-1/2*c+1/2)*e^3-1/4/d
*b^2*ln(d*x+c-1)^2*c*e^2-1/4/d*b^2*ln(d*x+c+1)^2*c*e^2-5/3/d^3*a*b*f^2*c^2-
1/6/d^3*b^2*f^2*ln(d*x+c-1)*ln(1/2+1/2*d*x+1/2*c)+1/6/d^3*b^2*f^2*ln(d*x+c+
1)*ln(-1/2*d*x-1/2*c+1/2)-1/d^3*b^2*f^2*dilog(1/2+1/2*d*x+1/2*c)*c^2-1/12/d
^3*b^2*f^2*ln(d*x+c+1)^2*c^3+1/3/d^3*a*b*f^2*ln(d*x+c-1)-1/4/d^3*b^2*f^2*ln
(d*x+c+1)^2*c^2-1/3*a*b/f*ln(d*x+c+1)*e^3+2/3*a*b/f*arctanh(d*x+c)*e^3+1/6*
b^2/f*ln(1/2+1/2*d*x+1/2*c)*ln(-1/2*d*x-1/2*c+1/2)*e^3+1/3*b^2/f*arctanh(d*
x+c)*ln(d*x+c-1)*e^3+1/d^2*a*b*f*ln(d*x+c-1)*c^2*e-2/d^2*a*b*f*ln(d*x+c+1)*
c*e-2/d^2*b^2*f*arctanh(d*x+c)*ln(d*x+c+1)*c*e-2/d^2*b^2*f*arctanh(d*x+c)*l
n(d*x+c-1)*c*e-1/d^2*b^2*f*arctanh(d*x+c)*ln(d*x+c+1)*c^2*e-1/2/d^2*b^2*f*l
n(d*x+c-1)*ln(1/2+1/2*d*x+1/2*c)*c^2*e+1/4/d^2*b^2*f*ln(d*x+c+1)^2*c^2*e+1/
d^2*a*b*f*ln(d*x+c-1)*e-1/d^2*a*b*f*ln(d*x+c+1)*e+1/2/d^2*b^2*f*ln(d*x+c+1)
^2*c*e+1/2/d*b^2*ln(d*x+c+1)*ln(-1/2*d*x-1/2*c+1/2)*c*e^2-1/2/d*b^2*ln(1/2+
1/2*d*x+1/2*c)*ln(-1/2*d*x-1/2*c+1/2)*c*e^2+1/2/d*b^2*ln(d*x+c-1)*ln(1/2+1/
2*d*x+1/2*c)*c*e^2

```

maxima [B] time = 0.62, size = 806, normalized size = 2.16

$$\frac{1}{3} a^2 f^2 x^3 + a^2 e f x^2 + \left(2x^2 \operatorname{artanh}(dx + c) + d \left(\frac{2x}{d^2} - \frac{(c^2 + 2c + 1) \log(dx + c + 1)}{d^3} + \frac{(c^2 - 2c + 1) \log(dx + c - 1)}{d^3} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x+e)^2*(a+b*arctanh(d*x+c))^2,x, algorithm="maxima")

```

[Out] 1/3*a^2*f^2*x^3 + a^2*e*f*x^2 + (2*x^2*arctanh(d*x + c) + d*(2*x/d^2 - (c^2
+ 2*c + 1)*log(d*x + c + 1)/d^3 + (c^2 - 2*c + 1)*log(d*x + c - 1)/d^3))*a
*b*e*f + 1/3*(2*x^3*arctanh(d*x + c) + d*((d*x^2 - 4*c*x)/d^3 + (c^3 + 3*c^
2 + 3*c + 1)*log(d*x + c + 1)/d^4 - (c^3 - 3*c^2 + 3*c - 1)*log(d*x + c - 1
)/d^4))*a*b*f^2 + a^2*e^2*x + (2*(d*x + c)*arctanh(d*x + c) + log(-(d*x + c
)^2 + 1))*a*b*e^2/d + 1/3*(3*d^2*e^2 - 6*c*d*e*f + 3*c^2*f^2 + f^2)*(log(d*
x + c + 1)*log(-1/2*d*x - 1/2*c + 1/2) + dilog(1/2*d*x + 1/2*c + 1/2))*b^2/
d^3 - 1/6*(5*c^2*f^2 - 6*d*e*f - 6*(d*e*f - f^2)*c + f^2)*b^2*log(d*x + c +
1)/d^3 + 1/6*(5*c^2*f^2 + 6*d*e*f - 6*(d*e*f + f^2)*c + f^2)*b^2*log(d*x +
c - 1)/d^3 + 1/12*(4*b^2*d*f^2*x + (b^2*d^3*f^2*x^3 + 3*b^2*d^3*e*f*x^2 +

```

$3*b^2*d^3*e^2*x + (c^3*f^2 + 3*d^2*e^2 - 3*(d*e*f - f^2)*c^2 - 3*d*e*f + 3*(d^2*e^2 - 2*d*e*f + f^2)*c + f^2)*b^2)*\log(d*x + c + 1)^2 + (b^2*d^3*f^2*x^3 + 3*b^2*d^3*e*f*x^2 + 3*b^2*d^3*e^2*x + (c^3*f^2 - 3*d^2*e^2 - 3*(d*e*f + f^2)*c^2 - 3*d*e*f + 3*(d^2*e^2 + 2*d*e*f + f^2)*c - f^2)*b^2)*\log(-d*x - c + 1)^2 + 2*(b^2*d^2*f^2*x^2 + 2*(3*d^2*e*f - 2*c*d*f^2)*b^2*x)*\log(d*x + c + 1) - 2*(b^2*d^2*f^2*x^2 + 2*(3*d^2*e*f - 2*c*d*f^2)*b^2*x + (b^2*d^3*f^2*x^3 + 3*b^2*d^3*e*f*x^2 + 3*b^2*d^3*e^2*x + (c^3*f^2 + 3*d^2*e^2 - 3*(d*e*f - f^2)*c^2 - 3*d*e*f + 3*(d^2*e^2 - 2*d*e*f + f^2)*c + f^2)*b^2)*\log(d*x + c + 1))*\log(-d*x - c + 1))/d^3$

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (e + f x)^2 (a + b \operatorname{atanh}(c + d x))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e + f*x)^2*(a + b*atanh(c + d*x))^2,x)

[Out] int((e + f*x)^2*(a + b*atanh(c + d*x))^2, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \operatorname{atanh}(c + d x))^2 (e + f x)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x+e)**2*(a+b*atanh(d*x+c))**2,x)

[Out] Integral((a + b*atanh(c + d*x))**2*(e + f*x)**2, x)

3.40 $\int (e + fx) \left(a + b \tanh^{-1}(c + dx) \right)^2 dx$

Optimal. Leaf size=221

$$\frac{\left((c^2 + 1)f^2 - 2cdef + d^2e^2 \right) \left(a + b \tanh^{-1}(c + dx) \right)^2}{2d^2f} + \frac{(de - cf) \left(a + b \tanh^{-1}(c + dx) \right)^2}{d^2} - \frac{2b(de - cf) \log\left(\frac{2}{-c-dx}\right)}{d^2}$$

[Out] a*b*f*x/d+b^2*f*(d*x+c)*arctanh(d*x+c)/d^2+(-c*f+d*e)*(a+b*arctanh(d*x+c))^2/d^2-1/2*(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)*(a+b*arctanh(d*x+c))^2/d^2/f+1/2*(f*x+e)^2*(a+b*arctanh(d*x+c))^2/f-2*b*(-c*f+d*e)*(a+b*arctanh(d*x+c))*ln(2/(-d*x-c+1))/d^2+1/2*b^2*f*ln(1-(d*x+c)^2)/d^2-b^2*(-c*f+d*e)*polylog(2,(-d*x-c-1)/(-d*x-c+1))/d^2

Rubi [A] time = 0.46, antiderivative size = 220, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$, Rules used = {6111, 5928, 5910, 260, 6048, 5948, 5984, 5918, 2402, 2315}

$$\frac{b^2(de - cf) \text{PolyLog}\left(2, -\frac{c+dx+1}{-c-dx+1}\right)}{d^2} + \frac{\left(-\frac{(c^2+1)f}{d} + 2ce - \frac{de^2}{f}\right) \left(a + b \tanh^{-1}(c + dx)\right)^2}{2d} + \frac{(de - cf) \left(a + b \tanh^{-1}(c + dx)\right)^2}{d^2}$$

Antiderivative was successfully verified.

[In] Int[(e + f*x)*(a + b*ArcTanh[c + d*x])^2,x]

[Out] (a*b*f*x)/d + (b^2*f*(c + d*x)*ArcTanh[c + d*x])/d^2 + ((d*e - c*f)*(a + b*ArcTanh[c + d*x])^2)/d^2 + ((2*c*e - (d*e^2)/f - ((1 + c^2)*f)/d)*(a + b*ArcTanh[c + d*x])^2)/(2*d) + ((e + f*x)^2*(a + b*ArcTanh[c + d*x])^2)/(2*f) - (2*b*(d*e - c*f)*(a + b*ArcTanh[c + d*x])*Log[2/(1 - c - d*x)])/d^2 + (b^2*f*Log[1 - (c + d*x)^2])/(2*d^2) - (b^2*(d*e - c*f)*PolyLog[2, -((1 + c + d*x)/(1 - c - d*x))])/d^2

Rule 260

Int[(x_)^(m_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 2315

Int[Log[(c_)*(x_)]/((d_) + (e_)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 - c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] := -Dist[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)])*(b_.))^p, x_Symbol] := Simp[x*(a + b*ArcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)])*(b_.))^p/((d_) + (e_.)*(x_)), x_Symbol] := -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/((1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0]
```

Rule 5928

```
Int[((a_.) + ArcTanh[(c_.)*(x_)])*(b_.))^p*((d_) + (e_.)*(x_)^q), x_Symbol] := Simp[((d + e*x)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(e*(q + 1)), x] - Dist[(b*c*p)/(e*(q + 1)), Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^(p - 1), (d + e*x)^(q + 1)/(1 - c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)])*(b_.))^p/((d_) + (e_.)*(x_)^2), x_Symbol] := Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 5984

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)])*(b_.))^p*(x_)/((d_) + (e_.)*(x_)^2), x_Symbol] := Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 6048

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)])*(b_.))^p*((f_) + (g_.)*(x_)^m)/((d_) + (e_.)*(x_)^2), x_Symbol] := Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p/(d + e*x^2), (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && I
```

GtQ[p, 0] && EqQ[c^2*d + e, 0] && IGtQ[m, 0]

Rule 6111

Int[((a_.) + ArcTanh[(c_) + (d_.)*(x_)])*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*ArcTanh[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]

Rubi steps

$$\begin{aligned}
\int (e + fx) (a + b \tanh^{-1}(c + dx))^2 dx &= \frac{\text{Subst} \left(\int \left(\frac{de - cf}{d} + \frac{fx}{d} \right) (a + b \tanh^{-1}(x))^2 dx, x, c + dx \right)}{d} \\
&= \frac{(e + fx)^2 (a + b \tanh^{-1}(c + dx))^2}{2f} - \frac{b \text{Subst} \left(\int \left(-\frac{f^2 (a + b \tanh^{-1}(x))}{d^2} + \frac{(d^2)}{1-x^2} \right) dx, x, c + dx \right)}{d^2} \\
&= \frac{(e + fx)^2 (a + b \tanh^{-1}(c + dx))^2}{2f} - \frac{b \text{Subst} \left(\int \frac{(d^2 e^2 - 2cdef + (1+c^2)f^2 + 2f(d^2 - c^2))}{1-x^2} dx, x, c + dx \right)}{d^2} \\
&= \frac{abfx}{d} + \frac{(e + fx)^2 (a + b \tanh^{-1}(c + dx))^2}{2f} - \frac{b \text{Subst} \left(\int \left(\frac{d^2 e^2 \left(1 + \frac{f(-2cde + f)}{d^2 e^2} \right)}{1-x^2} \right) dx, x, c + dx \right)}{d^2} \\
&= \frac{abfx}{d} + \frac{b^2 f (c + dx) \tanh^{-1}(c + dx)}{d^2} + \frac{(e + fx)^2 (a + b \tanh^{-1}(c + dx))^2}{2f} \\
&= \frac{abfx}{d} + \frac{b^2 f (c + dx) \tanh^{-1}(c + dx)}{d^2} + \frac{(de - cf) (a + b \tanh^{-1}(c + dx))^2}{d^2} \\
&= \frac{abfx}{d} + \frac{b^2 f (c + dx) \tanh^{-1}(c + dx)}{d^2} + \frac{(de - cf) (a + b \tanh^{-1}(c + dx))^2}{d^2} \\
&= \frac{abfx}{d} + \frac{b^2 f (c + dx) \tanh^{-1}(c + dx)}{d^2} + \frac{(de - cf) (a + b \tanh^{-1}(c + dx))^2}{d^2} \\
&= \frac{abfx}{d} + \frac{b^2 f (c + dx) \tanh^{-1}(c + dx)}{d^2} + \frac{(de - cf) (a + b \tanh^{-1}(c + dx))^2}{d^2}
\end{aligned}$$

Mathematica [A] time = 0.54, size = 271, normalized size = 1.23

$$\frac{-a^2 c^2 f + 2a^2 cde + 2a^2 d^2 ex + a^2 d^2 fx^2 + 2b \tanh^{-1}(c + dx) \left(-((c + dx)(acf - ad(2e + fx) - bf)) - 2b(de - cf) \right)}{d^2}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(e + f*x)*(a + b*ArcTanh[c + d*x])^2,x]

```
[Out] (2*a^2*c*d*e + 2*a*b*c*f - a^2*c^2*f + 2*a^2*d^2*e*x + 2*a*b*d*f*x + a^2*d^2*f*x^2 + b^2*(-1 + c + d*x)*(2*d*e + f - c*f + d*f*x)*ArcTanh[c + d*x]^2 + 2*b*ArcTanh[c + d*x]*(-(c + d*x)*(-(b*f) + a*c*f - a*d*(2*e + f*x))) - 2*b*(d*e - c*f)*Log[1 + E^(-2*ArcTanh[c + d*x])] + a*b*f*Log[1 - c - d*x] - a*b*f*Log[1 + c + d*x] - 4*a*b*d*e*Log[1/Sqrt[1 - (c + d*x)^2]] - 2*b^2*f*Log[1/Sqrt[1 - (c + d*x)^2]] + 4*a*b*c*f*Log[1/Sqrt[1 - (c + d*x)^2]] + 2*b^2*(d*e - c*f)*PolyLog[2, -E^(-2*ArcTanh[c + d*x])])/(2*d^2)
```

fricas [F] time = 0.52, size = 0, normalized size = 0.00

$$\int (a^2fx + a^2e + (b^2fx + b^2e) \operatorname{artanh}(dx + c)^2 + 2(abfx + abe) \operatorname{artanh}(dx + c), x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(a+b*arctanh(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral(a^2*f*x + a^2*e + (b^2*f*x + b^2*e)*arctanh(d*x + c)^2 + 2*(a*b*f*x + a*b*e)*arctanh(d*x + c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (fx + e)(b \operatorname{artanh}(dx + c) + a)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(a+b*arctanh(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((f*x + e)*(b*arctanh(d*x + c) + a)^2, x)
```

maple [B] time = 0.06, size = 857, normalized size = 3.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*(a+b*arctanh(d*x+c))^2,x)
```

```
[Out] a*b*f*x/d+1/4/d*b^2*ln(d*x+c-1)^2*e-1/4/d*b^2*ln(d*x+c+1)^2*e-1/d*b^2*dilog(1/2+1/2*d*x+1/2*c)*e+1/2*b^2*arctanh(d*x+c)^2*f*x^2+arctanh(d*x+c)^2*x*b^2*e+1/2/d^2*b^2*ln(d*x+c-1)*f+1/2/d^2*b^2*ln(d*x+c+1)*f+1/8/d^2*b^2*ln(d*x+c+1)^2*f+1/8/d^2*b^2*ln(d*x+c-1)^2*f+1/d*a^2*c*e-1/2/d^2*a^2*c^2*f+1/2/d^2*b^2*arctanh(d*x+c)*ln(d*x+c-1)*f-1/2/d^2*b^2*arctanh(d*x+c)*ln(d*x+c+1)*f-1/d^2*a*b*ln(d*x+c+1)*c*f-1/d^2*a*b*ln(d*x+c-1)*c*f-1/d^2*b^2*arctanh(d*x+c)*ln(d*x+c-1)*c*f-1/2/d^2*b^2*ln(d*x+c+1)*ln(-1/2*d*x-1/2*c+1/2)*c*f+1/2/d^2*b^2*ln(d*x+c-1)*ln(1/2+1/2*d*x+1/2*c)*c*f+1/2/d^2*b^2*ln(1/2+1/2*d*x+1/2*c)*ln(-1/2*d*x-1/2*c+1/2)*c*f-1/d^2*b^2*arctanh(d*x+c)*ln(d*x+c+1)*c*f-1/d^2*
```



```

a*b*arctanh(d*x+c)*f*c^2+2/d*arctanh(d*x+c)*a*b*c*e-1/4/d^2*b^2*ln(d*x+c+1)
*ln(-1/2*d*x-1/2*c+1/2)*f+1/d*arctanh(d*x+c)^2*b^2*c*e-1/2/d*e*b^2*ln(d*x+c
-1)*ln(1/2+1/2*d*x+1/2*c)-1/2/d^2*b^2*arctanh(d*x+c)^2*c^2*f+1/d^2*b^2*dilo
g(1/2+1/2*d*x+1/2*c)*c*f-1/4/d^2*b^2*ln(d*x+c-1)^2*c*f+a*b*arctanh(d*x+c)*f
*x^2+2*arctanh(d*x+c)*x*a*b*e+1/4/d^2*b^2*ln(1/2+1/2*d*x+1/2*c)*ln(-1/2*d*x
-1/2*c+1/2)*f+1/d*b^2*arctanh(d*x+c)*ln(d*x+c+1)*e+1/2/d*b^2*ln(d*x+c+1)*ln
(-1/2*d*x-1/2*c+1/2)*e+1/d*b^2*arctanh(d*x+c)*ln(d*x+c-1)*e-1/2/d*b^2*ln(1/
2+1/2*d*x+1/2*c)*ln(-1/2*d*x-1/2*c+1/2)*e+1/d*a*b*ln(d*x+c-1)*e+1/d*a*b*ln(
d*x+c+1)*e+1/2*a^2*x^2*f+x*e*a^2+1/d^2*a*b*c*f+1/d*b^2*arctanh(d*x+c)*f*x+1
/d^2*b^2*arctanh(d*x+c)*f*c+1/4/d^2*b^2*ln(d*x+c+1)^2*c*f-1/4/d^2*b^2*ln(d*
x+c-1)*ln(1/2+1/2*d*x+1/2*c)*f+1/2/d^2*a*b*ln(d*x+c-1)*f-1/2/d^2*a*b*ln(d*x
+c+1)*f

```

maxima [B] time = 0.62, size = 415, normalized size = 1.88

$$\frac{1}{2} a^2 f x^2 + \frac{1}{2} \left(2 x^2 \operatorname{artanh}(d x + c) + d \left(\frac{2 x}{d^2} - \frac{(c^2 + 2 c + 1) \log(d x + c + 1)}{d^3} + \frac{(c^2 - 2 c + 1) \log(d x + c - 1)}{d^3} \right) \right) a b$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x+e)*(a+b*arctanh(d*x+c))^2,x, algorithm="maxima")

```

[Out] 1/2*a^2*f*x^2 + 1/2*(2*x^2*arctanh(d*x + c) + d*(2*x/d^2 - (c^2 + 2*c + 1)*
log(d*x + c + 1)/d^3 + (c^2 - 2*c + 1)*log(d*x + c - 1)/d^3))*a*b*f + a^2*e
*x + (2*(d*x + c)*arctanh(d*x + c) + log(-(d*x + c)^2 + 1))*a*b*e/d + (d*e
- c*f)*(log(d*x + c + 1)*log(-1/2*d*x - 1/2*c + 1/2) + dilog(1/2*d*x + 1/2*
c + 1/2))*b^2/d^2 + 1/2*(c*f + f)*b^2*log(d*x + c + 1)/d^2 - 1/2*(c*f - f)*
b^2*log(d*x + c - 1)/d^2 + 1/8*(4*b^2*d*f*x*log(d*x + c + 1) + (b^2*d^2*f*x
^2 + 2*b^2*d^2*e*x - (c^2*f - 2*(d*e - f)*c - 2*d*e + f)*b^2)*log(d*x + c +
1)^2 + (b^2*d^2*f*x^2 + 2*b^2*d^2*e*x - (c^2*f - 2*(d*e + f)*c + 2*d*e + f
)*b^2)*log(-d*x - c + 1)^2 - 2*(2*b^2*d*f*x + (b^2*d^2*f*x^2 + 2*b^2*d^2*e
*x - (c^2*f - 2*(d*e - f)*c - 2*d*e + f)*b^2)*log(d*x + c + 1))*log(-d*x - c
+ 1))/d^2

```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (e + f x) (a + b \operatorname{atanh}(c + d x))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e + f*x)*(a + b*atanh(c + d*x))^2,x)

[Out] int((e + f*x)*(a + b*atanh(c + d*x))^2, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \operatorname{atanh}(c + d x))^2 (e + f x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(a+b*atanh(d*x+c))**2,x)
```

```
[Out] Integral((a + b*atanh(c + d*x))**2*(e + f*x), x)
```

3.41 $\int (a + b \tanh^{-1}(c + dx))^2 dx$

Optimal. Leaf size=97

$$\frac{(c + dx)(a + b \tanh^{-1}(c + dx))^2}{d} + \frac{(a + b \tanh^{-1}(c + dx))^2}{d} - \frac{2b \log\left(\frac{2}{-c-dx+1}\right)(a + b \tanh^{-1}(c + dx))}{d} - \frac{b^2 \text{Li}_2\left(-\frac{c}{-c-dx+1}\right)}{d}$$

[Out] (a+b*arctanh(d*x+c))^2/d+(d*x+c)*(a+b*arctanh(d*x+c))^2/d-2*b*(a+b*arctanh(d*x+c))*ln(2/(-d*x-c+1))/d-b^2*polylog(2,(-d*x-c-1)/(-d*x-c+1))/d

Rubi [A] time = 0.12, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6103, 5910, 5984, 5918, 2402, 2315}

$$-\frac{b^2 \text{PolyLog}\left(2, -\frac{c+dx+1}{-c-dx+1}\right)}{d} + \frac{(c + dx)(a + b \tanh^{-1}(c + dx))^2}{d} + \frac{(a + b \tanh^{-1}(c + dx))^2}{d} - \frac{2b \log\left(\frac{2}{-c-dx+1}\right)(a + b \tanh^{-1}(c + dx))}{d}$$

Antiderivative was successfully verified.

[In] Int[(a + b*ArcTanh[c + d*x])^2, x]

[Out] (a + b*ArcTanh[c + d*x])^2/d + ((c + d*x)*(a + b*ArcTanh[c + d*x])^2)/d - (2*b*(a + b*ArcTanh[c + d*x])*Log[2/(1 - c - d*x)]/d - (b^2*PolyLog[2, -(1 + c + d*x)/(1 - c - d*x)]))/d

Rule 2315

Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 - c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]

Rule 2402

Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dist[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]

Rule 5910

Int[((a_.) + ArcTanh[(c_.)*(x_)])*(b_.))^p, x_Symbol] :> Simp[x*(a + b*ArcTanh[c*x])^p, x] - Dist[b*c^p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_.)]*(b_.))^(p_.)/((d_.) + (e_.)*(x_.)), x_Symbol]
:= -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/((1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0]
```

Rule 5984

```
Int[(((a_.) + ArcTanh[(c_.)*(x_.)]*(b_.))^(p_.)*(x_.))/((d_.) + (e_.)*(x_.)^2), x_Symbol]
:= Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 6103

```
Int[((a_.) + ArcTanh[(c_.) + (d_.)*(x_.)]*(b_.))^(p_.), x_Symbol] := Dist[1/d, Subst[Int[(a + b*ArcTanh[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
 \int (a + b \tanh^{-1}(c + dx))^2 dx &= \frac{\text{Subst}\left(\int (a + b \tanh^{-1}(x))^2 dx, x, c + dx\right)}{d} \\
 &= \frac{(c + dx)(a + b \tanh^{-1}(c + dx))^2}{d} - \frac{(2b) \text{Subst}\left(\int \frac{x^{(a+b \tanh^{-1}(x))}}{1-x^2} dx, x, c + dx\right)}{d} \\
 &= \frac{(a + b \tanh^{-1}(c + dx))^2}{d} + \frac{(c + dx)(a + b \tanh^{-1}(c + dx))^2}{d} - \frac{(2b) \text{Subst}\left(\int \frac{a+b}{1-x^2} dx, x, c + dx\right)}{d} \\
 &= \frac{(a + b \tanh^{-1}(c + dx))^2}{d} + \frac{(c + dx)(a + b \tanh^{-1}(c + dx))^2}{d} - \frac{2b(a + b \tanh^{-1}(c + dx))}{d} \\
 &= \frac{(a + b \tanh^{-1}(c + dx))^2}{d} + \frac{(c + dx)(a + b \tanh^{-1}(c + dx))^2}{d} - \frac{2b(a + b \tanh^{-1}(c + dx))}{d} \\
 &= \frac{(a + b \tanh^{-1}(c + dx))^2}{d} + \frac{(c + dx)(a + b \tanh^{-1}(c + dx))^2}{d} - \frac{2b(a + b \tanh^{-1}(c + dx))}{d}
 \end{aligned}$$

Mathematica [A] time = 0.19, size = 107, normalized size = 1.10

$$\frac{a(dx + (b - bc) \log(-c - dx + 1) + b(c + 1) \log(c + dx + 1)) + 2b \tanh^{-1}(c + dx) \left(adx - b \log \left(e^{-2 \tanh^{-1}(c+dx)} + \dots \right) \right)}{d}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b*ArcTanh[c + d*x])^2, x]

[Out] (b^2*(-1 + c + d*x)*ArcTanh[c + d*x]^2 + 2*b*ArcTanh[c + d*x]*(a*d*x - b*Log[1 + E^(-2*ArcTanh[c + d*x])]) + a*(a*d*x + (b - b*c)*Log[1 - c - d*x] + b*(1 + c)*Log[1 + c + d*x]) + b^2*PolyLog[2, -E^(-2*ArcTanh[c + d*x])])/d

fricas [F] time = 0.42, size = 0, normalized size = 0.00

$$\text{integral} \left(b^2 \operatorname{artanh}(dx + c)^2 + 2ab \operatorname{artanh}(dx + c) + a^2, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arctanh(d*x+c))^2,x, algorithm="fricas")

[Out] integral(b^2*arctanh(d*x + c)^2 + 2*a*b*arctanh(d*x + c) + a^2, x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \operatorname{artanh}(dx + c) + a)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arctanh(d*x+c))^2,x, algorithm="giac")

[Out] integrate((b*arctanh(d*x + c) + a)^2, x)

maple [A] time = 0.23, size = 174, normalized size = 1.79

$$\operatorname{arctanh}(dx + c)^2 x b^2 + \frac{\operatorname{arctanh}(dx + c)^2 b^2 c}{d} + 2 \operatorname{arctanh}(dx + c) x a b + \frac{b^2 \operatorname{arctanh}(dx + c)^2}{d} - \frac{2 \operatorname{arctanh}(dx + c)}{d} + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*arctanh(d*x+c))^2,x)

[Out] arctanh(d*x+c)^2*x*b^2+1/d*arctanh(d*x+c)^2*b^2*c+2*arctanh(d*x+c)*x*a*b+1/d*b^2*arctanh(d*x+c)^2-2/d*arctanh(d*x+c)*ln(1+(d*x+c+1)^2/(1-(d*x+c)^2))*b^2+2/d*arctanh(d*x+c)*a*b*c+a^2*x+1/d*a*b*ln(1-(d*x+c)^2)-1/d*polylog(2,-(d*x+c+1)^2/(1-(d*x+c)^2))*b^2+1/d*a^2*c

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{4} \left(cd \left(\frac{(c+1) \log(dx+c+1)}{d^2} - \frac{(c-1) \log(dx+c-1)}{d^2} \right) + d^2 \left(\frac{2x}{d^2} - \frac{(c^2+2c+1) \log(dx+c+1)}{d^3} + \frac{(c^2-2c+1) \log(dx+c-1)}{d^3} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arctanh(d*x+c))^2,x, algorithm="maxima")

[Out] $-1/4*(c*d*((c+1)*\log(d*x+c+1)/d^2 - (c-1)*\log(d*x+c-1)/d^2) + d^2*(2*x/d^2 - (c^2+2*c+1)*\log(d*x+c+1)/d^3 + (c^2-2*c+1)*\log(d*x+c-1)/d^3) - 2*c*d*\text{integrate}(x*\log(d*x+c+1)/(d^2*x^2+2*c*d*x+c^2-1), x) - 2*c^2*\text{integrate}(\log(d*x+c+1)/(d^2*x^2+2*c*d*x+c^2-1), x) + d*((c+1)*\log(d*x+c+1)/d^2 - (c-1)*\log(d*x+c-1)/d^2) - 6*d*\text{integrate}(x*\log(d*x+c+1)/(d^2*x^2+2*c*d*x+c^2-1), x) - 4*c*\text{integrate}(\log(d*x+c+1)/(d^2*x^2+2*c*d*x+c^2-1), x) - (d*x+c-1)*(\log(-d*x-c+1)^2 - 2*\log(-d*x-c+1) + 2)/d - (d*x*\log(d*x+c+1)^2 + 2*(d*x - (d*x+c+1)*\log(d*x+c+1))*\log(-d*x-c+1))/d - 2*\text{integrate}(\log(d*x+c+1)/(d^2*x^2+2*c*d*x+c^2-1), x))*b^2 + a^2*x + (2*(d*x+c)*\text{arctanh}(d*x+c) + \log(-(d*x+c)^2+1))*a*b/d$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (a + b \operatorname{atanh}(c + dx))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*atanh(c + d*x))^2,x)

[Out] int((a + b*atanh(c + d*x))^2, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \operatorname{atanh}(c + dx))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*atanh(d*x+c))**2,x)

[Out] Integral((a + b*atanh(c + d*x))**2, x)

$$3.42 \quad \int \frac{(a+b \tanh^{-1}(c+dx))^2}{e+fx} dx$$

Optimal. Leaf size=214

$$\frac{b(a+b \tanh^{-1}(c+dx)) \operatorname{Li}_2\left(1 - \frac{2d(e+fx)}{(de-cf+f)(c+dx+1)}\right)}{f} + \frac{(a+b \tanh^{-1}(c+dx))^2 \log\left(\frac{2d(e+fx)}{(c+dx+1)(-cf+de+f)}\right)}{f} + \frac{b \operatorname{Li}_2\left(1 - \frac{2d(e+fx)}{(de-cf+f)(c+dx+1)}\right)}{f}$$

[Out] $-(a+b \operatorname{arctanh}(d*x+c))^2 \ln(2/(d*x+c+1))/f + (a+b \operatorname{arctanh}(d*x+c))^2 \ln(2*d*(f*x+e)/(-c*f+d*e+f)/(d*x+c+1))/f + b*(a+b \operatorname{arctanh}(d*x+c))*\operatorname{polylog}(2, 1-2/(d*x+c+1))/f - b*(a+b \operatorname{arctanh}(d*x+c))*\operatorname{polylog}(2, 1-2*d*(f*x+e)/(-c*f+d*e+f)/(d*x+c+1))/f + 1/2*b^2*\operatorname{polylog}(3, 1-2/(d*x+c+1))/f - 1/2*b^2*\operatorname{polylog}(3, 1-2*d*(f*x+e)/(-c*f+d*e+f)/(d*x+c+1))/f$

Rubi [A] time = 0.15, antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {6111, 5922}

$$\frac{b(a+b \tanh^{-1}(c+dx)) \operatorname{PolyLog}\left(2, 1 - \frac{2d(e+fx)}{(c+dx+1)(-cf+de+f)}\right)}{f} + \frac{b \operatorname{PolyLog}\left(2, 1 - \frac{2}{c+dx+1}\right)(a+b \tanh^{-1}(c+dx))}{f}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(a + b \operatorname{ArcTanh}[c + d*x])^2/(e + f*x), x]$

[Out] $-\left(\frac{(a + b \operatorname{ArcTanh}[c + d*x])^2 \operatorname{Log}\left[\frac{2}{1 + c + d*x}\right]}{f}\right) + \left(\frac{(a + b \operatorname{ArcTanh}[c + d*x])^2 \operatorname{Log}\left[\frac{2*d*(e + f*x)}{(d*e + f - c*f)*(1 + c + d*x)}\right]}{f}\right) + \left(\frac{b*(a + b \operatorname{ArcTanh}[c + d*x])*\operatorname{PolyLog}\left[2, 1 - \frac{2}{(1 + c + d*x)}\right]}{f}\right) - \left(\frac{b*(a + b \operatorname{ArcTanh}[c + d*x])*\operatorname{PolyLog}\left[2, 1 - \frac{2*d*(e + f*x)}{(d*e + f - c*f)*(1 + c + d*x)}\right]}{f}\right) + \left(\frac{b^2*\operatorname{PolyLog}\left[3, 1 - \frac{2}{(1 + c + d*x)}\right]}{2*f}\right) - \left(\frac{b^2*\operatorname{PolyLog}\left[3, 1 - \frac{2*d*(e + f*x)}{(d*e + f - c*f)*(1 + c + d*x)}\right]}{2*f}\right)$

Rule 5922

$\operatorname{Int}[(a + b \operatorname{ArcTanh}[c + d*x])^2/(e + f*x), x] \rightarrow -\operatorname{Simp}\left[\frac{(a + b \operatorname{ArcTanh}[c*x])^2 \operatorname{Log}\left[\frac{2}{1 + c*x}\right]}{e}, x\right] + \operatorname{Simp}\left[\frac{(a + b \operatorname{ArcTanh}[c*x])^2 \operatorname{Log}\left[\frac{2*c*(d + e*x)}{(c*d + e)*(1 + c*x)}\right]}{e}, x\right] + \operatorname{Simp}\left[\frac{b*(a + b \operatorname{ArcTanh}[c*x])*\operatorname{PolyLog}\left[2, 1 - \frac{2}{(1 + c*x)}\right]}{e}, x\right] - \operatorname{Simp}\left[\frac{b*(a + b \operatorname{ArcTanh}[c*x])*\operatorname{PolyLog}\left[2, 1 - \frac{2*c*(d + e*x)}{(c*d + e)*(1 + c*x)}\right]}{e}, x\right] + \operatorname{Simp}\left[\frac{b^2*\operatorname{PolyLog}\left[3, 1 - \frac{2}{(1 + c*x)}\right]}{2*e}, x\right] - \operatorname{Simp}\left[\frac{b^2*\operatorname{PolyLog}\left[3, 1 - \frac{2*c*(d + e*x)}{(c*d + e)*(1 + c*x)}\right]}{2*e}, x\right] /; \operatorname{FreeQ}\{a, b, c, d, e, x\} \ \&\& \operatorname{NeQ}[c^2*d^2 - e^2, 0]$

Rule 6111

Int[((a_.) + ArcTanh[(c_.) + (d_.)*(x_.)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_.))^(m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*ArcTanh[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]

Rubi steps

$$\int \frac{(a + b \tanh^{-1}(c + dx))^2}{e + fx} dx = \frac{\text{Subst}\left(\int \frac{(a + b \tanh^{-1}(x))^2}{\frac{de - cf}{d} + \frac{fx}{d}} dx, x, c + dx\right)}{d}$$

$$= -\frac{(a + b \tanh^{-1}(c + dx))^2 \log\left(\frac{2}{1 + c + dx}\right)}{f} + \frac{(a + b \tanh^{-1}(c + dx))^2 \log\left(\frac{2d(e + f)}{(de + f - cf)(1 + c + dx)}\right)}{f}$$

Mathematica [C] time = 24.28, size = 1757, normalized size = 8.21

result too large to display

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b*ArcTanh[c + d*x])^2/(e + f*x), x]

[Out] (a^2*Log[e + f*x])/f - ((2*I)*a*b*(I*ArcTanh[c + d*x]*(-Log[1/Sqrt[1 - (c + d*x)^2]] + Log[I*Sinh[ArcTanh[(d*e - c*f)/f] + ArcTanh[c + d*x]]]) + ((-I)*(I*ArcTanh[(d*e - c*f)/f] + I*ArcTanh[c + d*x])^2 - (I/4)*(Pi - (2*I)*ArcTanh[c + d*x])^2 + 2*(I*ArcTanh[(d*e - c*f)/f] + I*ArcTanh[c + d*x])*Log[1 - E^((2*I)*(I*ArcTanh[(d*e - c*f)/f] + I*ArcTanh[c + d*x]))] + (Pi - (2*I)*ArcTanh[c + d*x])*Log[1 - E^(I*(Pi - (2*I)*ArcTanh[c + d*x]))] - (Pi - (2*I)*ArcTanh[c + d*x])*Log[2*Sin[(Pi - (2*I)*ArcTanh[c + d*x])/2]] - 2*(I*ArcTanh[(d*e - c*f)/f] + I*ArcTanh[c + d*x])*Log[(2*I)*Sinh[ArcTanh[(d*e - c*f)/f] + ArcTanh[c + d*x]]) - I*PolyLog[2, E^((2*I)*(I*ArcTanh[(d*e - c*f)/f] + I*ArcTanh[c + d*x]))] - I*PolyLog[2, E^(I*(Pi - (2*I)*ArcTanh[c + d*x]))]/2))/f + (b^2*(d*e - c*f + f*(c + d*x))*((2*ArcTanh[c + d*x]^2*(d*e*ArcTanh[c + d*x] - (1 + c)*f*ArcTanh[c + d*x] + 3*(d*e - c*f)*Log[1 + E^(-2*ArcTanh[c + d*x])]) + (-6*d*e*ArcTanh[c + d*x] + 6*c*f*ArcTanh[c + d*x])*PolyLog[2, -E^(-2*ArcTanh[c + d*x])] + (-3*d*e + 3*c*f)*PolyLog[3, -E^(-2*ArcTanh[c + d*x])])/(6*f*(-(d*e) + c*f)) - (((-d*e) - f + c*f)*(-(d*e) + f + c*f)*(-3*d*e*ArcTanh[c + d*x]^3 + f*ArcTanh[c + d*x]^3 + 3*c*f*ArcTanh[c + d*x]^3 - (2*Sqrt[1 - c^2 - (d^2*e^2)/f^2 + (2*c*d*e)/f]*f*ArcTanh[c + d*x]^3)/E^ArcTanh[(d*e - c*f)/f] - (3*I)*d*e*Pi*ArcTanh[c + d*x])*Log[(1 + E^(2*ArcTanh[

$$\frac{c + dx)}{(2E^{\text{ArcTanh}[c + dx]})} + (3I) * c * f * \text{Pi} * \text{ArcTanh}[c + dx] * \text{Log}[(1 + E^{(2 * \text{ArcTanh}[c + dx])}) / (2E^{\text{ArcTanh}[c + dx]})] + 3 * d * e * \text{ArcTanh}[c + dx]^2 * \text{Log}[1 - E^{\text{ArcTanh}[(d * e - c * f) / f] + \text{ArcTanh}[c + dx]}] - 3 * c * f * \text{ArcTanh}[c + dx]^2 * \text{Log}[1 - E^{\text{ArcTanh}[(d * e - c * f) / f] + \text{ArcTanh}[c + dx]}] + 3 * d * e * \text{ArcTanh}[c + dx]^2 * \text{Log}[1 + E^{\text{ArcTanh}[(d * e - c * f) / f] + \text{ArcTanh}[c + dx]}] - 3 * c * f * \text{ArcTanh}[c + dx]^2 * \text{Log}[1 + E^{\text{ArcTanh}[(d * e - c * f) / f] + \text{ArcTanh}[c + dx]}] + 6 * d * e * \text{ArcTanh}[(d * e - c * f) / f] * \text{ArcTanh}[c + dx] * \text{Log}[(1/2) * E^{-\text{ArcTanh}[(d * e - c * f) / f] - \text{ArcTanh}[c + dx]}] * (-1 + E^{(2 * (\text{ArcTanh}[(d * e - c * f) / f] + \text{ArcTanh}[c + dx]))}] - 6 * c * f * \text{ArcTanh}[(d * e - c * f) / f] * \text{ArcTanh}[c + dx] * \text{Log}[(1/2) * E^{-\text{ArcTanh}[(d * e - c * f) / f] - \text{ArcTanh}[c + dx]}] * (-1 + E^{(2 * (\text{ArcTanh}[(d * e - c * f) / f] + \text{ArcTanh}[c + dx]))}] + 3 * d * e * \text{ArcTanh}[c + dx]^2 * \text{Log}[(d * e * (1 + E^{(2 * \text{ArcTanh}[c + dx])}) - (1 + c - E^{(2 * \text{ArcTanh}[c + dx])}) + c * E^{(2 * \text{ArcTanh}[c + dx])}) * f] / (2 * E^{\text{ArcTanh}[c + dx]})] - 3 * c * f * \text{ArcTanh}[c + dx]^2 * \text{Log}[(d * e * (1 + E^{(2 * \text{ArcTanh}[c + dx])}) - (1 + c - E^{(2 * \text{ArcTanh}[c + dx])}) + c * E^{(2 * \text{ArcTanh}[c + dx])}) * f] / (2 * E^{\text{ArcTanh}[c + dx]})] + (3I) * d * e * \text{Pi} * \text{ArcTanh}[c + dx] * \text{Log}[1 / \text{Sqrt}[1 - (c + dx)^2]] - (3I) * c * f * \text{Pi} * \text{ArcTanh}[c + dx] * \text{Log}[1 / \text{Sqrt}[1 - (c + dx)^2]] - 3 * d * e * \text{ArcTanh}[c + dx]^2 * \text{Log}[(d * e) / \text{Sqrt}[1 - (c + dx)^2] - (c * f) / \text{Sqrt}[1 - (c + dx)^2] + (f * (c + dx)) / \text{Sqrt}[1 - (c + dx)^2]] + 3 * c * f * \text{ArcTanh}[c + dx]^2 * \text{Log}[(d * e) / \text{Sqrt}[1 - (c + dx)^2] - (c * f) / \text{Sqrt}[1 - (c + dx)^2] + (f * (c + dx)) / \text{Sqrt}[1 - (c + dx)^2]] - 6 * d * e * \text{ArcTanh}[(d * e - c * f) / f] * \text{ArcTanh}[c + dx] * \text{Log}[I * \text{Sinh}[\text{ArcTanh}[(d * e - c * f) / f] + \text{ArcTanh}[c + dx]]] + 6 * c * f * \text{ArcTanh}[(d * e - c * f) / f] * \text{ArcTanh}[c + dx] * \text{Log}[I * \text{Sinh}[\text{ArcTanh}[(d * e - c * f) / f] + \text{ArcTanh}[c + dx]]] + 6 * (d * e - c * f) * \text{ArcTanh}[c + dx] * \text{PolyLog}[2, -E^{\text{ArcTanh}[(d * e - c * f) / f] + \text{ArcTanh}[c + dx]}] + 6 * (d * e - c * f) * \text{ArcTanh}[c + dx] * \text{PolyLog}[2, E^{\text{ArcTanh}[(d * e - c * f) / f] + \text{ArcTanh}[c + dx]}] - 6 * d * e * \text{PolyLog}[3, -E^{\text{ArcTanh}[(d * e - c * f) / f] + \text{ArcTanh}[c + dx]}] + 6 * c * f * \text{PolyLog}[3, -E^{\text{ArcTanh}[(d * e - c * f) / f] + \text{ArcTanh}[c + dx]}] - 6 * d * e * \text{PolyLog}[3, E^{\text{ArcTanh}[(d * e - c * f) / f] + \text{ArcTanh}[c + dx]}] + 6 * c * f * \text{PolyLog}[3, E^{\text{ArcTanh}[(d * e - c * f) / f] + \text{ArcTanh}[c + dx]}]) / (3 * f * (-d * e) + c * f) * (d^2 * e^2 - 2 * c * d * e * f + (-1 + c^2) * f^2)) / (d * (e + f * x))$$

fricas [F] time = 0.59, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^2 \operatorname{artanh}(dx + c)^2 + 2ab \operatorname{artanh}(dx + c) + a^2}{fx + e}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arctanh(dx+c))^2/(f*x+e),x, algorithm="fricas")

[Out] integral((b^2*arctanh(dx + c)^2 + 2*a*b*arctanh(dx + c) + a^2)/(f*x + e), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{artanh}(dx + c) + a)^2}{fx + e} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(d*x+c))^2/(f*x+e),x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(d*x + c) + a)^2/(f*x + e), x)
```

maple [C] time = 0.97, size = 1984, normalized size = 9.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(d*x+c))^2/(f*x+e),x)
```

```
[Out] a*b/f*ln((d*x+c)*f-c*f+d*e)*ln(((d*x+c)*f-f)/(c*f-d*e-f))-a*b/f*ln((d*x+c)*
f-c*f+d*e)*ln(((d*x+c)*f+f)/(c*f-d*e+f))+b^2*c/(c*f-d*e-f)*arctanh(d*x+c)^2
*ln(1-(c*f-d*e-f)*(d*x+c+1)^2/(1-(d*x+c)^2)/(-c*f+d*e-f))+b^2*c/(c*f-d*e-f)
*arctanh(d*x+c)*polylog(2,(c*f-d*e-f)*(d*x+c+1)^2/(1-(d*x+c)^2)/(-c*f+d*e-f
))-I*b^2/f*Pi*arctanh(d*x+c)^2+1/2*I*b^2/f*arctanh(d*x+c)^2*Pi*csgn(I/(1+(d
*x+c+1)^2/(1-(d*x+c)^2)))*csgn(I*(c*f*(1+(d*x+c+1)^2/(1-(d*x+c)^2))+(-(d*x+
c+1)^2/(1-(d*x+c)^2)-1)*e*d+(1-(d*x+c+1)^2/(1-(d*x+c)^2))*f))*csgn(I*(c*f*(
1+(d*x+c+1)^2/(1-(d*x+c)^2))+(-(d*x+c+1)^2/(1-(d*x+c)^2)-1)*e*d+(1-(d*x+c+1
)^2/(1-(d*x+c)^2))*f)/(1+(d*x+c+1)^2/(1-(d*x+c)^2)))+b^2*ln((d*x+c)*f-c*f+d
*e)/f*arctanh(d*x+c)^2-b^2/f*arctanh(d*x+c)^2*ln(c*f*(1+(d*x+c+1)^2/(1-(d*x
+c)^2))+(-(d*x+c+1)^2/(1-(d*x+c)^2)-1)*e*d+(1-(d*x+c+1)^2/(1-(d*x+c)^2))*f)
-b^2/f*arctanh(d*x+c)*polylog(2,-(d*x+c+1)^2/(1-(d*x+c)^2))-b^2/(c*f-d*e-f)
*arctanh(d*x+c)*polylog(2,(c*f-d*e-f)*(d*x+c+1)^2/(1-(d*x+c)^2)/(-c*f+d*e-f
))+a*b/f*dilog(((d*x+c)*f-f)/(c*f-d*e-f))-b^2/(c*f-d*e-f)*arctanh(d*x+c)^2*
ln(1-(c*f-d*e-f)*(d*x+c+1)^2/(1-(d*x+c)^2)/(-c*f+d*e-f))-a*b/f*dilog(((d*x+
c)*f+f)/(c*f-d*e+f))-1/2*b^2*c/(c*f-d*e-f)*polylog(3,(c*f-d*e-f)*(d*x+c+1)^
2/(1-(d*x+c)^2)/(-c*f+d*e-f))+2*a*b*ln((d*x+c)*f-c*f+d*e)/f*arctanh(d*x+c)+
1/2*d*b^2/f*e/(c*f-d*e-f)*polylog(3,(c*f-d*e-f)*(d*x+c+1)^2/(1-(d*x+c)^2)/(-
c*f+d*e-f))+I*b^2/f*arctanh(d*x+c)^2*Pi*csgn(I*(c*f*(1+(d*x+c+1)^2/(1-(d*x
+c)^2))+(-(d*x+c+1)^2/(1-(d*x+c)^2)-1)*e*d+(1-(d*x+c+1)^2/(1-(d*x+c)^2))*f)
/(1+(d*x+c+1)^2/(1-(d*x+c)^2)))^2-1/2*I*b^2/f*arctanh(d*x+c)^2*Pi*csgn(I*(c
*f*(1+(d*x+c+1)^2/(1-(d*x+c)^2))+(-(d*x+c+1)^2/(1-(d*x+c)^2)-1)*e*d+(1-(d*x
+c+1)^2/(1-(d*x+c)^2))*f)/(1+(d*x+c+1)^2/(1-(d*x+c)^2)))^3-1/2*I*b^2/f*arct
anh(d*x+c)^2*Pi*csgn(I/(1+(d*x+c+1)^2/(1-(d*x+c)^2)))*csgn(I*(c*f*(1+(d*x+c
+1)^2/(1-(d*x+c)^2))+(-(d*x+c+1)^2/(1-(d*x+c)^2)-1)*e*d+(1-(d*x+c+1)^2/(1-
d*x+c)^2))*f)/(1+(d*x+c+1)^2/(1-(d*x+c)^2)))^2-d*b^2/f*e/(c*f-d*e-f)*arctan
h(d*x+c)^2*ln(1-(c*f-d*e-f)*(d*x+c+1)^2/(1-(d*x+c)^2)/(-c*f+d*e-f))-d*b^2/f
*e/(c*f-d*e-f)*arctanh(d*x+c)*polylog(2,(c*f-d*e-f)*(d*x+c+1)^2/(1-(d*x+c)^
2)/(-c*f+d*e-f))-1/2*I*b^2/f*arctanh(d*x+c)^2*Pi*csgn(I*(c*f*(1+(d*x+c+1)^2
/(1-(d*x+c)^2))+(-(d*x+c+1)^2/(1-(d*x+c)^2)-1)*e*d+(1-(d*x+c+1)^2/(1-(d*x+c
)^2))*f))*csgn(I*(c*f*(1+(d*x+c+1)^2/(1-(d*x+c)^2))+(-(d*x+c+1)^2/(1-(d*x+c
)^2)-1)*e*d+(1-(d*x+c+1)^2/(1-(d*x+c)^2))*f)/(1+(d*x+c+1)^2/(1-(d*x+c)^2)))
```

$\wedge 2 + 1/2 * b^2 / (c * f - d * e - f) * \text{polylog}(3, (c * f - d * e - f) * (d * x + c + 1)^2 / (1 - (d * x + c)^2) / (-c * f + d * e - f)) + a^2 * \ln((d * x + c) * f - c * f + d * e) / f + 1/2 * b^2 / f * \text{polylog}(3, -(d * x + c + 1)^2 / (1 - (d * x + c)^2))$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{a^2 \log(fx + e)}{f} + \int \frac{b^2 (\log(dx + c + 1) - \log(-dx - c + 1))^2}{4(fx + e)} + \frac{ab(\log(dx + c + 1) - \log(-dx - c + 1))}{fx + e} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arctanh(d*x+c))^2/(f*x+e),x, algorithm="maxima")

[Out] a^2*log(f*x + e)/f + integrate(1/4*b^2*(log(d*x + c + 1) - log(-d*x - c + 1))^2/(f*x + e) + a*b*(log(d*x + c + 1) - log(-d*x - c + 1))/(f*x + e), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + b \operatorname{atanh}(c + dx))^2}{e + fx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*atanh(c + d*x))^2/(e + f*x),x)

[Out] int((a + b*atanh(c + d*x))^2/(e + f*x), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{atanh}(c + dx))^2}{e + fx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*atanh(d*x+c))**2/(f*x+e),x)

[Out] Integral((a + b*atanh(c + d*x))**2/(e + f*x), x)

$$3.43 \quad \int \frac{(a+b \tanh^{-1}(c+dx))^2}{(e+fx)^2} dx$$

Optimal. Leaf size=480

$$\frac{2abd \log(e+fx)}{f^2 - (de - cf)^2} - \frac{abd \log(-c - dx + 1)}{f(-cf + de + f)} + \frac{abd \log(c + dx + 1)}{f(-cf + de - f)} - \frac{(a + b \tanh^{-1}(c + dx))^2}{f(e + fx)} + \frac{b^2 d \operatorname{Li}_2\left(-\frac{c+dx+1}{-c-dx+1}\right)}{2f(-cf + de + f)} + \frac{b^2 d \operatorname{Li}_2\left(-\frac{c+dx+1}{-c-dx+1}\right)}{2f(-cf + de - f)}$$

[Out] $-(a+b*\operatorname{arctanh}(d*x+c))^2/f/(f*x+e)+b^2*d*\operatorname{arctanh}(d*x+c)*\ln(2/(-d*x-c+1))/f/(-c*f+d*e+f)-a*b*d*\ln(-d*x-c+1)/f/(-c*f+d*e+f)-b^2*d*\operatorname{arctanh}(d*x+c)*\ln(2/(d*x+c+1))/f/(-c*f+d*e-f)+2*b^2*d*\operatorname{arctanh}(d*x+c)*\ln(2/(d*x+c+1))/(-c*f+d*e-f)/(-c*f+d*e+f)+a*b*d*\ln(d*x+c+1)/f/(-c*f+d*e-f)+2*a*b*d*\ln(f*x+e)/(f^2-(c*f+d*e)^2)-2*b^2*d*\operatorname{arctanh}(d*x+c)*\ln(2*d*(f*x+e)/(-c*f+d*e+f)/(d*x+c+1))/(-c*f+d*e-f)/(-c*f+d*e+f)+1/2*b^2*d*\operatorname{polylog}(2,(-d*x-c-1)/(-d*x-c+1))/f/(-c*f+d*e+f)+1/2*b^2*d*\operatorname{polylog}(2,1-2/(d*x+c+1))/f/(-c*f+d*e-f)-b^2*d*\operatorname{polylog}(2,1-2/(d*x+c+1))/(-c*f+d*e-f)/(-c*f+d*e+f)+b^2*d*\operatorname{polylog}(2,1-2*d*(f*x+e)/(-c*f+d*e+f)/(d*x+c+1))/(-c*f+d*e-f)/(-c*f+d*e+f)$

Rubi [A] time = 1.71, antiderivative size = 485, normalized size of antiderivative = 1.01, number of steps used = 21, number of rules used = 19, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.950$, Rules used = {6109, 1982, 705, 31, 632, 6741, 6121, 706, 633, 6688, 12, 6725, 72, 6742, 5918, 2402, 2315, 5920, 2447}

$$\frac{b^2 d \operatorname{PolyLog}\left(2, -\frac{c+dx+1}{-c-dx+1}\right)}{2f(-cf + de + f)} + \frac{b^2 d \operatorname{PolyLog}\left(2, 1 - \frac{2}{c+dx+1}\right)}{2f(-cf + de - f)} - \frac{b^2 d \operatorname{PolyLog}\left(2, 1 - \frac{2}{c+dx+1}\right)}{(-cf + de + f)(de - (c + 1)f)} + \frac{b^2 d \operatorname{PolyLog}\left(2, 1 - \frac{2}{c+dx+1}\right)}{(-cf + de + f)(de - (c + 1)f)}$$

Antiderivative was successfully verified.

[In] Int[(a + b*ArcTanh[c + d*x])^2/(e + f*x)^2, x]

[Out] $-\left(\frac{(a + b*\operatorname{ArcTanh}[c + d*x])^2}{(f*(e + f*x))} + (b^2*d*\operatorname{ArcTanh}[c + d*x]*\operatorname{Log}\left[\frac{2}{(1 - c - d*x)}\right]\right)/f*(d*e + f - c*f) - (a*b*d*\operatorname{Log}[1 - c - d*x])/f*(d*e + f - c*f) - (b^2*d*\operatorname{ArcTanh}[c + d*x]*\operatorname{Log}\left[\frac{2}{(1 + c + d*x)}\right])/f*(d*e - f - c*f) + (2*b^2*d*\operatorname{ArcTanh}[c + d*x]*\operatorname{Log}\left[\frac{2}{(1 + c + d*x)}\right])/((d*e + f - c*f)*(d*e - (1 + c)*f)) + (a*b*d*\operatorname{Log}[1 + c + d*x])/f*(d*e - f - c*f) - (2*a*b*d*\operatorname{Log}[e + f*x])/((d*e + f - c*f)*(d*e - (1 + c)*f)) - (2*b^2*d*\operatorname{ArcTanh}[c + d*x]*\operatorname{Log}\left[\frac{2*d*(e + f*x)}{(d*e + f - c*f)*(1 + c + d*x)}\right])/((d*e + f - c*f)*(d*e - (1 + c)*f)) + (b^2*d*\operatorname{PolyLog}[2, -((1 + c + d*x)/(1 - c - d*x))])/f*(d*e + f - c*f) + (b^2*d*\operatorname{PolyLog}[2, 1 - 2/(1 + c + d*x)])/f*(d*e - f - c*f) - (b^2*d*\operatorname{PolyLog}[2, 1 - 2/(1 + c + d*x)])/((d*e + f - c*f)*(d*e - (1 + c)*f)) + (b^2*d*\operatorname{PolyLog}[2, 1 - (2*d*(e + f*x))/(d*e + f - c*f)])/((d*e + f - c*f)*(1 + c + d*x)))/((d*e + f - c*f)*(d*e - (1 + c)*f))$

Rule 12

`Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]`

Rule 31

`Int[((a_) + (b_)*(x_))^-1, x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]`

Rule 72

`Int[((e_) + (f_)*(x_))^(p_)/(((a_) + (b_)*(x_))*((c_) + (d_)*(x_))), x_Symbol] := Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]`

Rule 632

`Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && NiceSqrtQ[b^2 - 4*a*c]`

Rule 633

`Int[((d_) + (e_)*(x_))/((a_) + (c_)*(x_)^2), x_Symbol] := With[{q = Rt[-(a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x), x], x] + Dist[e/2 - (c*d)/(2*q), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[-(a*c)]`

Rule 705

`Int[1/(((d_) + (e_)*(x_))*((a_) + (b_)*(x_) + (c_)*(x_)^2)), x_Symbol] := Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0]`

Rule 706

`Int[1/(((d_) + (e_)*(x_))*((a_) + (c_)*(x_)^2)), x_Symbol] := Dist[e^2/(c*d^2 + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d^2 + a*e^2), Int[(c*d - c*e*x)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0]`

Rule 1982

```
Int[(u_)^(m_)*(v_)^(p_), x_Symbol] := Int[ExpandToSum[u, x]^m*ExpandToSum
[v, x]^p, x] /; FreeQ[{m, p}, x] && LinearQ[u, x] && QuadraticQ[v, x] && !
(LinearMatchQ[u, x] && QuadraticMatchQ[v, x])
```

Rule 2315

```
Int[Log[(c_)*(x_)]/((d_) + (e_)*(x_)), x_Symbol] := -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2402

```
Int[Log[(c_)/((d_) + (e_)*(x_))]/((f_) + (g_)*(x_)^2), x_Symbol] := -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_), x_Symbol] := With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
x][[2]], Expon[Pq, x]]
```

Rule 5918

```
Int[((a_) + ArcTanh[(c_)*(x_)]*(b_))/((d_) + (e_)*(x_)), x_Symbol
] := -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)]/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)]/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 5920

```
Int[((a_) + ArcTanh[(c_)*(x_)]*(b_))/((d_) + (e_)*(x_)), x_Symbol] := -
Simp[((a + b*ArcTanh[c*x])*Log[2/(1 + c*x)]/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x))
/((c*d + e)*(1 + c*x))]/(1 - c^2*x^2), x], x] + Simp[((a + b*ArcTanh[c*x])*
Log[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))]/e, x)) /; FreeQ[{a, b, c, d, e}
, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rule 6109

```
Int[((a_) + ArcTanh[(c_) + (d_)*(x_)]*(b_))^(p_)*((e_) + (f_)*(x_))^(
m_), x_Symbol] := Simp[((e + f*x)^(m + 1)*(a + b*ArcTanh[c + d*x])^p)/(f*(m
```

```
+ 1)), x] - Dist[(b*d*p)/(f*(m + 1)), Int[((e + f*x)^(m + 1)*(a + b*ArcTanh[c + d*x])^(p - 1))/(1 - (c + d*x)^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && ILtQ[m, -1]
```

Rule 6121

```
Int[((a_.) + ArcTanh[(c_) + (d_.)*(x_)])*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m_.)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)^(q_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(-(C/d^2) + (C*x^2)/d^2)^q*(a + b*ArcTanh[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m, p, q}, x] && EqQ[B*(1 - c^2) + 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

Rule 6688

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; SimplifyIntegrandQ[v, u, x]]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionExpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ[n, 0]
```

Rule 6741

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v != u]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a + b \tanh^{-1}(c + dx))^2}{(e + fx)^2} dx &= -\frac{(a + b \tanh^{-1}(c + dx))^2}{f(e + fx)} + \frac{(2bd) \int \frac{a + b \tanh^{-1}(c + dx)}{(e + fx)(1 - (c + dx)^2)} dx}{f} \\
&= -\frac{(a + b \tanh^{-1}(c + dx))^2}{f(e + fx)} + \frac{(2bd) \int \frac{a + b \tanh^{-1}(c + dx)}{(e + fx)(1 - c^2 - 2cdx - d^2x^2)} dx}{f} \\
&= -\frac{(a + b \tanh^{-1}(c + dx))^2}{f(e + fx)} + \frac{(2b) \text{Subst} \left(\int \frac{a + b \tanh^{-1}(x)}{\left(\frac{de - cf}{d} + \frac{fx}{d}\right)(1 - x^2)} dx, x, c + dx \right)}{f} \\
&= -\frac{(a + b \tanh^{-1}(c + dx))^2}{f(e + fx)} + \frac{(2b) \text{Subst} \left(\int \frac{d(a + b \tanh^{-1}(x))}{(de - cf + fx)(1 - x^2)} dx, x, c + dx \right)}{f} \\
&= -\frac{(a + b \tanh^{-1}(c + dx))^2}{f(e + fx)} + \frac{(2bd) \text{Subst} \left(\int \frac{a + b \tanh^{-1}(x)}{(de - cf + fx)(1 - x^2)} dx, x, c + dx \right)}{f} \\
&= -\frac{(a + b \tanh^{-1}(c + dx))^2}{f(e + fx)} + \frac{(2bd) \text{Subst} \left(\int \left(-\frac{a}{(-1 + x)(1 + x)(de - cf + fx)} - \frac{b \tanh^{-1}(x)}{(-1 + x)(1 + x)(de - cf + fx)} \right) dx, x, c + dx \right)}{f} \\
&= -\frac{(a + b \tanh^{-1}(c + dx))^2}{f(e + fx)} - \frac{(2abd) \text{Subst} \left(\int \frac{1}{(-1 + x)(1 + x)(de - cf + fx)} dx, x, c + dx \right)}{f} \\
&= -\frac{(a + b \tanh^{-1}(c + dx))^2}{f(e + fx)} - \frac{(2abd) \text{Subst} \left(\int \left(\frac{1}{2(de + f - cf)(-1 + x)} + \frac{1}{2(-de + (1 + c)f(1 + x))} \right) dx, x, c + dx \right)}{f} \\
&= -\frac{(a + b \tanh^{-1}(c + dx))^2}{f(e + fx)} - \frac{abd \log(1 - c - dx)}{f(de + f - cf)} + \frac{abd \log(1 + c + dx)}{f(de - f - cf)} - \frac{2abd}{f(de + f - cf)} \\
&= -\frac{(a + b \tanh^{-1}(c + dx))^2}{f(e + fx)} + \frac{b^2d \tanh^{-1}(c + dx) \log\left(\frac{2}{1 - c - dx}\right)}{f(de + f - cf)} - \frac{abd \log(1 - c - dx)}{f(de + f - cf)} \\
&= -\frac{(a + b \tanh^{-1}(c + dx))^2}{f(e + fx)} + \frac{b^2d \tanh^{-1}(c + dx) \log\left(\frac{2}{1 - c - dx}\right)}{f(de + f - cf)} - \frac{abd \log(1 - c - dx)}{f(de + f - cf)} \\
&= -\frac{(a + b \tanh^{-1}(c + dx))^2}{f(e + fx)} + \frac{b^2d \tanh^{-1}(c + dx) \log\left(\frac{2}{1 - c - dx}\right)}{f(de + f - cf)} - \frac{abd \log(1 - c - dx)}{f(de + f - cf)}
\end{aligned}$$

Mathematica [C] time = 7.53, size = 425, normalized size = 0.89

$$\frac{a^2}{f} + \frac{2ab \left(\tanh^{-1}(c+dx)(c^2(-f)+cd(e-fx)+d^2ex+f) - d(e+fx) \log \left(\frac{d(e+fx)}{\sqrt{1-(c+dx)^2}} \right) \right)}{(-cf+de+f)(de-(c+1)f)} + \frac{b^2 d(e+fx) \left(\text{Li}_2 \left(\exp \left(-2 \left(\tanh^{-1} \left(\frac{de-cf}{f} \right) + \tanh^{-1}(c+dx) \right) \right) \right) \right)}{(-cf+de+f)(de-(c+1)f)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b*ArcTanh[c + d*x])^2/(e + f*x)^2, x]

[Out]
$$\begin{aligned} & \left(-\frac{a^2}{f} + \frac{2ab \left(\tanh^{-1}(c+dx)(c^2(-f)+cd(e-fx)+d^2ex+f) - d(e+fx) \log \left(\frac{d(e+fx)}{\sqrt{1-(c+dx)^2}} \right) \right)}{(-cf+de+f)(de-(c+1)f)} + \frac{b^2 d(e+fx) \left(\text{Li}_2 \left(\exp \left(-2 \left(\tanh^{-1} \left(\frac{de-cf}{f} \right) + \tanh^{-1}(c+dx) \right) \right) \right) \right)}{(-cf+de+f)(de-(c+1)f)} \right) \\ & - \frac{d(e+fx) \log \left(\frac{d(e+fx)}{\sqrt{1-(c+dx)^2}} \right)}{(de-cf)(de-(c+1)f)} + \frac{b^2 d(e+fx) \left(-\text{ArcTanh}[c+dx]^2 / (E^{\text{ArcTanh}[(de-cf)/f]} * f * \sqrt{1-(de-cf)^2/f^2}) \right) + ((c+dx) * \text{ArcTanh}[c+dx]^2) / (d(e+fx)) + ((de-cf) * (I * \pi * \log[1 + E^{2 * \text{ArcTanh}[c+dx]})] - 2 * \text{ArcTanh}[c+dx] * \log[1 - E^{-2 * (\text{ArcTanh}[(de-cf)/f] + \text{ArcTanh}[c+dx])}]) - I * \pi * (\text{ArcTanh}[c+dx] + \log[1/\sqrt{1-(c+dx)^2}]) - 2 * \text{ArcTanh}[(de-cf)/f] * (\text{ArcTanh}[c+dx] + \log[1 - E^{-2 * (\text{ArcTanh}[(de-cf)/f] + \text{ArcTanh}[c+dx])}]) - \log[I * \sinh[\text{ArcTanh}[(de-cf)/f] + \text{ArcTanh}[c+dx]]) + \text{PolyLog}[2, E^{-2 * (\text{ArcTanh}[(de-cf)/f] + \text{ArcTanh}[c+dx])}] \right)}{(d^2 * e^2 - 2 * c * d * e * f + (-1 + c^2) * f^2)} \end{aligned}$$

fricas [F] time = 0.51, size = 0, normalized size = 0.00

$$\text{integral} \left(\frac{b^2 \operatorname{artanh}(dx+c)^2 + 2ab \operatorname{artanh}(dx+c) + a^2}{f^2 x^2 + 2efx + e^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arctanh(d*x+c))^2/(f*x+e)^2,x, algorithm="fricas")

[Out] integral((b^2*arctanh(d*x + c)^2 + 2*a*b*arctanh(d*x + c) + a^2)/(f^2*x^2 + 2*e*f*x + e^2), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{artanh}(dx+c) + a)^2}{(fx+e)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arctanh(d*x+c))^2/(f*x+e)^2,x, algorithm="giac")

[Out] integrate((b*arctanh(d*x + c) + a)^2/(f*x + e)^2, x)

maple [A] time = 0.08, size = 783, normalized size = 1.63

$$\frac{d a^2}{(d f x + d e) f} - \frac{d b^2 \operatorname{arctanh}(d x + c)^2}{(d f x + d e) f} - \frac{2 d b^2 \operatorname{arctanh}(d x + c) \ln((d x + c) f - c f + d e)}{(c f - d e - f)(c f - d e + f)} + \frac{2 d b^2 \operatorname{arctanh}(d x + c) \ln((d x + c) f + c f - d e)}{f(2 c f - 2 d e - f)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*arctanh(d*x+c))^2/(f*x+e)^2,x)

[Out] $-d*a^2/(d*f*x+d*e)/f-d*b^2/(d*f*x+d*e)/f*\operatorname{arctanh}(d*x+c)^2-2*d*b^2*\operatorname{arctanh}(d*x+c)/(c*f-d*e-f)/(c*f-d*e+f)*\ln((d*x+c)*f-c*f+d*e)+2*d*b^2/f*\operatorname{arctanh}(d*x+c)/(2*c*f-2*d*e-2*f)*\ln(d*x+c-1)-2*d*b^2/f*\operatorname{arctanh}(d*x+c)/(2*c*f-2*d*e+2*f)*\ln(d*x+c+1)-d*b^2/(c*f-d*e-f)/(c*f-d*e+f)*\ln((d*x+c)*f-c*f+d*e)*\ln(((d*x+c)*f-f)/(c*f-d*e-f))-d*b^2/(c*f-d*e-f)/(c*f-d*e+f)*\operatorname{dilog}(((d*x+c)*f-f)/(c*f-d*e-f))+d*b^2/(c*f-d*e-f)/(c*f-d*e+f)*\ln((d*x+c)*f-c*f+d*e)*\ln(((d*x+c)*f+f)/(c*f-d*e+f))+d*b^2/(c*f-d*e-f)/(c*f-d*e+f)*\operatorname{dilog}(((d*x+c)*f+f)/(c*f-d*e+f))+1/4*d*b^2/f/(c*f-d*e-f)*\ln(d*x+c-1)^2-1/2*d*b^2/f/(c*f-d*e-f)*\operatorname{dilog}(1/2+1/2*d*x+1/2*c)-1/2*d*b^2/f/(c*f-d*e-f)*\ln(d*x+c-1)*\ln(1/2+1/2*d*x+1/2*c)+1/4*d*b^2/f/(c*f-d*e-f)*\ln(d*x+c+1)^2-1/2*d*b^2/f/(c*f-d*e-f)*\ln(-1/2*d*x-1/2*c+1/2)*\ln(d*x+c+1)+1/2*d*b^2/f/(c*f-d*e-f)*\ln(-1/2*d*x-1/2*c+1/2)*\ln(1/2+1/2*d*x+1/2*c)+1/2*d*b^2/f/(c*f-d*e-f)*\operatorname{dilog}(1/2+1/2*d*x+1/2*c)-2*d*a*b/(d*f*x+d*e)/f*\operatorname{arctanh}(d*x+c)-2*d*a*b/(c*f-d*e-f)/(c*f-d*e+f)*\ln((d*x+c)*f-c*f+d*e)+2*d*a*b/f/(2*c*f-2*d*e-2*f)*\ln(d*x+c-1)-2*d*a*b/f/(2*c*f-2*d*e+2*f)*\ln(d*x+c+1)$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\left(d \left(\frac{\log(dx + c + 1)}{def - (c + 1)f^2} - \frac{\log(dx + c - 1)}{def - (c - 1)f^2} - \frac{2 \log(fx + e)}{d^2e^2 - 2cdef + (c^2 - 1)f^2} \right) - \frac{2 \operatorname{artanh}(dx + c)}{f^2x + ef} \right) ab - \frac{1}{4} b^2 \left(\frac{\log(-dx - c)}{f^2x + ef} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arctanh(d*x+c))^2/(f*x+e)^2,x, algorithm="maxima")

[Out] $(d*(\log(d*x + c + 1)/(d*e*f - (c + 1)*f^2) - \log(d*x + c - 1)/(d*e*f - (c - 1)*f^2) - 2*\log(f*x + e)/(d^2*e^2 - 2*c*d*e*f + (c^2 - 1)*f^2)) - 2*\operatorname{arctanh}(d*x + c)/(f^2*x + e*f))*a*b - 1/4*b^2*(\log(-d*x - c + 1)^2/(f^2*x + e*f) + \operatorname{integrate}(-((d*f*x + c*f - f)*\log(d*x + c + 1)^2 + 2*(d*f*x + d*e - (d*f*x + c*f - f)*\log(d*x + c + 1))*\log(-d*x - c + 1))/(d*f^3*x^3 + c*e^2*f - e^2*f + (2*d*e*f^2 + c*f^3 - f^3)*x^2 + (d*e^2*f + 2*c*e*f^2 - 2*e*f^2)*x), x)) - a^2/(f^2*x + e*f)$

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + b \operatorname{atanh}(c + dx))^2}{(e + fx)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*atanh(c + d*x))^2/(e + f*x)^2, x)

[Out] int((a + b*atanh(c + d*x))^2/(e + f*x)^2, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{atanh}(c + dx))^2}{(e + fx)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*atanh(d*x+c))**2/(f*x+e)**2, x)

[Out] Integral((a + b*atanh(c + d*x))**2/(e + f*x)**2, x)

$$3.44 \quad \int \frac{(a+b \tanh^{-1}(c+dx))^2}{(e+fx)^3} dx$$

Optimal. Leaf size=750

$$-\frac{abd^2 \log(-c-dx+1)}{2f(-cf+de+f)^2} + \frac{abd^2 \log(c+dx+1)}{2f(-cf+de-f)^2} - \frac{2abd^2(de-cf) \log(e+fx)}{(-cf+de+f)^2(de-(c+1)f)^2} - \frac{abd}{(e+fx)(f^2-(de-cf)^2)} - \frac{(a+}{$$

[Out] $-a*b*d/(f^2-(c*f+d*e)^2)/(f*x+e)+b^2*d*arctanh(d*x+c)/(-c*f+d*e-f)/(-c*f+d*e+f)/(f*x+e)-1/2*(a+b*arctanh(d*x+c))^2/f/(f*x+e)^2+1/2*b^2*d^2*arctanh(d*x+c)*ln(2/(-d*x-c+1))/f/(-c*f+d*e+f)^2-1/2*a*b*d^2*ln(-d*x-c+1)/f/(-c*f+d*e+f)^2+1/2*b^2*d^2*ln(-d*x-c+1)/(-c*f+d*e+f)^2/(d*e-(1+c)*f)-1/2*b^2*d^2*arctanh(d*x+c)*ln(2/(d*x+c+1))/f/(-c*f+d*e-f)^2+2*b^2*d^2*(-c*f+d*e)*arctanh(d*x+c)*ln(2/(d*x+c+1))/(-c*f+d*e+f)^2/(d*e-(1+c)*f)^2+1/2*a*b*d^2*ln(d*x+c+1)/f/(-c*f+d*e-f)^2-1/2*b^2*d^2*ln(d*x+c+1)/(-c*f+d*e+f)/(d*e-(1+c)*f)^2+b^2*d^2*f*ln(f*x+e)/(-c*f+d*e+f)^2/(d*e-(1+c)*f)^2-2*a*b*d^2*(-c*f+d*e)*ln(f*x+e)/(-c*f+d*e+f)^2/(d*e-(1+c)*f)^2-2*b^2*d^2*(-c*f+d*e)*arctanh(d*x+c)*ln(2*d*(f*x+e)/(-c*f+d*e+f)/(d*x+c+1))/(-c*f+d*e+f)^2/(d*e-(1+c)*f)^2+1/4*b^2*d^2^2*polylog(2,(-d*x-c-1)/(-d*x-c+1))/f/(-c*f+d*e+f)^2+1/4*b^2*d^2*polylog(2,1-2/(d*x+c+1))/f/(-c*f+d*e-f)^2-b^2*d^2*(-c*f+d*e)*polylog(2,1-2/(d*x+c+1))/(-c*f+d*e+f)^2/(d*e-(1+c)*f)^2+b^2*d^2*(-c*f+d*e)*polylog(2,1-2*d*(f*x+e)/(-c*f+d*e+f)/(d*x+c+1))/(-c*f+d*e+f)^2/(d*e-(1+c)*f)^2$

Rubi [A] time = 2.13, antiderivative size = 750, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 18, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.900$, Rules used = {6109, 1982, 709, 800, 6741, 6121, 710, 801, 6725, 5918, 2402, 2315, 5926, 706, 31, 633, 5920, 2447}

$$\frac{b^2 d^2 \text{PolyLog}\left(2, \frac{c+dx+1}{-c-dx+1}\right)}{4f(-cf+de+f)^2} + \frac{b^2 d^2 \text{PolyLog}\left(2, 1 - \frac{2}{c+dx+1}\right)}{4f(-cf+de-f)^2} - \frac{b^2 d^2 (de-cf) \text{PolyLog}\left(2, 1 - \frac{2}{c+dx+1}\right)}{(-cf+de+f)^2(de-(c+1)f)^2} + \frac{b^2 d^2 (de-cf)}{(-cf$$

Antiderivative was successfully verified.

[In] Int[(a + b*ArcTanh[c + d*x])^2/(e + f*x)^3, x]

[Out] $-((a*b*d)/((f^2-(d*e-c*f)^2)*(e+f*x)))+(b^2*d*ArcTanh[c+d*x])/((d*e+f-c*f)*(d*e-(1+c)*f)*(e+f*x))-(a+b*ArcTanh[c+d*x])^2/(2*f*(e+f*x)^2)+(b^2*d^2*ArcTanh[c+d*x]*Log[2/(1-c-d*x)])/(2*f*(d*e+f-c*f)^2)-(a*b*d^2*Log[1-c-d*x])/(2*f*(d*e+f-c*f)^2)+(b^2*d^2*Log[1-c-d*x])/(2*(d*e+f-c*f)^2*(d*e-(1+c)*f))-(b^2*d^2*ArcTanh[c+d*x]*Log[2/(1+c+d*x)])/(2*f*(d*e-f-c*f)^2)+(2*b^2*d^2*(d*e-c*f)*ArcTanh[c+d*x]*Log[2/(1+c+d*x)])/((d*e+f-c*f)^2*(d*e-$

$$\begin{aligned} & (1 + c)*f)^2) + (a*b*d^2*\text{Log}[1 + c + d*x])/(2*f*(d*e - f - c*f)^2) - (b^2*d \\ & ^2*\text{Log}[1 + c + d*x])/(2*(d*e + f - c*f)*(d*e - (1 + c)*f)^2) + (b^2*d^2*f*L \\ & \text{og}[e + f*x])/((d*e + f - c*f)^2*(d*e - (1 + c)*f)^2) - (2*a*b*d^2*(d*e - c* \\ & f)*\text{Log}[e + f*x])/((d*e + f - c*f)^2*(d*e - (1 + c)*f)^2) - (2*b^2*d^2*(d*e \\ & - c*f)*\text{ArcTanh}[c + d*x]*\text{Log}[(2*d*(e + f*x))/((d*e + f - c*f)*(1 + c + d*x)) \\ &])/((d*e + f - c*f)^2*(d*e - (1 + c)*f)^2) + (b^2*d^2*\text{PolyLog}[2, -((1 + c + \\ & d*x)/(1 - c - d*x))])/(4*f*(d*e + f - c*f)^2) + (b^2*d^2*\text{PolyLog}[2, 1 - 2/ \\ & (1 + c + d*x)])/(4*f*(d*e - f - c*f)^2) - (b^2*d^2*(d*e - c*f)*\text{PolyLog}[2, 1 \\ & - 2/(1 + c + d*x)])/((d*e + f - c*f)^2*(d*e - (1 + c)*f)^2) + (b^2*d^2*(d*e \\ & - c*f)*\text{PolyLog}[2, 1 - (2*d*(e + f*x))/((d*e + f - c*f)*(1 + c + d*x))])/(\\ & (d*e + f - c*f)^2*(d*e - (1 + c)*f)^2) \end{aligned}$$

Rule 31

$\text{Int}[(a + (b \cdot x)^{-1}), x_Symbol] \rightarrow \text{Simp}[\text{Log}[\text{RemoveContent}[a + b*x, x]]/b, x] \text{ ; FreeQ}\{a, b\}, x]$

Rule 633

$\text{Int}[(d + (e \cdot x))/(a + (c \cdot x)^2), x_Symbol] \rightarrow \text{With}\{q = \text{Rt}[-(a*c), 2]\}, \text{Dist}[e/2 + (c*d)/(2*q), \text{Int}[1/(-q + c*x), x], x] + \text{Dist}[e/2 - (c*d)/(2*q), \text{Int}[1/(q + c*x), x], x]] \text{ ; FreeQ}\{a, c, d, e\}, x \ \&\& \ \text{NiceSqrtQ}[-(a*c)]$

Rule 706

$\text{Int}[1/(((d + (e \cdot x)) \cdot ((a + (c \cdot x)^2))), x_Symbol] \rightarrow \text{Dist}[e^2/(c*d^2 + a*e^2), \text{Int}[1/(d + e*x), x], x] + \text{Dist}[1/(c*d^2 + a*e^2), \text{Int}[(c*d - c*e*x)/(a + c*x^2), x], x] \text{ ; FreeQ}\{a, c, d, e\}, x \ \&\& \ \text{NeQ}[c*d^2 + a*e^2, 0]$

Rule 709

$\text{Int}[(d + (e \cdot x))^m/(a + (b \cdot x) + (c \cdot x)^2), x_Symbol] \rightarrow \text{Simp}[(e*(d + e*x)^{(m+1)})/((m+1)*(c*d^2 - b*d*e + a*e^2)), x] + \text{Dist}[1/(c*d^2 - b*d*e + a*e^2), \text{Int}[(d + e*x)^{(m+1)}*\text{Simp}[c*d - b*e - c*e*x, x])/(a + b*x + c*x^2), x], x] \text{ ; FreeQ}\{a, b, c, d, e, m\}, x \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0] \ \&\& \ \text{NeQ}[c*d^2 - b*d*e + a*e^2, 0] \ \&\& \ \text{NeQ}[2*c*d - b*e, 0] \ \&\& \ \text{LtQ}[m, -1]$

Rule 710

$\text{Int}[(d + (e \cdot x))^m/(a + (c \cdot x)^2), x_Symbol] \rightarrow \text{Simp}[(e*(d + e*x)^{(m+1)})/((m+1)*(c*d^2 + a*e^2)), x] + \text{Dist}[c/(c*d^2 + a*e^2), \text{Int}[(d + e*x)^{(m+1)}*(d - e*x)/(a + c*x^2), x], x] \text{ ; FreeQ}\{a, c, d, e, m$

}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]

Rule 800

Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]

Rule 801

Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2), x_Symbol] := Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]

Rule 1982

Int[(u_)^(m_.)*(v_)^(p_.), x_Symbol] := Int[ExpandToSum[u, x]^m*ExpandToSum[v, x]^p, x] /; FreeQ[{m, p}, x] && LinearQ[u, x] && QuadraticQ[v, x] && ! (LinearMatchQ[u, x] && QuadraticMatchQ[v, x])

Rule 2315

Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] := -Simp[PolyLog[2, 1 - c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]

Rule 2402

Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] := -Dist[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]

Rule 2447

Int[Log[u_]*(Pq_)^(m_.), x_Symbol] := With[{C = FullSimplify[(Pq^m*(1 - u))/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x] /; IntegerQ[m] && PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u, x][[2]], Expon[Pq, x]]

Rule 5918

Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol] := -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)]/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0]

]

Rule 5920

```
Int[((a_.) + ArcTanh[(c_.)*(x_.)]*(b_.))/((d_.) + (e_.)*(x_.)), x_Symbol] := -
Simp[((a + b*ArcTanh[c*x])*Log[2/(1 + c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x))
/(c*d + e)*(1 + c*x)]/(1 - c^2*x^2), x], x] + Simp[((a + b*ArcTanh[c*x])*
Log[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e, x]) /; FreeQ[{a, b, c, d, e}
, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rule 5926

```
Int[((a_.) + ArcTanh[(c_.)*(x_.)]*(b_.))*((d_.) + (e_.)*(x_.))^(q_.), x_Symbol
] := Simp[((d + e*x)^(q + 1)*(a + b*ArcTanh[c*x]))/(e*(q + 1)), x] - Dist[(
b*c)/(e*(q + 1)), Int[(d + e*x)^(q + 1)/(1 - c^2*x^2), x], x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[q, -1]
```

Rule 6109

```
Int[((a_.) + ArcTanh[(c_.) + (d_.)*(x_.)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_.))^(
m_), x_Symbol] := Simp[((e + f*x)^(m + 1)*(a + b*ArcTanh[c + d*x])^p)/(f*(m
+ 1)), x] - Dist[(b*d*p)/(f*(m + 1)), Int[((e + f*x)^(m + 1)*(a + b*ArcTan
h[c + d*x])^(p - 1))/(1 - (c + d*x)^2), x], x] /; FreeQ[{a, b, c, d, e, f},
x] && IGtQ[p, 0] && ILtQ[m, -1]
```

Rule 6121

```
Int[((a_.) + ArcTanh[(c_.) + (d_.)*(x_.)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_.))^(
m_.)*((A_.) + (B_.)*(x_.) + (C_.)*(x_.)^2)^(q_.), x_Symbol] := Dist[1/d, Subs
t[Int[((d*e - c*f)/d + (f*x)/d)^m*(-(C/d^2) + (C*x^2)/d^2)^q*(a + b*ArcTanh
[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m, p, q},
x] && EqQ[B*(1 - c^2) + 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 6741

```
Int[u_, x_Symbol] := With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v !=
= u]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a + b \tanh^{-1}(c + dx))^2}{(e + fx)^3} dx &= -\frac{(a + b \tanh^{-1}(c + dx))^2}{2f(e + fx)^2} + \frac{(bd) \int \frac{a + b \tanh^{-1}(c + dx)}{(e + fx)^2(1 - (c + dx)^2)} dx}{f} \\
&= -\frac{(a + b \tanh^{-1}(c + dx))^2}{2f(e + fx)^2} + \frac{(bd) \int \frac{a + b \tanh^{-1}(c + dx)}{(e + fx)^2(1 - c^2 - 2cdx - d^2x^2)} dx}{f} \\
&= -\frac{(a + b \tanh^{-1}(c + dx))^2}{2f(e + fx)^2} + \frac{b \operatorname{Subst}\left(\int \frac{a + b \tanh^{-1}(x)}{\left(\frac{de - cf}{d} + \frac{fx}{d}\right)^2(1 - x^2)} dx, x, c + dx\right)}{f} \\
&= -\frac{(a + b \tanh^{-1}(c + dx))^2}{2f(e + fx)^2} + \frac{b \operatorname{Subst}\left(\int \left(-\frac{ad^2}{(de - cf + fx)^2(-1 + x^2)} - \frac{bd^2 \tanh^{-1}(x)}{(de - cf + fx)^2(-1 + x^2)}\right) dx, x, c + dx\right)}{f} \\
&= -\frac{(a + b \tanh^{-1}(c + dx))^2}{2f(e + fx)^2} - \frac{(abd^2) \operatorname{Subst}\left(\int \frac{1}{(de - cf + fx)^2(-1 + x^2)} dx, x, c + dx\right)}{f} - \frac{(b^2d^2) \operatorname{Subst}\left(\int \frac{\tanh^{-1}(x)}{-1 - x^2} dx, x, c + dx\right)}{f} \\
&= -\frac{abd}{(f^2 - (de - cf)^2)(e + fx)} - \frac{(a + b \tanh^{-1}(c + dx))^2}{2f(e + fx)^2} - \frac{(b^2d^2) \operatorname{Subst}\left(\int \frac{\tanh^{-1}(x)}{-1 - x^2} dx, x, c + dx\right)}{f} \\
&= -\frac{abd}{(f^2 - (de - cf)^2)(e + fx)} - \frac{(a + b \tanh^{-1}(c + dx))^2}{2f(e + fx)^2} - \frac{(b^2d^2) \operatorname{Subst}\left(\int \frac{\tanh^{-1}(x)}{-1 - x^2} dx, x, c + dx\right)}{f} \\
&= -\frac{abd}{(f^2 - (de - cf)^2)(e + fx)} + \frac{b^2d \tanh^{-1}(c + dx)}{(de + f - cf)(de - (1 + c)f)(e + fx)} - \frac{(a + b \tanh^{-1}(c + dx))^2}{2f(e + fx)^2} \\
&= -\frac{abd}{(f^2 - (de - cf)^2)(e + fx)} + \frac{b^2d \tanh^{-1}(c + dx)}{(de + f - cf)(de - (1 + c)f)(e + fx)} - \frac{(a + b \tanh^{-1}(c + dx))^2}{2f(e + fx)^2} \\
&= -\frac{abd}{(f^2 - (de - cf)^2)(e + fx)} + \frac{b^2d \tanh^{-1}(c + dx)}{(de + f - cf)(de - (1 + c)f)(e + fx)} - \frac{(a + b \tanh^{-1}(c + dx))^2}{2f(e + fx)^2} \\
&= -\frac{abd}{(f^2 - (de - cf)^2)(e + fx)} + \frac{b^2d \tanh^{-1}(c + dx)}{(de + f - cf)(de - (1 + c)f)(e + fx)} - \frac{(a + b \tanh^{-1}(c + dx))^2}{2f(e + fx)^2}
\end{aligned}$$

Mathematica [C] time = 14.79, size = 1968, normalized size = 2.62

result too large to display

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b*ArcTanh[c + d*x])^2/(e + f*x)^3,x]

[Out]
$$-1/2*a^2/(f*(e + f*x)^2) + (a*b*(d*e - c*f + f*(c + d*x))^3*((f*(2 + ((d*e + f - c*f)*(d*e - (1 + c)*f)))/((d*e - c*f)/\text{Sqrt}[1 - (c + d*x)^2] + (f*(c + d*x))/\text{Sqrt}[1 - (c + d*x)^2]))^2*\text{ArcTanh}[c + d*x])/((d*e + f - c*f)^2*(-(d*e) + f + c*f)^2) - ((c + d*x)*(f - 2*d*e*\text{ArcTanh}[c + d*x] + 2*c*f*\text{ArcTanh}[c + d*x]))/((d*e - c*f)*(d*e + f - c*f)*(d*e - (1 + c)*f)*\text{Sqrt}[1 - (c + d*x)^2]*((d*e - c*f)/\text{Sqrt}[1 - (c + d*x)^2] + (f*(c + d*x))/\text{Sqrt}[1 - (c + d*x)^2])) - (2*(d*e - c*f)*\text{Log}[(d*e)/\text{Sqrt}[1 - (c + d*x)^2] - (c*f)/\text{Sqrt}[1 - (c + d*x)^2] + (f*(c + d*x))/\text{Sqrt}[1 - (c + d*x)^2]])/(d^2*e^2 - 2*c*d*e*f + (-1 + c^2)*f^2)^2)/(d*(e + f*x)^3) + (b^2*(d*e - c*f + f*(c + d*x))^3*((f*(1 - (c + d*x)^2)^(3/2)*((d*e)/\text{Sqrt}[1 - (c + d*x)^2] - (c*f)/\text{Sqrt}[1 - (c + d*x)^2] + (f*(c + d*x))/\text{Sqrt}[1 - (c + d*x)^2]))^3*\text{ArcTanh}[c + d*x]^2)/(2*(d*e - f - c*f)*(d*e + f - c*f)*(d*e - c*f + f*(c + d*x))^3*(-((d*e)/\text{Sqrt}[1 - (c + d*x)^2]) + (c*f)/\text{Sqrt}[1 - (c + d*x)^2] - (f*(c + d*x))/\text{Sqrt}[1 - (c + d*x)^2]))^2) + ((1 - (c + d*x)^2)^(3/2)*((d*e)/\text{Sqrt}[1 - (c + d*x)^2] - (c*f)/\text{Sqrt}[1 - (c + d*x)^2] + (f*(c + d*x))/\text{Sqrt}[1 - (c + d*x)^2]))^3*((f*(c + d*x)*\text{ArcTanh}[c + d*x])/(\text{Sqrt}[1 - (c + d*x)^2] - (d*e*(c + d*x)*\text{ArcTanh}[c + d*x]^2)/\text{Sqrt}[1 - (c + d*x)^2] + (c*f*(c + d*x)*\text{ArcTanh}[c + d*x]^2)/\text{Sqrt}[1 - (c + d*x)^2])))/((d*e - c*f)*(d*e - f - c*f)*(d*e + f - c*f)*(d*e - c*f + f*(c + d*x))^3*(-((d*e)/\text{Sqrt}[1 - (c + d*x)^2]) + (c*f)/\text{Sqrt}[1 - (c + d*x)^2] - (f*(c + d*x))/\text{Sqrt}[1 - (c + d*x)^2])) + (f*(1 - (c + d*x)^2)^(3/2)*((d*e)/\text{Sqrt}[1 - (c + d*x)^2] - (c*f)/\text{Sqrt}[1 - (c + d*x)^2] + (f*(c + d*x))/\text{Sqrt}[1 - (c + d*x)^2]))^3*(-(f*\text{ArcTanh}[c + d*x]) + (d*e - c*f)*\text{Log}[(d*e - c*f)/\text{Sqrt}[1 - (c + d*x)^2] + (f*(c + d*x))/\text{Sqrt}[1 - (c + d*x)^2]]))/((d*e - c*f)*(d*e - f - c*f)*(d*e + f - c*f)*(-f^2 + (d*e - c*f)^2)*(d*e - c*f + f*(c + d*x))^3) + (c*(1 - (c + d*x)^2)^(3/2)*((d*e)/\text{Sqrt}[1 - (c + d*x)^2] - (c*f)/\text{Sqrt}[1 - (c + d*x)^2] + (f*(c + d*x))/\text{Sqrt}[1 - (c + d*x)^2]))^3*(\text{ArcTanh}[c + d*x]^2/E^\text{ArcTanh}[(d*e - c*f)/f] - (I*(d*e - c*f)*(-((-Pi + (2*I)*\text{ArcTanh}[(d*e - c*f)/f])*\text{ArcTanh}[c + d*x]) - 2*(I*\text{ArcTanh}[(d*e - c*f)/f] + I*\text{ArcTanh}[c + d*x])*\text{Log}[1 - E^((2*I)*(I*\text{ArcTanh}[(d*e - c*f)/f] + I*\text{ArcTanh}[c + d*x]))] - Pi*\text{Log}[1 + E^(2*\text{ArcTanh}[c + d*x])] + Pi*\text{Log}[1/\text{Sqrt}[1 - (c + d*x)^2]] + (2*I)*\text{ArcTanh}[(d*e - c*f)/f]*\text{Log}[I*\text{Sinh}[\text{ArcTanh}[(d*e - c*f)/f] + \text{ArcTanh}[c + d*x]]] + I*\text{PolyLog}[2, E^((2*I)*(I*\text{ArcTanh}[(d*e - c*f)/f] + I*\text{ArcTanh}[c + d*x]))])))/((f*\text{Sqrt}[1 - (d*e - c*f)^2/f^2])))/((d*e - c*f)*(d*e - f - c*f)*(d*e + f - c*f)*\text{Sqrt}[(f^2 - (d*e - c*f)^2)/f^2]*(d*e - c*f + f*(c + d*x))^3) - (d*e*(1 - (c + d*x)^2)^(3/2)*((d*e)/\text{Sqrt}[1 - (c + d*x)^2] - (c*f)/\text{Sqrt}[1 - (c + d*x)^2] + (f*(c + d*x))/\text{Sqrt}[1 - (c + d*x)^2]))^3*(\text{ArcTanh}[c + d*x]^2/E^\text{ArcTanh}[(d*e - c*f)/f] - (I*(d*e - c*f)*(-((-Pi + (2*I)*\text{ArcTanh}[(d*e - c*f)/f])*\text{Ar$$

$c \operatorname{Tanh}[c + d*x] - 2*(I*\operatorname{ArcTanh}[(d*e - c*f)/f] + I*\operatorname{ArcTanh}[c + d*x])* \operatorname{Log}[1 - E^{((2*I)*(I*\operatorname{ArcTanh}[(d*e - c*f)/f] + I*\operatorname{ArcTanh}[c + d*x]))}] - \operatorname{Pi}*\operatorname{Log}[1 + E^{(2*\operatorname{ArcTanh}[c + d*x])}] + \operatorname{Pi}*\operatorname{Log}[1/\operatorname{Sqrt}[1 - (c + d*x)^2]] + (2*I)*\operatorname{ArcTanh}[(d*e - c*f)/f]* \operatorname{Log}[I*\operatorname{Sinh}[\operatorname{ArcTanh}[(d*e - c*f)/f] + \operatorname{ArcTanh}[c + d*x]]] + I*\operatorname{PolyLog}[2, E^{((2*I)*(I*\operatorname{ArcTanh}[(d*e - c*f)/f] + I*\operatorname{ArcTanh}[c + d*x]))}]/(f*\operatorname{Sqrt}[1 - (d*e - c*f)^2/f^2])]/(f*(d*e - c*f)*(d*e - f - c*f)*(d*e + f - c*f)*\operatorname{Sqrt}[(f^2 - (d*e - c*f)^2)/f^2]*(d*e - c*f + f*(c + d*x))^3)]/(d*(e + f*x)^3)$

fricas [F] time = 0.55, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{b^2 \operatorname{artanh}(dx + c)^2 + 2ab \operatorname{artanh}(dx + c) + a^2}{f^3 x^3 + 3ef^2 x^2 + 3e^2 fx + e^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arctanh(d*x+c))^2/(f*x+e)^3,x, algorithm="fricas")`

[Out] `integral((b^2*arctanh(d*x + c)^2 + 2*a*b*arctanh(d*x + c) + a^2)/(f^3*x^3 + 3*e*f^2*x^2 + 3*e^2*f*x + e^3), x)`

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{artanh}(dx + c) + a)^2}{(fx + e)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arctanh(d*x+c))^2/(f*x+e)^3,x, algorithm="giac")`

[Out] `integrate((b*arctanh(d*x + c) + a)^2/(f*x + e)^3, x)`

maple [A] time = 0.09, size = 1428, normalized size = 1.90

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b*arctanh(d*x+c))^2/(f*x+e)^3,x)`

[Out] `1/2*d^2*a*b/f/(c*f-d*e+f)^2*ln(d*x+c+1)+d^2*b^2*f/(c*f-d*e-f)^2/(c*f-d*e+f)^2*ln((d*x+c)*f-c*f+d*e)-d^2*b^2/(c*f-d*e-f)/(c*f-d*e+f)/(2*c*f-2*d*e-2*f)*ln(d*x+c-1)+d^2*b^2/(c*f-d*e-f)/(c*f-d*e+f)/(2*c*f-2*d*e+2*f)*ln(d*x+c+1)-1/2*d^2*a^2/(d*f*x+d*e)^2/f+d^2*b^2*f/(c*f-d*e-f)^2/(c*f-d*e+f)^2*ln((d*x+c)*f-c*f+d*e)*ln(((d*x+c)*f-f)/(c*f-d*e-f))*c-d^2*b^2*f/(c*f-d*e-f)^2/(c*f-d*e+f)^2*ln((d*x+c)*f-c*f+d*e)*ln(((d*x+c)*f+f)/(c*f-d*e+f))*c+2*d^2*a*b*f/(c`

```

*f-d*e-f)^2/(c*f-d*e+f)^2*ln((d*x+c)*f-c*f+d*e)*c+2*d^2*b^2*f*arctanh(d*x+c
)/(c*f-d*e-f)^2/(c*f-d*e+f)^2*ln((d*x+c)*f-c*f+d*e)*c+1/4*d^2*b^2/f/(c*f-d*
e-f)^2*ln(d*x+c-1)*ln(1/2+1/2*d*x+1/2*c)-1/4*d^2*b^2/f/(c*f-d*e+f)^2*ln(-1/
2*d*x-1/2*c+1/2)*ln(1/2+1/2*d*x+1/2*c)+1/4*d^2*b^2/f/(c*f-d*e+f)^2*ln(-1/2*
d*x-1/2*c+1/2)*ln(d*x+c+1)-1/2*d^2*a*b/f/(c*f-d*e-f)^2*ln(d*x+c-1)+d^2*a*b/
(c*f-d*e-f)/(c*f-d*e+f)/(d*f*x+d*e)+d^3*b^2/(c*f-d*e-f)^2/(c*f-d*e+f)^2*dil
og(((d*x+c)*f+f)/(c*f-d*e+f))*e+1/4*d^2*b^2/f/(c*f-d*e-f)^2*dilog(1/2+1/2*d
*x+1/2*c)-1/8*d^2*b^2/f/(c*f-d*e+f)^2*ln(d*x+c+1)^2-1/4*d^2*b^2/f/(c*f-d*e+
f)^2*dilog(1/2+1/2*d*x+1/2*c)-1/8*d^2*b^2/f/(c*f-d*e-f)^2*ln(d*x+c-1)^2-d^3
*b^2/(c*f-d*e-f)^2/(c*f-d*e+f)^2*dilog(((d*x+c)*f-f)/(c*f-d*e-f))*e+1/2*d^2
*b^2/f*arctanh(d*x+c)/(c*f-d*e+f)^2*ln(d*x+c+1)+d^2*b^2*arctanh(d*x+c)/(c*f
-d*e-f)/(c*f-d*e+f)/(d*f*x+d*e)-d^2*a*b/(d*f*x+d*e)^2/f*arctanh(d*x+c)-1/2*
d^2*b^2/f*arctanh(d*x+c)/(c*f-d*e-f)^2*ln(d*x+c-1)-1/2*d^2*b^2/(d*f*x+d*e)^
2/f*arctanh(d*x+c)^2-d^2*b^2*f/(c*f-d*e-f)^2/(c*f-d*e+f)^2*dilog(((d*x+c)*f
+f)/(c*f-d*e+f))*c-2*d^3*a*b/(c*f-d*e-f)^2/(c*f-d*e+f)^2*ln((d*x+c)*f-c*f+d
*e)*e-d^3*b^2/(c*f-d*e-f)^2/(c*f-d*e+f)^2*ln((d*x+c)*f-c*f+d*e)*ln(((d*x+c)
*f-f)/(c*f-d*e-f))*e+d^3*b^2/(c*f-d*e-f)^2/(c*f-d*e+f)^2*ln((d*x+c)*f-c*f+d
*e)*ln(((d*x+c)*f+f)/(c*f-d*e+f))*e-2*d^3*b^2*arctanh(d*x+c)/(c*f-d*e-f)^2/
(c*f-d*e+f)^2*ln((d*x+c)*f-c*f+d*e)*e+d^2*b^2*f/(c*f-d*e-f)^2/(c*f-d*e+f)^2
*dilog(((d*x+c)*f-f)/(c*f-d*e-f))*c

```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2} \left(d \left(\frac{d \log(dx + c + 1)}{d^2 e^2 f - 2(c + 1)def^2 + (c^2 + 2c + 1)f^3} - \frac{d \log(dx + c - 1)}{d^2 e^2 f - 2(c - 1)def^2 + (c^2 - 2c + 1)f^3} - \frac{1}{d^4 e^4 - 4cd^3 e^3 f + 2} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arctanh(d*x+c))^2/(f*x+e)^3,x, algorithm="maxima")

```

[Out] 1/2*(d*(d*log(d*x + c + 1)/(d^2*e^2*f - 2*(c + 1)*d*e*f^2 + (c^2 + 2*c + 1)
*f^3) - d*log(d*x + c - 1)/(d^2*e^2*f - 2*(c - 1)*d*e*f^2 + (c^2 - 2*c + 1)
*f^3) - 4*(d^2*e - c*d*f)*log(f*x + e)/(d^4*e^4 - 4*c*d^3*e^3*f + 2*(3*c^2
- 1)*d^2*e^2*f^2 - 4*(c^3 - c)*d*e*f^3 + (c^4 - 2*c^2 + 1)*f^4) + 2/(d^2*e^
3 - 2*c*d*e^2*f + (c^2 - 1)*e*f^2 + (d^2*e^2*f - 2*c*d*e*f^2 + (c^2 - 1)*f^
3)*x)) - 2*arctanh(d*x + c)/(f^3*x^2 + 2*e*f^2*x + e^2*f))*a*b - 1/8*b^2*(l
og(-d*x - c + 1)^2/(f^3*x^2 + 2*e*f^2*x + e^2*f) + 2*integrate(-((d*f*x + c
*f - f)*log(d*x + c + 1)^2 + (d*f*x + d*e - 2*(d*f*x + c*f - f)*log(d*x + c
+ 1))*log(-d*x - c + 1))/(d*f^4*x^4 + c*e^3*f - e^3*f + (3*d*e*f^3 + c*f^4
- f^4)*x^3 + 3*(d*e^2*f^2 + c*e*f^3 - e*f^3)*x^2 + (d*e^3*f + 3*c*e^2*f^2
- 3*e^2*f^2)*x), x)) - 1/2*a^2/(f^3*x^2 + 2*e*f^2*x + e^2*f)

```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + b \operatorname{atanh}(c + dx))^2}{(e + fx)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b*atanh(c + d*x))^2/(e + f*x)^3,x)`

[Out] `int((a + b*atanh(c + d*x))^2/(e + f*x)^3, x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{atanh}(c + dx))^2}{(e + fx)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*atanh(d*x+c))**2/(f*x+e)**3,x)`

[Out] `Integral((a + b*atanh(c + d*x))**2/(e + f*x)**3, x)`

3.45 $\int (e + fx)^2 \left(a + b \tanh^{-1}(c + dx) \right)^3 dx$

Optimal. Leaf size=546

$$\frac{b^2 \left((3c^2 + 1) f^2 - 6cdef + 3d^2 e^2 \right) \operatorname{Li}_2 \left(1 - \frac{2}{-c-dx+1} \right) \left(a + b \tanh^{-1}(c + dx) \right) - 6b^2 f (de - cf) \log \left(\frac{2}{-c-dx+1} \right) \left(a + b \tanh^{-1}(c + dx) \right)}{d^3}$$

[Out] $a*b^2*f^2*x/d^2 + b^3*f^2*(d*x+c)*\operatorname{arctanh}(d*x+c)/d^3 - 1/2*b*f^2*(a+b*\operatorname{arctanh}(d*x+c))^2/d^3 + 3*b*f*(-c*f+d*e)*(a+b*\operatorname{arctanh}(d*x+c))^2/d^3 + 3*b*f*(-c*f+d*e)*(d*x+c)*(a+b*\operatorname{arctanh}(d*x+c))^2/d^3 + 1/2*b*f^2*(d*x+c)^2*(a+b*\operatorname{arctanh}(d*x+c))^2/d^3 - 1/3*(-c*f+d*e)*(d^2*e^2 - 2*c*d*e*f + (c^2+3)*f^2)*(a+b*\operatorname{arctanh}(d*x+c))^3/d^3 + f + 1/3*(3*d^2*e^2 - 6*c*d*e*f + (3*c^2+1)*f^2)*(a+b*\operatorname{arctanh}(d*x+c))^3/d^3 + 1/3*(f*x+e)^3*(a+b*\operatorname{arctanh}(d*x+c))^3/f - 6*b^2*f*(-c*f+d*e)*(a+b*\operatorname{arctanh}(d*x+c))*\ln(2/(-d*x-c+1))/d^3 - b*(3*d^2*e^2 - 6*c*d*e*f + (3*c^2+1)*f^2)*(a+b*\operatorname{arctanh}(d*x+c))^2*\ln(2/(-d*x-c+1))/d^3 + 1/2*b^3*f^2*\ln(1-(d*x+c)^2)/d^3 - 3*b^3*f*(-c*f+d*e)*\operatorname{polylog}(2, (-d*x-c-1)/(-d*x-c+1))/d^3 - b^2*(3*d^2*e^2 - 6*c*d*e*f + (3*c^2+1)*f^2)*(a+b*\operatorname{arctanh}(d*x+c))*\operatorname{polylog}(2, 1-2/(-d*x-c+1))/d^3 + 1/2*b^3*(3*d^2*e^2 - 6*c*d*e*f + (3*c^2+1)*f^2)*\operatorname{polylog}(3, 1-2/(-d*x-c+1))/d^3$

Rubi [A] time = 1.05, antiderivative size = 546, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 14, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.700$, Rules used = {6111, 5928, 5910, 5984, 5918, 2402, 2315, 5916, 5980, 260, 5948, 6048, 6058, 6610}

$$\frac{b^2 \left((3c^2 + 1) f^2 - 6cdef + 3d^2 e^2 \right) \operatorname{PolyLog} \left(2, 1 - \frac{2}{-c-dx+1} \right) \left(a + b \tanh^{-1}(c + dx) \right) - b^3 \left((3c^2 + 1) f^2 - 6cdef + 3d^2 e^2 \right) \operatorname{PolyLog} \left(3, 1 - \frac{2}{-c-dx+1} \right) \left(a + b \tanh^{-1}(c + dx) \right)}{d^3}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(e + f*x)^2*(a + b*\operatorname{ArcTanh}[c + d*x])^3, x]$

[Out] $(a*b^2*f^2*x)/d^2 + (b^3*f^2*(c + d*x)*\operatorname{ArcTanh}[c + d*x])/d^3 - (b*f^2*(a + b*\operatorname{ArcTanh}[c + d*x])^2)/(2*d^3) + (3*b*f*(d*e - c*f)*(a + b*\operatorname{ArcTanh}[c + d*x])^2)/d^3 + (3*b*f*(d*e - c*f)*(c + d*x)*(a + b*\operatorname{ArcTanh}[c + d*x])^2)/d^3 + (b*f^2*(c + d*x)^2*(a + b*\operatorname{ArcTanh}[c + d*x])^2)/(2*d^3) - ((d*e - c*f)*(d^2*e^2 - 2*c*d*e*f + (3 + c^2)*f^2)*(a + b*\operatorname{ArcTanh}[c + d*x])^3)/(3*d^3*f) + ((3*d^2*e^2 - 6*c*d*e*f + (1 + 3*c^2)*f^2)*(a + b*\operatorname{ArcTanh}[c + d*x])^3)/(3*d^3) + ((e + f*x)^3*(a + b*\operatorname{ArcTanh}[c + d*x])^3)/(3*f) - (6*b^2*f*(d*e - c*f)*(a + b*\operatorname{ArcTanh}[c + d*x])*Log[2/(1 - c - d*x)])/d^3 - (b*(3*d^2*e^2 - 6*c*d*e*f + (1 + 3*c^2)*f^2)*(a + b*\operatorname{ArcTanh}[c + d*x])^2*Log[2/(1 - c - d*x)])/d^3 + (b^3*f^2*Log[1 - (c + d*x)^2])/(2*d^3) - (3*b^3*f*(d*e - c*f)*PolyLog[2, -((1 + c + d*x)/(1 - c - d*x))])/d^3 - (b^2*(3*d^2*e^2 - 6*c*d*e*f + (1 + 3*c^2)*f^2)*(a + b*\operatorname{ArcTanh}[c + d*x])*PolyLog[2, 1 - 2/(1 - c - d*x)])/d^3 + ($

$$b^3(3d^2e^2 - 6cd*ef + (1 + 3c^2)f^2)*PolyLog[3, 1 - 2/(1 - c - d*x)]/(2*d^3)$$

Rule 260

$$\text{Int}[(x_)^{(m_.)}/((a_) + (b_)*(x_)^{(n_)}), x_Symbol] \text{ :> } \text{Simp}[\text{Log}[\text{RemoveContent}[a + b*x^n, x]]/(b*n), x] \text{ /; } \text{FreeQ}[\{a, b, m, n\}, x] \ \&\& \ \text{EqQ}[m, n - 1]$$

Rule 2315

$$\text{Int}[\text{Log}[(c_)*(x_)]/((d_) + (e_)*(x_)), x_Symbol] \text{ :> } -\text{Simp}[\text{PolyLog}[2, 1 - c*x]/e, x] \text{ /; } \text{FreeQ}[\{c, d, e\}, x] \ \&\& \ \text{EqQ}[e + c*d, 0]$$

Rule 2402

$$\text{Int}[\text{Log}[(c_)/((d_) + (e_)*(x_))]/((f_) + (g_)*(x_)^2), x_Symbol] \text{ :> } -\text{Dist}[e/g, \text{Subst}[\text{Int}[\text{Log}[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] \text{ /; } \text{FreeQ}[\{c, d, e, f, g\}, x] \ \&\& \ \text{EqQ}[c, 2*d] \ \&\& \ \text{EqQ}[e^2*f + d^2*g, 0]$$

Rule 5910

$$\text{Int}[(a_ + \text{ArcTanh}[(c_)*(x_)]*(b_))^{(p_.)}, x_Symbol] \text{ :> } \text{Simp}[x*(a + b*\text{ArcTanh}[c*x])^p, x] - \text{Dist}[b*c*p, \text{Int}[(x*(a + b*\text{ArcTanh}[c*x])^{(p - 1)})/(1 - c^2*x^2), x], x] \text{ /; } \text{FreeQ}[\{a, b, c\}, x] \ \&\& \ \text{IGtQ}[p, 0]$$

Rule 5916

$$\text{Int}[(a_ + \text{ArcTanh}[(c_)*(x_)]*(b_))^{(p_.)}*((d_)*(x_))^{(m_.)}, x_Symbol] \text{ :> } \text{Simp}[(d*x)^{(m + 1)}*(a + b*\text{ArcTanh}[c*x])^p/(d*(m + 1)), x] - \text{Dist}[(b*c*p)/(d*(m + 1)), \text{Int}[(d*x)^{(m + 1)}*(a + b*\text{ArcTanh}[c*x])^{(p - 1)})/(1 - c^2*x^2), x], x] \text{ /; } \text{FreeQ}[\{a, b, c, d, m\}, x] \ \&\& \ \text{IGtQ}[p, 0] \ \&\& \ (\text{EqQ}[p, 1] \ || \ \text{IntegerQ}[m]) \ \&\& \ \text{NeQ}[m, -1]$$

Rule 5918

$$\text{Int}[(a_ + \text{ArcTanh}[(c_)*(x_)]*(b_))^{(p_.)}/((d_) + (e_)*(x_)), x_Symbol] \text{ :> } -\text{Simp}[(a + b*\text{ArcTanh}[c*x])^p*\text{Log}[2/(1 + (e*x)/d)]/e, x] + \text{Dist}[(b*c*p)/e, \text{Int}[(a + b*\text{ArcTanh}[c*x])^{(p - 1)}*\text{Log}[2/(1 + (e*x)/d)]/(1 - c^2*x^2), x], x] \text{ /; } \text{FreeQ}[\{a, b, c, d, e\}, x] \ \&\& \ \text{IGtQ}[p, 0] \ \&\& \ \text{EqQ}[c^2*d^2 - e^2, 0]$$

Rule 5928

$$\text{Int}[(a_ + \text{ArcTanh}[(c_)*(x_)]*(b_))^{(p_.)}*((d_) + (e_)*(x_))^{(q_.)}, x_Symbol] \text{ :> } \text{Simp}[(d + e*x)^{(q + 1)}*(a + b*\text{ArcTanh}[c*x])^p/(e*(q + 1)), x] - \text{Dist}[(b*c*p)/(e*(q + 1)), \text{Int}[\text{ExpandIntegrand}[(a + b*\text{ArcTanh}[c*x])^{(p - 1)}]]$$

, (d + e*x)^(q + 1)/(1 - c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]

Rule 5948

Int[((a_.) + ArcTanh[(c_.)*(x_.)]*(b_.))^(p_.)/((d_.) + (e_.)*(x_)^2), x_Symbol] := Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]

Rule 5980

Int[((a_.) + ArcTanh[(c_.)*(x_.)]*(b_.))^(p_.)*((f_.)*(x_)^(m_.))/((d_.) + (e_.)*(x_)^2), x_Symbol] := Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x])^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTanh[c*x])^p)/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]

Rule 5984

Int[((a_.) + ArcTanh[(c_.)*(x_.)]*(b_.))^(p_.)*(x_)/((d_.) + (e_.)*(x_)^2), x_Symbol] := Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]

Rule 6048

Int[((a_.) + ArcTanh[(c_.)*(x_.)]*(b_.))^(p_.)*((f_.) + (g_.)*(x_)^(m_.))/((d_.) + (e_.)*(x_)^2), x_Symbol] := Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p/(d + e*x^2), (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGtQ[p, 0] && EqQ[c^2*d + e, 0] && IGtQ[m, 0]

Rule 6058

Int[(Log[u]*((a_.) + ArcTanh[(c_.)*(x_.)]*(b_.))^(p_.))/((d_.) + (e_.)*(x_)^2), x_Symbol] := -Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d), x] + Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]

Rule 6111

Int[((a_.) + ArcTanh[(c_.) + (d_.)*(x_.)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_)^(m_.)), x_Symbol] := Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*ArcTanh[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGt

Q[p, 0]

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :=> With[{w = DerivativeDivides[v, u*v,  
x]}, Simp[w*PolyLog[n + 1, v], x] /; !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\begin{aligned}
\int (e + fx)^2 (a + b \tanh^{-1}(c + dx))^3 dx &= \frac{\text{Subst}\left(\int \left(\frac{de-cf}{d} + \frac{fx}{d}\right)^2 (a + b \tanh^{-1}(x))^3 dx, x, c + dx\right)}{d} \\
&= \frac{(e + fx)^3 (a + b \tanh^{-1}(c + dx))^3}{3f} - \frac{b \text{Subst}\left(\int \left(-\frac{3f^2(de-cf)(a+b \tanh^{-1}(x))^3}{d^3}\right) dx, x, c + dx\right)}{d^3} \\
&= \frac{(e + fx)^3 (a + b \tanh^{-1}(c + dx))^3}{3f} - \frac{b \text{Subst}\left(\int \frac{((de-cf)(d^2e^2 - 2cdef + 3f^2 + c^2d^2)) (a+b \tanh^{-1}(x))^3}{d^3} dx, x, c + dx\right)}{d^3} \\
&= \frac{3bf(de - cf)(c + dx) (a + b \tanh^{-1}(c + dx))^2}{d^3} + \frac{bf^2(c + dx)^2 (a + b \tanh^{-1}(c + dx))^2}{2d^3} \\
&= \frac{3bf(de - cf) (a + b \tanh^{-1}(c + dx))^2}{d^3} + \frac{3bf(de - cf)(c + dx) (a + b \tanh^{-1}(c + dx))^2}{d^3} \\
&= \frac{ab^2 f^2 x}{d^2} - \frac{bf^2 (a + b \tanh^{-1}(c + dx))^2}{2d^3} + \frac{3bf(de - cf) (a + b \tanh^{-1}(c + dx))^2}{d^3} \\
&= \frac{ab^2 f^2 x}{d^2} + \frac{b^3 f^2 (c + dx) \tanh^{-1}(c + dx)}{d^3} - \frac{bf^2 (a + b \tanh^{-1}(c + dx))^2}{2d^3} \\
&= \frac{ab^2 f^2 x}{d^2} + \frac{b^3 f^2 (c + dx) \tanh^{-1}(c + dx)}{d^3} - \frac{bf^2 (a + b \tanh^{-1}(c + dx))^2}{2d^3} \\
&= \frac{ab^2 f^2 x}{d^2} + \frac{b^3 f^2 (c + dx) \tanh^{-1}(c + dx)}{d^3} - \frac{bf^2 (a + b \tanh^{-1}(c + dx))^2}{2d^3}
\end{aligned}$$

Mathematica [B] time = 10.16, size = 1868, normalized size = 3.42

result too large to display

Warning: Unable to verify antiderivative.

[In] Integrate[(e + f*x)^2*(a + b*ArcTanh[c + d*x])^3,x]

```
[Out] (a^2*(a*d^2*e^2 + 3*b*d*e*f - 2*b*c*f^2)*x)/d^2 + (a^2*f*(2*a*d*e + b*f)*x^
2)/(2*d) + (a^3*f^2*x^3)/3 + a^2*b*x*(3*e^2 + 3*e*f*x + f^2*x^2)*ArcTanh[c
+ d*x] + ((3*a^2*b*d^2*e^2 - 3*a^2*b*c*d^2*e^2 + 3*a^2*b*d*e*f - 6*a^2*b*c*
d*e*f + 3*a^2*b*c^2*d*e*f + a^2*b*f^2 - 3*a^2*b*c*f^2 + 3*a^2*b*c^2*f^2 - a
^2*b*c^3*f^2)*Log[1 - c - d*x])/(2*d^3) + ((3*a^2*b*d^2*e^2 + 3*a^2*b*c*d^2
*e^2 - 3*a^2*b*d*e*f - 6*a^2*b*c*d*e*f - 3*a^2*b*c^2*d*e*f + a^2*b*f^2 + 3*
a^2*b*c*f^2 + 3*a^2*b*c^2*f^2 + a^2*b*c^3*f^2)*Log[1 + c + d*x])/(2*d^3) +
(3*a*b^2*e^2*(ArcTanh[c + d*x]*(-ArcTanh[c + d*x] + (c + d*x)*ArcTanh[c + d
*x] - 2*Log[1 + E^(-2*ArcTanh[c + d*x])])) + PolyLog[2, -E^(-2*ArcTanh[c + d
*x])]))/d - (3*a*b^2*e*f*((1 - (c + d*x)^2)*ArcTanh[c + d*x]^2 + 2*(-((c +
d*x)*ArcTanh[c + d*x]) - c*ArcTanh[c + d*x]^2 + c*(c + d*x)*ArcTanh[c + d*x
]^2 - 2*c*ArcTanh[c + d*x]*Log[1 + E^(-2*ArcTanh[c + d*x])]) + Log[1/Sqrt[1
- (c + d*x)^2]]) + 2*c*PolyLog[2, -E^(-2*ArcTanh[c + d*x])]))/d^2 + (b^3*e^
2*(ArcTanh[c + d*x]^2*(-ArcTanh[c + d*x] + (c + d*x)*ArcTanh[c + d*x] - 3*L
og[1 + E^(-2*ArcTanh[c + d*x])]) + 3*ArcTanh[c + d*x]*PolyLog[2, -E^(-2*Arc
Tanh[c + d*x])]) + (3*PolyLog[3, -E^(-2*ArcTanh[c + d*x])])/(2))/d + (b^3*e*f
*(-(ArcTanh[c + d*x]*(3*ArcTanh[c + d*x] - 2*c*ArcTanh[c + d*x]^2 + (1 - (c
+ d*x)^2)*ArcTanh[c + d*x]^2 + (c + d*x)*ArcTanh[c + d*x]*(-3 + 2*c*ArcTan
h[c + d*x]) + 6*Log[1 + E^(-2*ArcTanh[c + d*x])]) - 6*c*ArcTanh[c + d*x]*Log
[1 + E^(-2*ArcTanh[c + d*x])])) + (3 - 6*c*ArcTanh[c + d*x])*PolyLog[2, -E^
(-2*ArcTanh[c + d*x])]) - 3*c*PolyLog[3, -E^(-2*ArcTanh[c + d*x])]))/d^2 - (
a*b^2*f^2*(1 - (c + d*x)^2)^(3/2)*(-(c + d*x)/Sqrt[1 - (c + d*x)^2]) + (6*
c*(c + d*x)*ArcTanh[c + d*x])/Sqrt[1 - (c + d*x)^2] + (3*(c + d*x)*ArcTanh[
c + d*x]^2)/Sqrt[1 - (c + d*x)^2] - (3*c^2*(c + d*x)*ArcTanh[c + d*x]^2)/Sq
rt[1 - (c + d*x)^2] + ArcTanh[c + d*x]^2*Cosh[3*ArcTanh[c + d*x]] + 3*c^2*A
rcTanh[c + d*x]^2*Cosh[3*ArcTanh[c + d*x]] + 2*ArcTanh[c + d*x]*Cosh[3*ArcT
anh[c + d*x]]*Log[1 + E^(-2*ArcTanh[c + d*x])]) + 6*c^2*ArcTanh[c + d*x]*Cos
h[3*ArcTanh[c + d*x]]*Log[1 + E^(-2*ArcTanh[c + d*x])]) - 6*c*Cosh[3*ArcTan
h[c + d*x]]*Log[1/Sqrt[1 - (c + d*x)^2]] + (ArcTanh[c + d*x]*(4 + 3*(1 - 4*c
+ 3*c^2)*ArcTanh[c + d*x]) + 6*(ArcTanh[c + d*x] + 3*c^2*ArcTanh[c + d*x])
*Log[1 + E^(-2*ArcTanh[c + d*x])]) - 18*c*Log[1/Sqrt[1 - (c + d*x)^2]])/Sqrt
[1 - (c + d*x)^2] - (4*(1 + 3*c^2)*PolyLog[2, -E^(-2*ArcTanh[c + d*x])])/(1
- (c + d*x)^2)^(3/2) - Sinh[3*ArcTanh[c + d*x]] + 6*c*ArcTanh[c + d*x]*Sin
h[3*ArcTanh[c + d*x]] - ArcTanh[c + d*x]^2*Sinh[3*ArcTanh[c + d*x]] - 3*c^2
*ArcTanh[c + d*x]^2*Sinh[3*ArcTanh[c + d*x]])/(4*d^3) + (b^3*f^2*((-3*c +
ArcTanh[c + d*x] + 3*c^2*ArcTanh[c + d*x])*PolyLog[2, -E^(-2*ArcTanh[c + d*
x])]) - ((1 - (c + d*x)^2)^(3/2))*((-3*(c + d*x)*ArcTanh[c + d*x])/Sqrt[1 - (
c + d*x)^2] + (9*c*(c + d*x)*ArcTanh[c + d*x]^2)/Sqrt[1 - (c + d*x)^2] + (3
*(c + d*x)*ArcTanh[c + d*x]^3)/Sqrt[1 - (c + d*x)^2] - (3*c^2*(c + d*x)*Arc
Tanh[c + d*x]^3)/Sqrt[1 - (c + d*x)^2] - 9*c*ArcTanh[c + d*x]^2*Cosh[3*ArcT
anh[c + d*x]] + ArcTanh[c + d*x]^3*Cosh[3*ArcTanh[c + d*x]] + 3*c^2*ArcTanh
[c + d*x]^3*Cosh[3*ArcTanh[c + d*x]] - 18*c*ArcTanh[c + d*x]*Cosh[3*ArcTan
h[c + d*x]]*Log[1 + E^(-2*ArcTanh[c + d*x])]) + 3*ArcTanh[c + d*x]^2*Cosh[3*A
rcTanh[c + d*x]]*Log[1 + E^(-2*ArcTanh[c + d*x])]) + 9*c^2*ArcTanh[c + d*x]^
2*Cosh[3*ArcTanh[c + d*x]]*Log[1 + E^(-2*ArcTanh[c + d*x])]) + 3*Cosh[3*ArcT
```

$$\operatorname{anh}[c + d*x]] * \operatorname{Log}[1/\operatorname{Sqrt}[1 - (c + d*x)^2]] + (3 * (\operatorname{ArcTanh}[c + d*x]^2 * (2 - 9 * c + \operatorname{ArcTanh}[c + d*x] - 4 * c * \operatorname{ArcTanh}[c + d*x] + 3 * c^2 * \operatorname{ArcTanh}[c + d*x]) + 3 * \operatorname{ArcTanh}[c + d*x] * (-6 * c + \operatorname{ArcTanh}[c + d*x] + 3 * c^2 * \operatorname{ArcTanh}[c + d*x]) * \operatorname{Log}[1 + E^{(-2 * \operatorname{ArcTanh}[c + d*x])}] + 3 * \operatorname{Log}[1/\operatorname{Sqrt}[1 - (c + d*x)^2]])) / \operatorname{Sqrt}[1 - (c + d*x)^2] - (6 * (1 + 3 * c^2) * \operatorname{PolyLog}[3, -E^{(-2 * \operatorname{ArcTanh}[c + d*x])}]) / (1 - (c + d*x)^2)^{(3/2)} - 3 * \operatorname{ArcTanh}[c + d*x] * \operatorname{Sinh}[3 * \operatorname{ArcTanh}[c + d*x]] + 9 * c * \operatorname{ArcTanh}[c + d*x]^2 * \operatorname{Sinh}[3 * \operatorname{ArcTanh}[c + d*x]] - \operatorname{ArcTanh}[c + d*x]^3 * \operatorname{Sinh}[3 * \operatorname{ArcTanh}[c + d*x]] - 3 * c^2 * \operatorname{ArcTanh}[c + d*x]^3 * \operatorname{Sinh}[3 * \operatorname{ArcTanh}[c + d*x]])) / 12) / d^3$$

fricas [F] time = 0.71, size = 0, normalized size = 0.00

$$\operatorname{integral}(a^3 f^2 x^2 + 2 a^3 e f x + a^3 e^2 + (b^3 f^2 x^2 + 2 b^3 e f x + b^3 e^2) \operatorname{artanh}(dx + c)^3 + 3 (ab^2 f^2 x^2 + 2 ab^2 e f x + ab^2 e^2) \operatorname{artanh}(dx + c)^2, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x+e)^2*(a+b*arctanh(d*x+c))^3,x, algorithm="fricas")`

[Out] `integral(a^3*f^2*x^2 + 2*a^3*e*f*x + a^3*e^2 + (b^3*f^2*x^2 + 2*b^3*e*f*x + b^3*e^2)*arctanh(d*x + c)^3 + 3*(a*b^2*f^2*x^2 + 2*a*b^2*e*f*x + a*b^2*e^2)*arctanh(d*x + c)^2 + 3*(a^2*b*f^2*x^2 + 2*a^2*b*e*f*x + a^2*b*e^2)*arctanh(d*x + c), x)`

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x+e)^2*(a+b*arctanh(d*x+c))^3,x, algorithm="giac")`

[Out] Timed out

maple [C] time = 22.27, size = 12111, normalized size = 22.18

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((f*x+e)^2*(a+b*arctanh(d*x+c))^3,x)`

[Out] result too large to display

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x+e)^2*(a+b*arctanh(d*x+c))^3,x, algorithm="maxima")

[Out] $\frac{1}{3}a^3f^2x^3 + a^3e^2fx^2 + \frac{3}{2}(2x^2\operatorname{arctanh}(dx+c) + d(2x/d^2 - (c^2 + 2c + 1)\log(dx+c+1)/d^3 + (c^2 - 2c + 1)\log(dx+c-1)/d^3))a^2b^2ef + \frac{1}{2}(2x^3\operatorname{arctanh}(dx+c) + d((dx^2 - 4cx)/d^3 + (c^3 + 3c^2 + 3c + 1)\log(dx+c+1)/d^4 - (c^3 - 3c^2 + 3c - 1)\log(dx+c-1)/d^4))a^2b^2f^2 + a^3e^2x + \frac{3}{2}(2(dx+c)\operatorname{arctanh}(dx+c) + \log(-(dx+c)^2 + 1))a^2b^2e^2/d - \frac{1}{24}((b^3d^3f^2x^3 + 3b^3d^3e^2fx^2 + 3b^3d^3e^2x + (c^3f^2 - 3d^2e^2 - 3(d^2ef + f^2)c^2 - 3d^2ef + 3(d^2e^2 + 2d^2ef + f^2)c - f^2)b^3)\log(-dx-c+1)^3 - 3(2ab^2d^3f^2x^3 + (6ab^2d^3ef + b^3d^2f^2)x^2 + 2(3ab^2d^3e^2 + (3d^2ef - 2cd^2f^2)b^3)x + (b^3d^3f^2x^3 + 3b^3d^3e^2fx^2 + 3b^3d^3e^2x + (c^3f^2 + 3d^2e^2 - 3(d^2ef - f^2)c^2 - 3d^2ef + 3(d^2e^2 - 2d^2ef + f^2)c + f^2)b^3)\log(dx+c+1))\log(-dx-c+1)^2)/d^3 - \operatorname{integrate}(-1/8((b^3d^3f^2x^3 + (2d^3ef + cd^2f^2 - d^2f^2)b^3x^2 + (d^3e^2 + 2cd^2ef - 2d^2e^2f)b^3x + (cd^2e^2 - d^2e^2)b^3)\log(dx+c+1)^3 + 6(ab^2d^3f^2x^3 + (2d^3ef + cd^2f^2 - d^2f^2)ab^2x^2 + (d^3e^2 + 2cd^2ef - 2d^2e^2f)ab^2x + (cd^2e^2 - d^2e^2)ab^2)\log(dx+c+1)^2 - (4ab^2d^3f^2x^3 + 2(6ab^2d^3ef + b^3d^2f^2)x^2 + 3(b^3d^3f^2x^3 + (2d^3ef + cd^2f^2 - d^2f^2)b^3x^2 + (d^3e^2 + 2cd^2ef - 2d^2e^2f)b^3x + (cd^2e^2 - d^2e^2)b^3)\log(dx+c+1)^2 + 4(3ab^2d^3e^2 + (3d^2ef - 2cd^2f^2)b^3)x + 2(6(cd^2e^2 - d^2e^2)ab^2 + (c^3f^2 + 3d^2e^2 - 3(d^2ef - f^2)c^2 - 3d^2ef + 3(d^2e^2 - 2d^2ef + f^2)c + f^2)b^3 + (6ab^2d^3f^2 + b^3d^3f^2)x^3 + 3(b^3d^3e^2 + 2(2d^3ef + cd^2f^2 - d^2f^2)ab^2)x^2 + 3(b^3d^3e^2 + 2(d^3e^2 + 2cd^2ef - 2d^2e^2f)ab^2)x)\log(dx+c+1))\log(-dx-c+1))/(d^3x + cd^2 - d^2), x)$

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (e + fx)^2 (a + b \operatorname{atanh}(c + dx))^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e + f*x)^2*(a + b*atanh(c + d*x))^3,x)

[Out] int((e + f*x)^2*(a + b*atanh(c + d*x))^3, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \operatorname{atanh}(c + dx))^3 (e + fx)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x+e)**2*(a+b*atanh(d*x+c))**3,x)

[Out] Integral((a + b*atanh(c + d*x))**3*(e + f*x)**2, x)

3.46 $\int (e + fx) \left(a + b \tanh^{-1}(c + dx) \right)^3 dx$

Optimal. Leaf size=326

$$\frac{3b^2(de - cf)\text{Li}_2\left(1 - \frac{2}{-c-dx+1}\right)(a + b \tanh^{-1}(c + dx))}{d^2} - \frac{3b^2 f \log\left(\frac{2}{-c-dx+1}\right)(a + b \tanh^{-1}(c + dx))}{d^2} - \frac{((c^2 + 1) f^2 -$$

[Out] $3/2*b*f*(a+b*\text{arctanh}(d*x+c))^2/d^2+3/2*b*f*(d*x+c)*(a+b*\text{arctanh}(d*x+c))^2/d^2+(-c*f+d*e)*(a+b*\text{arctanh}(d*x+c))^3/d^2-1/2*(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)*(a+b*\text{arctanh}(d*x+c))^3/d^2/f+1/2*(f*x+e)^2*(a+b*\text{arctanh}(d*x+c))^3/f-3*b^2*f*(a+b*\text{arctanh}(d*x+c))*\ln(2/(-d*x-c+1))/d^2-3*b*(-c*f+d*e)*(a+b*\text{arctanh}(d*x+c))^2*\ln(2/(-d*x-c+1))/d^2-3/2*b^3*f*\text{polylog}(2,(-d*x-c-1)/(-d*x-c+1))/d^2-3*b^2*(-c*f+d*e)*(a+b*\text{arctanh}(d*x+c))*\text{polylog}(2,1-2/(-d*x-c+1))/d^2+3/2*b^3*(-c*f+d*e)*\text{polylog}(3,1-2/(-d*x-c+1))/d^2$

Rubi [A] time = 0.73, antiderivative size = 325, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 11, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.611$, Rules used = {6111, 5928, 5910, 5984, 5918, 2402, 2315, 6048, 5948, 6058, 6610}

$$\frac{3b^2(de - cf)\text{PolyLog}\left(2, 1 - \frac{2}{-c-dx+1}\right)(a + b \tanh^{-1}(c + dx))}{d^2} + \frac{3b^3(de - cf)\text{PolyLog}\left(3, 1 - \frac{2}{-c-dx+1}\right)}{2d^2} - \frac{3b^3 f \text{PolyLog}\left(2, 1 - \frac{2}{-c-dx+1}\right)}{d^2}$$

Antiderivative was successfully verified.

[In] `Int[(e + f*x)*(a + b*ArcTanh[c + d*x])^3,x]`

[Out] $(3*b*f*(a + b*\text{ArcTanh}[c + d*x])^2)/(2*d^2) + (3*b*f*(c + d*x)*(a + b*\text{ArcTanh}[c + d*x])^2)/(2*d^2) + ((d*e - c*f)*(a + b*\text{ArcTanh}[c + d*x])^3)/d^2 + ((2*c*e - (d*e^2)/f - ((1 + c^2)*f)/d)*(a + b*\text{ArcTanh}[c + d*x])^3)/(2*d) + ((e + f*x)^2*(a + b*\text{ArcTanh}[c + d*x])^3)/(2*f) - (3*b^2*f*(a + b*\text{ArcTanh}[c + d*x]))*\text{Log}[2/(1 - c - d*x)]/d^2 - (3*b*(d*e - c*f)*(a + b*\text{ArcTanh}[c + d*x]))^2*\text{Log}[2/(1 - c - d*x)]/d^2 - (3*b^3*f*\text{PolyLog}[2, -((1 + c + d*x)/(1 - c - d*x))])/(2*d^2) - (3*b^2*(d*e - c*f)*(a + b*\text{ArcTanh}[c + d*x]))*\text{PolyLog}[2, 1 - 2/(1 - c - d*x)]/d^2 + (3*b^3*(d*e - c*f)*\text{PolyLog}[3, 1 - 2/(1 - c - d*x)])/(2*d^2)$

Rule 2315

`Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 - c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]`

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] := -Dist[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^p, x_Symbol] := Simp[x*(a + b*ArcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^p/((d_) + (e_.)*(x_)), x_Symbol] := -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/((1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0]
```

Rule 5928

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^p*((d_) + (e_.)*(x_)^q), x_Symbol] := Simp[((d + e*x)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(e*(q + 1)), x] - Dist[(b*c*p)/(e*(q + 1)), Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^(p - 1), (d + e*x)^(q + 1)/(1 - c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^p/((d_) + (e_.)*(x_)^2), x_Symbol] := Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 5984

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^p*(x_))/((d_) + (e_.)*(x_)^2), x_Symbol] := Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 6048

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^p*((f_) + (g_.)*(x_)^m)/((d_) + (e_.)*(x_)^2), x_Symbol] := Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p/(d + e*x^2), (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && I
```

GtQ[p, 0] && EqQ[c^2*d + e, 0] && IGtQ[m, 0]

Rule 6058

Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)])*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2), x_Symbol] := -Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d), x] + Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]

Rule 6111

Int[((a_.) + ArcTanh[(c_) + (d_.)*(x_)])*(b_.))^(p_.)*((e_.) + (f_.)*(x_)^(m_.)), x_Symbol] := Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*ArcTanh[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]

Rule 6610

Int[(u_)*PolyLog[n_, v_], x_Symbol] := With[{w = DerivativeDivides[v, u*v, x]}, Simp[w*PolyLog[n + 1, v], x] /; !FalseQ[w]] /; FreeQ[n, x]

Rubi steps

$$\begin{aligned}
\int (e + fx) (a + b \tanh^{-1}(c + dx))^3 dx &= \frac{\text{Subst} \left(\int \left(\frac{de - cf}{d} + \frac{fx}{d} \right) (a + b \tanh^{-1}(x))^3 dx, x, c + dx \right)}{d} \\
&= \frac{(e + fx)^2 (a + b \tanh^{-1}(c + dx))^3}{2f} - \frac{(3b) \text{Subst} \left(\int \left(-\frac{f^2 (a + b \tanh^{-1}(x))^2}{d^2} + \right) \right)}{2d} \\
&= \frac{(e + fx)^2 (a + b \tanh^{-1}(c + dx))^3}{2f} - \frac{(3b) \text{Subst} \left(\int \frac{(d^2 e^2 - 2cdef + (1 + c^2) f^2 + 2)}{1} \right)}{2d} \\
&= \frac{3bf(c + dx) (a + b \tanh^{-1}(c + dx))^2}{2d^2} + \frac{(e + fx)^2 (a + b \tanh^{-1}(c + dx))^2}{2f} \\
&= \frac{3bf (a + b \tanh^{-1}(c + dx))^2}{2d^2} + \frac{3bf(c + dx) (a + b \tanh^{-1}(c + dx))^2}{2d^2} + \\
&= \frac{3bf (a + b \tanh^{-1}(c + dx))^2}{2d^2} + \frac{3bf(c + dx) (a + b \tanh^{-1}(c + dx))^2}{2d^2} + \\
&= \frac{3bf (a + b \tanh^{-1}(c + dx))^2}{2d^2} + \frac{3bf(c + dx) (a + b \tanh^{-1}(c + dx))^2}{2d^2} + \\
&= \frac{3bf (a + b \tanh^{-1}(c + dx))^2}{2d^2} + \frac{3bf(c + dx) (a + b \tanh^{-1}(c + dx))^2}{2d^2} + \\
&= \frac{3bf (a + b \tanh^{-1}(c + dx))^2}{2d^2} + \frac{3bf(c + dx) (a + b \tanh^{-1}(c + dx))^2}{2d^2} + \\
&= \frac{3bf (a + b \tanh^{-1}(c + dx))^2}{2d^2} + \frac{3bf(c + dx) (a + b \tanh^{-1}(c + dx))^2}{2d^2} +
\end{aligned}$$

Mathematica [A] time = 1.15, size = 566, normalized size = 1.74

$$2a^3 f(c + dx)^2 + 2a^2(c + dx)(-2acf + 2ade + 3bf) + 3a^2b(-2cf + 2de + f) \log(-c - dx + 1) + 3a^2b(2de - (2c +$$

Warning: Unable to verify antiderivative.

[In] Integrate[(e + f*x)*(a + b*ArcTanh[c + d*x])^3,x]

[Out] $(2*a^2*(2*a*d*e + 3*b*f - 2*a*c*f)*(c + d*x) + 2*a^3*f*(c + d*x)^2 - 6*a^2*b*(c + d*x)*(c*f - d*(2*e + f*x))*\text{ArcTanh}[c + d*x] + 3*a^2*b*(2*d*e + f - 2*c*f)*\text{Log}[1 - c - d*x] + 3*a^2*b*(2*d*e - (1 + 2*c)*f)*\text{Log}[1 + c + d*x] + 12*a*b^2*f*((c + d*x)*\text{ArcTanh}[c + d*x] - ((1 - (c + d*x)^2)*\text{ArcTanh}[c + d*x]^2)/2 - \text{Log}[1/\text{Sqrt}[1 - (c + d*x)^2]]) + 12*a*b^2*d*e*(\text{ArcTanh}[c + d*x]*((-1 + c + d*x)*\text{ArcTanh}[c + d*x] - 2*\text{Log}[1 + E^{(-2*\text{ArcTanh}[c + d*x])}])) + \text{PolyLog}[2, -E^{(-2*\text{ArcTanh}[c + d*x])}] - 12*a*b^2*c*f*(\text{ArcTanh}[c + d*x]*((-1 + c + d*x)*\text{ArcTanh}[c + d*x] - 2*\text{Log}[1 + E^{(-2*\text{ArcTanh}[c + d*x])}])) + \text{PolyLog}[2, -E^{(-2*\text{ArcTanh}[c + d*x])}] + 2*b^3*f*(\text{ArcTanh}[c + d*x]*(3*(-1 + c + d*x)*\text{ArcTanh}[c + d*x] + (-1 + c^2 + 2*c*d*x + d^2*x^2)*\text{ArcTanh}[c + d*x]^2 - 6*\text{Log}[1 + E^{(-2*\text{ArcTanh}[c + d*x])}])) + 3*\text{PolyLog}[2, -E^{(-2*\text{ArcTanh}[c + d*x])}] + 4*b^3*d*e*(\text{ArcTanh}[c + d*x]^2*((-1 + c + d*x)*\text{ArcTanh}[c + d*x] - 3*\text{Log}[1 + E^{(-2*\text{ArcTanh}[c + d*x])}])) + 3*\text{ArcTanh}[c + d*x]*\text{PolyLog}[2, -E^{(-2*\text{ArcTanh}[c + d*x])}] + (3*\text{PolyLog}[3, -E^{(-2*\text{ArcTanh}[c + d*x])}]))/2) - 4*b^3*c*f*(\text{ArcTanh}[c + d*x]^2*((-1 + c + d*x)*\text{ArcTanh}[c + d*x] - 3*\text{Log}[1 + E^{(-2*\text{ArcTanh}[c + d*x])}])) + 3*\text{ArcTanh}[c + d*x]*\text{PolyLog}[2, -E^{(-2*\text{ArcTanh}[c + d*x])}] + (3*\text{PolyLog}[3, -E^{(-2*\text{ArcTanh}[c + d*x])}]))/2)/(4*d^2)$

fricas [F] time = 0.66, size = 0, normalized size = 0.00

integral $(a^3fx + a^3e + (b^3fx + b^3e) \operatorname{artanh}(dx + c)^3 + 3(ab^2fx + ab^2e) \operatorname{artanh}(dx + c)^2 + 3(a^2bfx + a^2be) \operatorname{artanh}(dx + c) + a^3e)$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x+e)*(a+b*arctanh(d*x+c))^3,x, algorithm="fricas")

[Out] integral $(a^3f*x + a^3e + (b^3f*x + b^3e)*\operatorname{arctanh}(d*x + c)^3 + 3*(a*b^2*f*x + a*b^2*e)*\operatorname{arctanh}(d*x + c)^2 + 3*(a^2*b*f*x + a^2*b*e)*\operatorname{arctanh}(d*x + c), x)$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (fx + e)(b \operatorname{artanh}(dx + c) + a)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x+e)*(a+b*arctanh(d*x+c))^3,x, algorithm="giac")

[Out] integrate((f*x + e)*(b*arctanh(d*x + c) + a)^3, x)

maple [C] time = 1.68, size = 20255, normalized size = 62.13

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f*x+e)*(a+b*arctanh(d*x+c))^3,x)

[Out] result too large to display

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2} a^3 f x^2 + \frac{3}{4} \left(2x^2 \operatorname{artanh}(dx + c) + d \left(\frac{2x}{d^2} - \frac{(c^2 + 2c + 1) \log(dx + c + 1)}{d^3} + \frac{(c^2 - 2c + 1) \log(dx + c - 1)}{d^3} \right) \right) a^2 b$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x+e)*(a+b*arctanh(d*x+c))^3,x, algorithm="maxima")

[Out] $\frac{1}{2} a^3 f x^2 + \frac{3}{4} (2x^2 \operatorname{arctanh}(dx + c) + d(2x/d^2 - (c^2 + 2c + 1) \log(dx + c + 1)/d^3 + (c^2 - 2c + 1) \log(dx + c - 1)/d^3)) a^2 b f + a^3 e x + \frac{3}{2} (2(d*x + c) \operatorname{arctanh}(d*x + c) + \log(-(d*x + c)^2 + 1)) a^2 b e/d - 1/16 * ((b^3 d^2 f x^2 + 2 b^3 d^2 e x - (c^2 f - 2(d e + f) c + 2 d e + f) b^3) \log(-d*x - c + 1)^3 - 3(2 a b^2 d^2 f x^2 + 2(2 a b^2 d^2 e + b^3 d f) x + (b^3 d^2 f x^2 + 2 b^3 d^2 e x - (c^2 f - 2(d e - f) c - 2 d e + f) b^3) \log(d*x + c + 1)) \log(-d*x - c + 1)^2 / d^2 - \operatorname{integrate}(-1/8 * ((b^3 d^2 f x^2 + (d^2 e + c d f - d f) b^3 x + (c d e - d e) b^3) \log(d*x + c + 1)^3 + 6(a b^2 d^2 f x^2 + (d^2 e + c d f - d f) a b^2 x + (c d e - d e) a b^2) \log(d*x + c + 1)^2 - 3(2 a b^2 d^2 f x^2 + (b^3 d^2 f x^2 + (d^2 e + c d f - d f) b^3 x + (c d e - d e) b^3) \log(d*x + c + 1)^2 + 2(2 a b^2 d^2 e + b^3 d f) x + (4(c d e - d e) a b^2 - (c^2 f - 2(d e - f) c - 2 d e + f) b^3 + (4 a b^2 d^2 f + b^3 d^2 f) x^2 + 2(b^3 d^2 e + 2(d^2 e + c d f - d f) a b^2) x) \log(d*x + c + 1)) \log(-d*x - c + 1)) / (d^2 x + c d - d), x)$

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (e + f x) (a + b \operatorname{atanh}(c + d x))^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e + f*x)*(a + b*atanh(c + d*x))^3,x)

[Out] int((e + f*x)*(a + b*atanh(c + d*x))^3, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \operatorname{atanh}(c + d x))^3 (e + f x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x+e)*(a+b*atanh(d*x+c))**3,x)

[Out] Integral((a + b*atanh(c + d*x))**3*(e + f*x), x)

3.47 $\int (a + b \tanh^{-1}(c + dx))^3 dx$

Optimal. Leaf size=132

$$\frac{3b^2 \operatorname{Li}_2\left(1 - \frac{2}{-c-dx+1}\right) (a + b \tanh^{-1}(c + dx))}{d} + \frac{(c + dx) (a + b \tanh^{-1}(c + dx))^3}{d} + \frac{(a + b \tanh^{-1}(c + dx))^3}{d} - \frac{3b \log\left(\frac{2}{-c-dx+1}\right) (a + b \tanh^{-1}(c + dx))^2}{d} + \frac{3b^3 \operatorname{PolyLog}\left(3, 1 - \frac{2}{-c-dx+1}\right) (a + b \tanh^{-1}(c + dx))}{2d}$$

[Out] (a+b*arctanh(d*x+c))^3/d+(d*x+c)*(a+b*arctanh(d*x+c))^3/d-3*b*(a+b*arctanh(d*x+c))^2*ln(2/(-d*x-c+1))/d-3*b^2*(a+b*arctanh(d*x+c))*polylog(2,1-2/(-d*x-c+1))/d+3/2*b^3*polylog(3,1-2/(-d*x-c+1))/d

Rubi [A] time = 0.23, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$, Rules used = {6103, 5910, 5984, 5918, 5948, 6058, 6610}

$$\frac{3b^2 \operatorname{PolyLog}\left(2, 1 - \frac{2}{-c-dx+1}\right) (a + b \tanh^{-1}(c + dx))}{d} + \frac{3b^3 \operatorname{PolyLog}\left(3, 1 - \frac{2}{-c-dx+1}\right) (a + b \tanh^{-1}(c + dx))}{2d} + \frac{(c + dx) (a + b \tanh^{-1}(c + dx))^3}{d}$$

Antiderivative was successfully verified.

[In] Int[(a + b*ArcTanh[c + d*x])^3, x]

[Out] (a + b*ArcTanh[c + d*x])^3/d + ((c + d*x)*(a + b*ArcTanh[c + d*x])^3)/d - (3*b*(a + b*ArcTanh[c + d*x])^2*Log[2/(1 - c - d*x)]/d - (3*b^2*(a + b*ArcTanh[c + d*x])*PolyLog[2, 1 - 2/(1 - c - d*x)]/d + (3*b^3*PolyLog[3, 1 - 2/(1 - c - d*x)])/(2*d))

Rule 5910

Int[((a_.) + ArcTanh[(c_.)*(x_.)]*(b_.))^p, x_Symbol] := Simp[x*(a + b*ArcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p-1))/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]

Rule 5918

Int[((a_.) + ArcTanh[(c_.)*(x_.)]*(b_.))^p/((d_.) + (e_.)*(x_.)), x_Symbol] := -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)]/e, x] + Dist[(b*c*p)/e, Int[((a + b*ArcTanh[c*x])^(p-1)*Log[2/(1 + (e*x)/d)]/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0]

Rule 5948

Int[((a_.) + ArcTanh[(c_.)*(x_.)]*(b_.))^p/((d_.) + (e_.)*(x_.)^2), x_Symbol] := Simp[(a + b*ArcTanh[c*x])^(p+1)/(b*c*d*(p+1)), x] /; FreeQ[{a, b

, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]

Rule 5984

Int[((a_.) + ArcTanh[(c_.)*(x_.)]*(b_.))^(p_.)*(x_.)/((d_.) + (e_.)*(x_.)^2), x_Symbol] := Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]

Rule 6058

Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_.)]*(b_.))^(p_.))/((d_.) + (e_.)*(x_.)^2), x_Symbol] := -Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d), x] + Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]

Rule 6103

Int[((a_.) + ArcTanh[(c_.) + (d_.)*(x_.)]*(b_.))^(p_.), x_Symbol] := Dist[1/d, Subst[Int[(a + b*ArcTanh[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[p, 0]

Rule 6610

Int[(u_)*PolyLog[n_, v_], x_Symbol] := With[{w = DerivativeDivides[v, u*v, x]}, Simp[w*PolyLog[n + 1, v], x] /; !FalseQ[w] /; FreeQ[n, x]

Rubi steps

$$\begin{aligned}
\int (a + b \tanh^{-1}(c + dx))^3 dx &= \frac{\text{Subst}\left(\int (a + b \tanh^{-1}(x))^3 dx, x, c + dx\right)}{d} \\
&= \frac{(c + dx)(a + b \tanh^{-1}(c + dx))^3}{d} - \frac{(3b) \text{Subst}\left(\int \frac{x(a + b \tanh^{-1}(x))^2}{1-x^2} dx, x, c + dx\right)}{d} \\
&= \frac{(a + b \tanh^{-1}(c + dx))^3}{d} + \frac{(c + dx)(a + b \tanh^{-1}(c + dx))^3}{d} - \frac{(3b) \text{Subst}\left(\int \frac{(a+b)}{1-x^2} dx, x, c + dx\right)}{d} \\
&= \frac{(a + b \tanh^{-1}(c + dx))^3}{d} + \frac{(c + dx)(a + b \tanh^{-1}(c + dx))^3}{d} - \frac{3b(a + b \tanh^{-1}(c + dx))}{d} \\
&= \frac{(a + b \tanh^{-1}(c + dx))^3}{d} + \frac{(c + dx)(a + b \tanh^{-1}(c + dx))^3}{d} - \frac{3b(a + b \tanh^{-1}(c + dx))}{d} \\
&= \frac{(a + b \tanh^{-1}(c + dx))^3}{d} + \frac{(c + dx)(a + b \tanh^{-1}(c + dx))^3}{d} - \frac{3b(a + b \tanh^{-1}(c + dx))}{d}
\end{aligned}$$

Mathematica [A] time = 0.27, size = 205, normalized size = 1.55

$$2a^3 dx - 3a^2 b(c-1) \log(-c-dx+1) + 3a^2 b(c+1) \log(c+dx+1) + 6a^2 b dx \tanh^{-1}(c+dx) + 6ab^2 \left(\text{Li}_2 \left(-e^{-2 \tanh^{-1}(c+dx)} \right) \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b*ArcTanh[c + d*x])^3, x]

[Out] (2*a^3*d*x + 6*a^2*b*d*x*ArcTanh[c + d*x] - 3*a^2*b*(-1 + c)*Log[1 - c - d*x] + 3*a^2*b*(1 + c)*Log[1 + c + d*x] + 6*a*b^2*(ArcTanh[c + d*x]*((-1 + c + d*x)*ArcTanh[c + d*x] - 2*Log[1 + E^(-2*ArcTanh[c + d*x])]) + PolyLog[2, -E^(-2*ArcTanh[c + d*x])]) + 2*b^3*(ArcTanh[c + d*x]^2*((-1 + c + d*x)*ArcTanh[c + d*x] - 3*Log[1 + E^(-2*ArcTanh[c + d*x])]) + 3*ArcTanh[c + d*x]*PolyLog[2, -E^(-2*ArcTanh[c + d*x])]) + (3*PolyLog[3, -E^(-2*ArcTanh[c + d*x])])/(2*d)))/(2*d)

fricas [F] time = 0.52, size = 0, normalized size = 0.00

$$\text{integral}(b^3 \text{artanh}(dx + c)^3 + 3ab^2 \text{artanh}(dx + c)^2 + 3a^2b \text{artanh}(dx + c) + a^3, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arctanh(d*x+c))^3,x, algorithm="fricas")

[Out] integral(b^3*arctanh(d*x + c)^3 + 3*a*b^2*arctanh(d*x + c)^2 + 3*a^2*b*arctanh(d*x + c) + a^3, x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \operatorname{artanh}(dx + c) + a)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arctanh(d*x+c))^3,x, algorithm="giac")

[Out] integrate((b*arctanh(d*x + c) + a)^3, x)

maple [B] time = 0.29, size = 346, normalized size = 2.62

$$a^3x + \frac{a^3c}{d} + \operatorname{arctanh}(dx + c)^3 x b^3 + \frac{\operatorname{arctanh}(dx + c)^3 b^3 c}{d} + \frac{b^3 \operatorname{arctanh}(dx + c)^3}{d} - \frac{3b^3 \operatorname{arctanh}(dx + c)^2 \ln\left(1 + \frac{d}{1-dx-c}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*arctanh(d*x+c))^3,x)

[Out] a^3*x+1/d*a^3*c+arctanh(d*x+c)^3*x*b^3+1/d*arctanh(d*x+c)^3*b^3*c+1/d*b^3*a*arctanh(d*x+c)^3-3/d*b^3*arctanh(d*x+c)^2*ln(1+(d*x+c+1)^2/(1-(d*x+c)^2))-3/d*b^3*arctanh(d*x+c)*polylog(2,-(d*x+c+1)^2/(1-(d*x+c)^2))+3/2/d*b^3*polylog(3,-(d*x+c+1)^2/(1-(d*x+c)^2))+3*arctanh(d*x+c)^2*x*a*b^2+3/d*arctanh(d*x+c)^2*a*b^2*c+3/d*a*b^2*arctanh(d*x+c)^2-6/d*arctanh(d*x+c)*ln(1+(d*x+c+1)^2/(1-(d*x+c)^2))*a*b^2-3/d*polylog(2,-(d*x+c+1)^2/(1-(d*x+c)^2))*a*b^2+3*arctanh(d*x+c)*x*a^2*b+3/d*arctanh(d*x+c)*a^2*b*c+3/2/d*a^2*b*ln(1-(d*x+c)^2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$a^3x + \frac{3\left(2(dx + c) \operatorname{artanh}(dx + c) + \log\left(-\left(dx + c\right)^2 + 1\right)\right)a^2b}{2d} - \frac{\left(b^3dx + b^3(c - 1)\right) \log(-dx - c + 1)^3 - 3\left(2ab^2d\right)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arctanh(d*x+c))^3,x, algorithm="maxima")

[Out] a^3*x + 3/2*(2*(d*x + c)*arctanh(d*x + c) + log(-(d*x + c)^2 + 1))*a^2*b/d - 1/8*((b^3*d*x + b^3*(c - 1))*log(-d*x - c + 1)^3 - 3*(2*a*b^2*d*x + (b^3*d*x + b^3*(c + 1))*log(d*x + c + 1))*log(-d*x - c + 1)^2)/d - integrate(-1/

```
8*((b^3*d*x + b^3*(c - 1))*log(d*x + c + 1)^3 + 6*(a*b^2*d*x + a*b^2*(c - 1))
)*log(d*x + c + 1)^2 - 3*(4*a*b^2*d*x + (b^3*d*x + b^3*(c - 1))*log(d*x +
c + 1)^2 + 2*(b^3*(c + 1) + 2*a*b^2*(c - 1) + (2*a*b^2*d + b^3*d)*x)*log(d*
x + c + 1))*log(-d*x - c + 1))/(d*x + c - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (a + b \operatorname{atanh}(c + dx))^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c + d*x))^3, x)
```

```
[Out] int((a + b*atanh(c + d*x))^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \operatorname{atanh}(c + dx))^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(d*x+c))**3, x)
```

```
[Out] Integral((a + b*atanh(c + d*x))**3, x)
```


$$3.48 \quad \int \frac{(a+b \tanh^{-1}(c+dx))^3}{e+fx} dx$$

Optimal. Leaf size=308

$$\frac{3b^2 (a + b \tanh^{-1}(c + dx)) \operatorname{Li}_3\left(1 - \frac{2d(e+fx)}{(de-cf+f)(c+dx+1)}\right)}{2f} + \frac{3b^2 \operatorname{Li}_3\left(1 - \frac{2}{c+dx+1}\right) (a + b \tanh^{-1}(c + dx))}{2f} - \frac{3b (a + b \tanh^{-1}(c + dx))}{2f}$$

```
[Out] -(a+b*arctanh(d*x+c))^3*ln(2/(d*x+c+1))/f+(a+b*arctanh(d*x+c))^3*ln(2*d*(f*x+e)/(-c*f+d*e+f)/(d*x+c+1))/f+3/2*b*(a+b*arctanh(d*x+c))^2*polylog(2,1-2/(d*x+c+1))/f-3/2*b*(a+b*arctanh(d*x+c))^2*polylog(2,1-2*d*(f*x+e)/(-c*f+d*e+f)/(d*x+c+1))/f+3/2*b^2*(a+b*arctanh(d*x+c))*polylog(3,1-2/(d*x+c+1))/f-3/2*b^2*(a+b*arctanh(d*x+c))*polylog(3,1-2*d*(f*x+e)/(-c*f+d*e+f)/(d*x+c+1))/f+3/4*b^3*polylog(4,1-2/(d*x+c+1))/f-3/4*b^3*polylog(4,1-2*d*(f*x+e)/(-c*f+d*e+f)/(d*x+c+1))/f
```

Rubi [A] time = 0.19, antiderivative size = 308, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {6111, 5924}

$$\frac{3b^2 (a + b \tanh^{-1}(c + dx)) \operatorname{PolyLog}\left(3, 1 - \frac{2d(e+fx)}{(c+dx+1)(-cf+de+f)}\right)}{2f} + \frac{3b^2 \operatorname{PolyLog}\left(3, 1 - \frac{2}{c+dx+1}\right) (a + b \tanh^{-1}(c + dx))}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c + d*x])^3/(e + f*x), x]
```

```
[Out] -(((a + b*ArcTanh[c + d*x])^3*Log[2/(1 + c + d*x)])/f) + ((a + b*ArcTanh[c + d*x])^3*Log[(2*d*(e + f*x))/((d*e + f - c*f)*(1 + c + d*x))])/f + (3*b*(a + b*ArcTanh[c + d*x])^2*PolyLog[2, 1 - 2/(1 + c + d*x)])/(2*f) - (3*b*(a + b*ArcTanh[c + d*x])^2*PolyLog[2, 1 - (2*d*(e + f*x))/((d*e + f - c*f)*(1 + c + d*x))])/((2*f) + (3*b^2*(a + b*ArcTanh[c + d*x])*PolyLog[3, 1 - 2/(1 + c + d*x)])/(2*f) - (3*b^2*(a + b*ArcTanh[c + d*x])*PolyLog[3, 1 - (2*d*(e + f*x))/((d*e + f - c*f)*(1 + c + d*x))])/((2*f) + (3*b^3*PolyLog[4, 1 - 2/(1 + c + d*x)])/(4*f) - (3*b^3*PolyLog[4, 1 - (2*d*(e + f*x))/((d*e + f - c*f)*(1 + c + d*x))])/((4*f)
```

Rule 5924

```
Int[((a_.) + ArcTanh[(c_.)*(x_)])*(b_.)^3/((d_.) + (e_.)*(x_)), x_Symbol] := -Simp[((a + b*ArcTanh[c*x])^3*Log[2/(1 + c*x)]/e, x] + (Simp[(a + b*ArcTanh[c*x])^3*Log[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))]/e, x] + Simp[(3*b*(a + b*ArcTanh[c*x])^2*PolyLog[2, 1 - 2/(1 + c*x)]/(2*e), x] - Simp[(3*b*(a + b*ArcTanh[c*x])^2*PolyLog[2, 1 - (2*c*(d + e*x))/((c*d + e)*(1 + c*x))])
```

/(2*e), x] + Simp[(3*b^2*(a + b*ArcTanh[c*x])*PolyLog[3, 1 - 2/(1 + c*x)])/(2*e), x] - Simp[(3*b^2*(a + b*ArcTanh[c*x])*PolyLog[3, 1 - (2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/(2*e), x] + Simp[(3*b^3*PolyLog[4, 1 - 2/(1 + c*x)])/(4*e), x] - Simp[(3*b^3*PolyLog[4, 1 - (2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/(4*e), x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[c^2*d^2 - e^2, 0]

Rule 6111

Int[((a_.) + ArcTanh[(c_) + (d_.)*(x_)])*(b_.))^p_.*((e_.) + (f_.)*(x_))^(m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*ArcTanh[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]

Rubi steps

$$\int \frac{(a + b \tanh^{-1}(c + dx))^3}{e + fx} dx = \frac{\text{Subst}\left(\int \frac{(a+b \tanh^{-1}(x))^3}{\frac{de-cf}{d} + \frac{fx}{d}} dx, x, c + dx\right)}{d}$$

$$= -\frac{(a + b \tanh^{-1}(c + dx))^3 \log\left(\frac{2}{1+c+dx}\right)}{f} + \frac{(a + b \tanh^{-1}(c + dx))^3 \log\left(\frac{2d(e+f)}{(de+f-cf)(1+c+dx)}\right)}{f}$$

Mathematica [F] time = 22.91, size = 0, normalized size = 0.00

$$\int \frac{(a + b \tanh^{-1}(c + dx))^3}{e + fx} dx$$

Verification is Not applicable to the result.

[In] Integrate[(a + b*ArcTanh[c + d*x])^3/(e + f*x), x]

[Out] Integrate[(a + b*ArcTanh[c + d*x])^3/(e + f*x), x]

fricas [F] time = 0.57, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^3 \operatorname{artanh}(dx + c)^3 + 3 ab^2 \operatorname{artanh}(dx + c)^2 + 3 a^2 b \operatorname{artanh}(dx + c) + a^3}{fx + e}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arctanh(d*x+c))^3/(f*x+e),x, algorithm="fricas")

[Out] integral((b^3*arctanh(d*x + c)^3 + 3*a*b^2*arctanh(d*x + c)^2 + 3*a^2*b*arctanh(d*x + c) + a^3)/(f*x + e), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arctanh}(dx + c) + a)^3}{fx + e} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arctanh(d*x+c))^3/(f*x+e),x, algorithm="giac")

[Out] integrate((b*arctanh(d*x + c) + a)^3/(f*x + e), x)

maple [C] time = 0.87, size = 4064, normalized size = 13.19

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*arctanh(d*x+c))^3/(f*x+e),x)

[Out]
$$\begin{aligned} & -3*a*b^2/f*\operatorname{arctanh}(d*x+c)*\operatorname{polylog}(2, -(d*x+c+1)^2/(1-(d*x+c)^2)) - 3*a*b^2/(c*f-d*e-f)*\operatorname{arctanh}(d*x+c)^2*\ln(1-(c*f-d*e-f)*(d*x+c+1)^2/(1-(d*x+c)^2)/(-c*f+d*e-f)) \\ & - 3*a*b^2/(c*f-d*e-f)*\operatorname{arctanh}(d*x+c)*\operatorname{polylog}(2, (c*f-d*e-f)*(d*x+c+1)^2/(1-(d*x+c)^2)/(-c*f+d*e-f)) \\ & + 3*a^2*b*\ln((d*x+c)*f-c*f+d*e)/f*\operatorname{arctanh}(d*x+c) + 3/2*a^2*b/f*\ln((d*x+c)*f-c*f+d*e)*\ln(((d*x+c)*f-f)/(c*f-d*e-f)) \\ & - 3/2*a^2*b/f*\ln((d*x+c)*f-c*f+d*e)*\ln(((d*x+c)*f+f)/(c*f-d*e-f)) + b^3*c/(c*f-d*e-f)*\operatorname{arctanh}(d*x+c)^3*\ln(1-(c*f-d*e-f)*(d*x+c+1)^2/(1-(d*x+c)^2)/(-c*f+d*e-f)) \\ & + 3/2*b^3*c/(c*f-d*e-f)*\operatorname{arctanh}(d*x+c)^2*\operatorname{polylog}(2, (c*f-d*e-f)*(d*x+c+1)^2/(1-(d*x+c)^2)/(-c*f+d*e-f)) \\ & - 3/2*b^3*c/(c*f-d*e-f)*\operatorname{arctanh}(d*x+c)*\operatorname{polylog}(3, (c*f-d*e-f)*(d*x+c+1)^2/(1-(d*x+c)^2)/(-c*f+d*e-f)) \\ & - 3/2*a*b^2*c/(c*f-d*e-f)*\operatorname{polylog}(3, (c*f-d*e-f)*(d*x+c+1)^2/(1-(d*x+c)^2)/(-c*f+d*e-f)) \\ & + 3*a*b^2*\ln((d*x+c)*f-c*f+d*e)/f*\operatorname{arctanh}(d*x+c)^2 - I*b^3/f*\operatorname{Pi}*\operatorname{arctanh}(d*x+c)^3 - b^3/(c*f-d*e-f)*\operatorname{arctanh}(d*x+c)^3*\ln(1-(c*f-d*e-f)*(d*x+c+1)^2/(1-(d*x+c)^2)/(-c*f+d*e-f)) \\ & + 3/2*a*b^2/f*\operatorname{polylog}(3, -(d*x+c+1)^2/(1-(d*x+c)^2)) + 3/2*a*b^2/(c*f-d*e-f)*\operatorname{polylog}(3, (c*f-d*e-f)*(d*x+c+1)^2/(1-(d*x+c)^2)/(-c*f+d*e-f)) \\ & - 3/4*b^3/f*\operatorname{polylog}(4, -(d*x+c+1)^2/(1-(d*x+c)^2)) - 3/4*b^3/(c*f-d*e-f)*\operatorname{polylog}(4, (c*f-d*e-f)*(d*x+c+1)^2/(1-(d*x+c)^2)/(-c*f+d*e-f)) \\ & + a^3*\ln((d*x+c)*f-c*f+d*e)/f - 3*d*a*b^2/f*e/(c*f-d*e-f)*\operatorname{arctanh}(d*x+c)^2*\ln(1-(c*f-d*e-f)*(d*x+c+1)^2/(1-(d*x+c)^2)/(-c*f+d*e-f)) \\ & - 3*d*a*b^2/f*e/(c*f-d*e-f)*\operatorname{arctanh}(d*x+c)*\operatorname{polylog}(2, (c*f-d*e-f)*(d*x+c+1)^2/(1-(d*x+c)^2)/(-c*f+d*e-f)) \\ & + 1/2*I*b^3/f*\operatorname{arctanh}(d*x+c)^3*c*\operatorname{sgn}(I/(1+(d*x+c+1)^2/(1-(d*x+c)^2))) * \operatorname{sgn}(I*(c*f*(1+(d*x+c+1)^2/(1-(d*x+c)^2)) + (-d*x+c+1)^2/(1-(d*x+c)^2) - 1)*e*d + (1-(d*x+c+1)^2/(1-(d*x+c)^2))*f)) * \operatorname{sgn}(I*(c*f*(1+(d*x+c+1)^2/(1-(d*x+c)^2)) + (-d*x+c+1)^2/(1-(d*x+c)^2) - 1)*e*d + (1-(d*x+c+1)^2/(1-(d*x+c)^2))*f)/(1+(d*x+c+1)^2/(1-(d*x+c)^2))) * \operatorname{Pi} - 3/2*I*a* \end{aligned}$$

$$\begin{aligned}
& b^2/f*\operatorname{arctanh}(d*x+c)^2*c\operatorname{sgn}(I/(1+(d*x+c+1)^2/(1-(d*x+c)^2))) *c\operatorname{sgn}(I*(c*f*(1+(d*x+c+1)^2/(1-(d*x+c)^2))+(-d*x+c+1)^2/(1-(d*x+c)^2)-1)*e*d+(1-(d*x+c+1)^2/(1-(d*x+c)^2))*f)/(1+(d*x+c+1)^2/(1-(d*x+c)^2)))^2*\operatorname{Pi}-3/2*I*a*b^2/f*\operatorname{arctanh}(d*x+c)^2*c\operatorname{sgn}(I*(c*f*(1+(d*x+c+1)^2/(1-(d*x+c)^2))+(-d*x+c+1)^2/(1-(d*x+c)^2)-1)*e*d+(1-(d*x+c+1)^2/(1-(d*x+c)^2))*f)) *c\operatorname{sgn}(I*(c*f*(1+(d*x+c+1)^2/(1-(d*x+c)^2))+(-d*x+c+1)^2/(1-(d*x+c)^2)-1)*e*d+(1-(d*x+c+1)^2/(1-(d*x+c)^2))*f)/(1+(d*x+c+1)^2/(1-(d*x+c)^2)))^2*\operatorname{Pi}-3*a*b^2/f*\operatorname{arctanh}(d*x+c)^2*\ln(c*f*(1+(d*x+c+1)^2/(1-(d*x+c)^2))+(-d*x+c+1)^2/(1-(d*x+c)^2)-1)*e*d+(1-(d*x+c+1)^2/(1-(d*x+c)^2))*f)+3/4*b^3*c/(c*f-d*e-f)*\operatorname{polylog}(4,(c*f-d*e-f)*(d*x+c+1)^2/(1-(d*x+c)^2)/(-c*f+d*e-f))+3/2*a^2*b/f*\operatorname{dilog}(((d*x+c)*f-f)/(c*f-d*e-f))-3/2*a^2*b/f*\operatorname{dilog}(((d*x+c)*f+f)/(c*f-d*e-f))-3/2*b^3/(c*f-d*e-f)*\operatorname{arctanh}(d*x+c)^2*\operatorname{polylog}(2,(c*f-d*e-f)*(d*x+c+1)^2/(1-(d*x+c)^2)/(-c*f+d*e-f))+3/2*b^3/(c*f-d*e-f)*\operatorname{arctanh}(d*x+c)*\operatorname{polylog}(3,(c*f-d*e-f)*(d*x+c+1)^2/(1-(d*x+c)^2)/(-c*f+d*e-f))+b^3*\ln((d*x+c)*f-c*f+d*e)/f*\operatorname{arctanh}(d*x+c)^3-b^3/f*\operatorname{arctanh}(d*x+c)^3*\ln(c*f*(1+(d*x+c+1)^2/(1-(d*x+c)^2))+(-d*x+c+1)^2/(1-(d*x+c)^2)-1)*e*d+(1-(d*x+c+1)^2/(1-(d*x+c)^2))*f)-3/2*b^3/f*\operatorname{arctanh}(d*x+c)^2*\operatorname{polylog}(2,-(d*x+c+1)^2/(1-(d*x+c)^2))+3/2*b^3/f*\operatorname{arctanh}(d*x+c)*\operatorname{polylog}(3,-(d*x+c+1)^2/(1-(d*x+c)^2))+3/2*I*a*b^2/f*\operatorname{arctanh}(d*x+c)^2*c\operatorname{sgn}(I/(1+(d*x+c+1)^2/(1-(d*x+c)^2))) *c\operatorname{sgn}(I*(c*f*(1+(d*x+c+1)^2/(1-(d*x+c)^2))+(-d*x+c+1)^2/(1-(d*x+c)^2)-1)*e*d+(1-(d*x+c+1)^2/(1-(d*x+c)^2))*f)) *c\operatorname{sgn}(I*(c*f*(1+(d*x+c+1)^2/(1-(d*x+c)^2))+(-d*x+c+1)^2/(1-(d*x+c)^2)-1)*e*d+(1-(d*x+c+1)^2/(1-(d*x+c)^2))*f)/(1+(d*x+c+1)^2/(1-(d*x+c)^2))) *c\operatorname{sgn}(I*(c*f*(1+(d*x+c+1)^2/(1-(d*x+c)^2))+(-d*x+c+1)^2/(1-(d*x+c)^2)-1)*e*d+(1-(d*x+c+1)^2/(1-(d*x+c)^2))*f)/(1+(d*x+c+1)^2/(1-(d*x+c)^2)))^2*\operatorname{Pi}-3/4*d*b^3/f*e/(c*f-d*e-f)*\operatorname{polylog}(4,(c*f-d*e-f)*(d*x+c+1)^2/(1-(d*x+c)^2)/(-c*f+d*e-f))+3*a*b^2*c/(c*f-d*e-f)*\operatorname{arctanh}(d*x+c)^2*\ln(1-(c*f-d*e-f)*(d*x+c+1)^2/(1-(d*x+c)^2)/(-c*f+d*e-f))+3*a*b^2*c/(c*f-d*e-f)*\operatorname{arctanh}(d*x+c)*\operatorname{polylog}(2,(c*f-d*e-f)*(d*x+c+1)^2/(1-(d*x+c)^2)/(-c*f+d*e-f))+I*b^3/f*\operatorname{arctanh}(d*x+c)^3*c\operatorname{sgn}(I*(c*f*(1+(d*x+c+1)^2/(1-(d*x+c)^2))+(-d*x+c+1)^2/(1-(d*x+c)^2)-1)*e*d+(1-(d*x+c+1)^2/(1-(d*x+c)^2))*f)/(1+(d*x+c+1)^2/(1-(d*x+c)^2)))^2*\operatorname{Pi}-1/2*I*b^3/f*\operatorname{arctanh}(d*x+c)^3*c\operatorname{sgn}(I*(c*f*(1+(d*x+c+1)^2/(1-(d*x+c)^2))+(-d*x+c+1)^2/(1-(d*x+c)^2)-1)*e*d+(1-(d*x+c+1)^2/(1-(d*x+c)^2))*f)/(1+(d*x+c+1)^2/(1-(d*x+c)^2)))^3*\operatorname{Pi}-3*I*a*b^2/f*\operatorname{Pi}*\operatorname{arctanh}(d*x+c)^2+3/2*d*a*b^2/f*e/(c*f-d*e-f)*\operatorname{polylog}(3,(c*f-d*e-f)*(d*x+c+1)^2/(1-(d*x+c)^2)/(-c*f+d*e-f))-1/2*I*b^3/f*\operatorname{arctanh}(d*x+c)^3*c\operatorname{sgn}(I/(1+(d*x+c+1)^2/(1-(d*x+c)^2))) *c\operatorname{sgn}(I*(c*f*(1+(d*x+c+1)^2/(1-(d*x+c)^2))+(-d*x+c+1)^2/(1-(d*x+c)^2)-1)*e*d+(1-(d*x+c+1)^2/(1-(d*x+c)^2))*f)/(1+(d*x+c+1)^2/(1-(d*x+c)^2)))^2*\operatorname{Pi}-1/2*I*b^3/f*\operatorname{arctanh}(d*x+c)^3*c\operatorname{sgn}(I*(c*f*(1+(d*x+c+1)^2/(1-(d*x+c)^2))+(-d*x+c+1)^2/(1-(d*x+c)^2)-1)*e*d+(1-(d*x+c+1)^2/(1-(d*x+c)^2))*f)/(1+(d*x+c+1)^2/(1-(d*x+c)^2)))^3*\operatorname{Pi}-d*b^3/f*e/(c*f-d*e-f)*\operatorname{arctanh}(d*x+c)^3*\ln(1-(c*f-d*e-f)*(d*x+c+1)^2/(1-(d*x+c)^2)/(-c*f+d*e-f))-3/2*d*b^3/f*e/(c*f-d*e-f)*\operatorname{arctanh}(d*x+c)^2*\operatorname{polylog}(2,(c
\end{aligned}$$

```
*f-d*e-f)*(d*x+c+1)^2/(1-(d*x+c)^2)/(-c*f+d*e-f))+3/2*d*b^3/f*e/(c*f-d*e-f)
*arctanh(d*x+c)*polylog(3,(c*f-d*e-f)*(d*x+c+1)^2/(1-(d*x+c)^2)/(-c*f+d*e-f
))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{a^3 \log(fx + e)}{f} + \int \frac{b^3 (\log(dx + c + 1) - \log(-dx - c + 1))^3}{8(fx + e)} + \frac{3ab^2 (\log(dx + c + 1) - \log(-dx - c + 1))^2}{4(fx + e)} + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(d*x+c))^3/(f*x+e),x, algorithm="maxima")
```

```
[Out] a^3*log(f*x + e)/f + integrate(1/8*b^3*(log(d*x + c + 1) - log(-d*x - c + 1))^3/(f*x + e) + 3/4*a*b^2*(log(d*x + c + 1) - log(-d*x - c + 1))^2/(f*x + e) + 3/2*a^2*b*(log(d*x + c + 1) - log(-d*x - c + 1))/(f*x + e), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + b \operatorname{atanh}(c + dx))^3}{e + fx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c + d*x))^3/(e + f*x),x)
```

```
[Out] int((a + b*atanh(c + d*x))^3/(e + f*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{atanh}(c + dx))^3}{e + fx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(d*x+c))**3/(f*x+e),x)
```

```
[Out] Integral((a + b*atanh(c + d*x))**3/(e + f*x), x)
```

$$3.49 \quad \int \frac{(a+b \tanh^{-1}(c+dx))^3}{(e+fx)^2} dx$$

Optimal. Leaf size=1089

$$\frac{3d \tanh^{-1}(c+dx)^2 \log\left(\frac{2}{-c-dx+1}\right) b^3}{2f(de-cf+f)} - \frac{3d \tanh^{-1}(c+dx)^2 \log\left(\frac{2}{c+dx+1}\right) b^3}{2f(de-cf-f)} + \frac{3d \tanh^{-1}(c+dx)^2 \log\left(\frac{2}{c+dx+1}\right) b^3}{(de-cf+f)(de-(c+1)f)} - \frac{3d \tanh^{-1}(c+dx)^2 \log\left(\frac{2}{-c-dx+1}\right) b^3}{(de-cf+f)(de-(c+1)f)}$$

[Out] $-(a+b*\operatorname{arctanh}(d*x+c))^3/f/(f*x+e)+3*a*b^2*d*\operatorname{arctanh}(d*x+c)*\ln(2/(-d*x-c+1))/f/(-c*f+d*e+f)+3/2*b^3*d*\operatorname{arctanh}(d*x+c)^2*\ln(2/(-d*x-c+1))/f/(-c*f+d*e+f)-3/2*a^2*b*d*\ln(-d*x-c+1)/f/(-c*f+d*e+f)-3*a*b^2*d*\operatorname{arctanh}(d*x+c)*\ln(2/(d*x+c+1))/f/(-c*f+d*e-f)+6*a*b^2*d*\operatorname{arctanh}(d*x+c)*\ln(2/(d*x+c+1))/(-c*f+d*e-f)/(-c*f+d*e+f)-3/2*b^3*d*\operatorname{arctanh}(d*x+c)^2*\ln(2/(d*x+c+1))/f/(-c*f+d*e-f)+3*b^3*d*\operatorname{arctanh}(d*x+c)^2*\ln(2/(d*x+c+1))/(-c*f+d*e-f)/(-c*f+d*e+f)+3/2*a^2*b*d*\ln(d*x+c+1)/f/(-c*f+d*e-f)+3*a^2*b*d*\ln(f*x+e)/(f^2-(-c*f+d*e)^2)-6*a*b^2*d*\operatorname{arctanh}(d*x+c)*\ln(2*d*(f*x+e)/(-c*f+d*e+f)/(d*x+c+1))/(-c*f+d*e-f)/(-c*f+d*e+f)-3*b^3*d*\operatorname{arctanh}(d*x+c)^2*\ln(2*d*(f*x+e)/(-c*f+d*e+f)/(d*x+c+1))/(-c*f+d*e-f)/(-c*f+d*e+f)+3/2*a*b^2*d*\operatorname{polylog}(2,(-d*x-c-1)/(-d*x-c+1))/f/(-c*f+d*e+f)+3/2*b^3*d*\operatorname{arctanh}(d*x+c)*\operatorname{polylog}(2,1-2/(-d*x-c+1))/f/(-c*f+d*e+f)+3/2*a*b^2*d*\operatorname{polylog}(2,1-2/(d*x+c+1))/f/(-c*f+d*e-f)-3*a*b^2*d*\operatorname{polylog}(2,1-2/(d*x+c+1))/(-c*f+d*e-f)/(-c*f+d*e+f)+3/2*b^3*d*\operatorname{arctanh}(d*x+c)*\operatorname{polylog}(2,1-2/(d*x+c+1))/f/(-c*f+d*e-f)-3*b^3*d*\operatorname{arctanh}(d*x+c)*\operatorname{polylog}(2,1-2/(d*x+c+1))/(-c*f+d*e-f)/(-c*f+d*e+f)+3*a*b^2*d*\operatorname{polylog}(2,1-2*d*(f*x+e)/(-c*f+d*e+f)/(d*x+c+1))/(-c*f+d*e-f)/(-c*f+d*e+f)+3*b^3*d*\operatorname{arctanh}(d*x+c)*\operatorname{polylog}(2,1-2*d*(f*x+e)/(-c*f+d*e+f)/(d*x+c+1))/(-c*f+d*e-f)/(-c*f+d*e+f)-3/4*b^3*d*\operatorname{polylog}(3,1-2/(-d*x-c+1))/f/(-c*f+d*e+f)+3/4*b^3*d*\operatorname{polylog}(3,1-2/(d*x+c+1))/f/(-c*f+d*e-f)-3/2*b^3*d*\operatorname{polylog}(3,1-2/(d*x+c+1))/(-c*f+d*e-f)/(-c*f+d*e+f)+3/2*b^3*d*\operatorname{polylog}(3,1-2*d*(f*x+e)/(-c*f+d*e+f)/(d*x+c+1))/(-c*f+d*e-f)/(-c*f+d*e+f)$

Rubi [A] time = 2.83, antiderivative size = 1094, normalized size of antiderivative = 1.00, number of steps used = 30, number of rules used = 18, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.900$, Rules used = {6109, 6741, 6121, 6688, 12, 6725, 72, 6742, 5918, 2402, 2315, 5920, 2447, 5948, 6058, 6610, 6056, 5922}

$$\frac{3d \tanh^{-1}(c+dx)^2 \log\left(\frac{2}{-c-dx+1}\right) b^3}{2f(de-cf+f)} - \frac{3d \tanh^{-1}(c+dx)^2 \log\left(\frac{2}{c+dx+1}\right) b^3}{2f(de-cf-f)} + \frac{3d \tanh^{-1}(c+dx)^2 \log\left(\frac{2}{c+dx+1}\right) b^3}{(de-cf+f)(de-(c+1)f)} - \frac{3d \tanh^{-1}(c+dx)^2 \log\left(\frac{2}{-c-dx+1}\right) b^3}{(de-cf+f)(de-(c+1)f)}$$

Antiderivative was successfully verified.

[In] Int[(a + b*ArcTanh[c + d*x])^3/(e + f*x)^2, x]

[Out] $-(a + b*\operatorname{ArcTanh}[c + d*x])^3/(f*(e + f*x)) + (3*a*b^2*d*\operatorname{ArcTanh}[c + d*x]*\operatorname{Log}[2/(1 - c - d*x)])/f*(d*e + f - c*f) + (3*b^3*d*\operatorname{ArcTanh}[c + d*x]^2*\operatorname{Log}[$

$$\begin{aligned} & 2/(1 - c - d*x)]/(2*f*(d*e + f - c*f)) - (3*a^2*b*d*Log[1 - c - d*x])/(2*f \\ & *(d*e + f - c*f)) - (3*a*b^2*d*ArcTanh[c + d*x]*Log[2/(1 + c + d*x)])/(f*(d \\ & *e - f - c*f)) + (6*a*b^2*d*ArcTanh[c + d*x]*Log[2/(1 + c + d*x)])/((d*e + \\ & f - c*f)*(d*e - (1 + c)*f)) - (3*b^3*d*ArcTanh[c + d*x]^2*Log[2/(1 + c + d* \\ & x)])/(2*f*(d*e - f - c*f)) + (3*b^3*d*ArcTanh[c + d*x]^2*Log[2/(1 + c + d*x \\ &)])/((d*e + f - c*f)*(d*e - (1 + c)*f)) + (3*a^2*b*d*Log[1 + c + d*x])/(2*f \\ & *(d*e - f - c*f)) - (3*a^2*b*d*Log[e + f*x])/((d*e + f - c*f)*(d*e - (1 + c \\ &)*f)) - (6*a*b^2*d*ArcTanh[c + d*x]*Log[(2*d*(e + f*x))]/((d*e + f - c*f)*(1 \\ & + c + d*x)))/((d*e + f - c*f)*(d*e - (1 + c)*f)) - (3*b^3*d*ArcTanh[c + d \\ & *x]^2*Log[(2*d*(e + f*x))]/((d*e + f - c*f)*(1 + c + d*x)))/((d*e + f - c*f \\ &)*(d*e - (1 + c)*f)) + (3*a*b^2*d*PolyLog[2, -((1 + c + d*x)/(1 - c - d*x)) \\ &])/(2*f*(d*e + f - c*f)) + (3*b^3*d*ArcTanh[c + d*x]*PolyLog[2, 1 - 2/(1 - \\ & c - d*x)])/(2*f*(d*e + f - c*f)) + (3*a*b^2*d*PolyLog[2, 1 - 2/(1 + c + d*x \\ &)])/(2*f*(d*e - f - c*f)) - (3*a*b^2*d*PolyLog[2, 1 - 2/(1 + c + d*x)])/((d \\ & *e + f - c*f)*(d*e - (1 + c)*f)) + (3*b^3*d*ArcTanh[c + d*x]*PolyLog[2, 1 - \\ & 2/(1 + c + d*x)])/(2*f*(d*e - f - c*f)) - (3*b^3*d*ArcTanh[c + d*x]*PolyLo \\ & g[2, 1 - 2/(1 + c + d*x)])/((d*e + f - c*f)*(d*e - (1 + c)*f)) + (3*a*b^2*d \\ & *PolyLog[2, 1 - (2*d*(e + f*x))]/((d*e + f - c*f)*(1 + c + d*x)))/((d*e + f \\ & - c*f)*(d*e - (1 + c)*f)) + (3*b^3*d*ArcTanh[c + d*x]*PolyLog[2, 1 - (2*d* \\ & (e + f*x))]/((d*e + f - c*f)*(1 + c + d*x)))/((d*e + f - c*f)*(d*e - (1 + c \\ &)*f)) - (3*b^3*d*PolyLog[3, 1 - 2/(1 - c - d*x)])/(4*f*(d*e + f - c*f)) + (\\ & 3*b^3*d*PolyLog[3, 1 - 2/(1 + c + d*x)])/(4*f*(d*e - f - c*f)) - (3*b^3*d*P \\ & olyLog[3, 1 - 2/(1 + c + d*x)])/(2*(d*e + f - c*f)*(d*e - (1 + c)*f)) + (3* \\ & b^3*d*PolyLog[3, 1 - (2*d*(e + f*x))]/((d*e + f - c*f)*(1 + c + d*x)))/(2*(\\ & d*e + f - c*f)*(d*e - (1 + c)*f)) \end{aligned}$$

Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_)] /; FreeQ[b, x]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] := Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] := -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] := -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
```

c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]

Rule 2447

Int[Log[u_]*(Pq_)^(m_), x_Symbol] := With[{C = FullSimplify[(Pq^m*(1 - u))/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x] /; IntegerQ[m] && PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u, x][[2]], Expon[Pq, x]]

Rule 5918

Int[((a_.) + ArcTanh[(c_.)*(x_.)]*(b_.))^((p_.)/((d_.) + (e_.)*(x_.)), x_Symbol] := -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)]]/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0]

Rule 5920

Int[((a_.) + ArcTanh[(c_.)*(x_.)]*(b_.))/((d_.) + (e_.)*(x_.)), x_Symbol] := -Simp[((a + b*ArcTanh[c*x])*Log[2/(1 + c*x)])/e, x] + (Dist[(b*c)/e, Int[Log[2/(1 + c*x)]]/(1 - c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))]]/(1 - c^2*x^2), x], x] + Simp[((a + b*ArcTanh[c*x])*Log[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e, x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[c^2*d^2 - e^2, 0]

Rule 5922

Int[((a_.) + ArcTanh[(c_.)*(x_.)]*(b_.))^2/((d_.) + (e_.)*(x_.)), x_Symbol] := -Simp[((a + b*ArcTanh[c*x])^2*Log[2/(1 + c*x)])/e, x] + (Simp[((a + b*ArcTanh[c*x])^2*Log[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e, x] + Simp[(b*(a + b*ArcTanh[c*x])*PolyLog[2, 1 - 2/(1 + c*x)])/e, x] - Simp[(b*(a + b*ArcTanh[c*x])*PolyLog[2, 1 - (2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e, x] + Simp[(b^2*PolyLog[3, 1 - 2/(1 + c*x)]]/(2*e), x] - Simp[(b^2*PolyLog[3, 1 - (2*c*(d + e*x))/((c*d + e)*(1 + c*x))]]/(2*e), x]) /; FreeQ[{a, b, c, d, e}, x] && NeQ[c^2*d^2 - e^2, 0]

Rule 5948

Int[((a_.) + ArcTanh[(c_.)*(x_.)]*(b_.))^((p_.)/((d_.) + (e_.)*(x_)^2), x_Symbol] := Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]

Rule 6056


```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)])*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] := Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d), x
] - Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d +
e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e
, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6058

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)])*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] := -Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6109

```
Int[((a_.) + ArcTanh[(c_) + (d_.)*(x_)])*(b_.))^(p_.)*((e_.) + (f_.)*(x_)^(
m_)), x_Symbol] := Simp[((e + f*x)^(m + 1)*(a + b*ArcTanh[c + d*x])^p)/(f*(m
+ 1)), x] - Dist[(b*d*p)/(f*(m + 1)), Int[((e + f*x)^(m + 1)*(a + b*ArcTan
h[c + d*x])^(p - 1))/(1 - (c + d*x)^2), x], x] /; FreeQ[{a, b, c, d, e, f},
x] && IGtQ[p, 0] && ILtQ[m, -1]
```

Rule 6121

```
Int[((a_.) + ArcTanh[(c_) + (d_.)*(x_)])*(b_.))^(p_.)*((e_.) + (f_.)*(x_)^(
m_.))*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)^(q_.), x_Symbol] := Dist[1/d, Subs
t[Int[((d*e - c*f)/d + (f*x)/d)^m*(-(C/d^2) + (C*x^2)/d^2)^q*(a + b*ArcTanh
[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m, p, q},
x] && EqQ[B*(1 - c^2) + 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] := With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /; !FalseQ[w]] /; FreeQ[n, x]
```

Rule 6688

```
Int[u_, x_Symbol] := With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
```

[n, 0]

Rule 6741

`Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v != u]`

Rule 6742

`Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]`
`]`

Rubi steps

$$\begin{aligned}
\int \frac{(a + b \tanh^{-1}(c + dx))^3}{(e + fx)^2} dx &= -\frac{(a + b \tanh^{-1}(c + dx))^3}{f(e + fx)} + \frac{(3bd) \int \frac{(a+b \tanh^{-1}(c+dx))^2}{(e+fx)(1-(c+dx)^2)} dx}{f} \\
&= -\frac{(a + b \tanh^{-1}(c + dx))^3}{f(e + fx)} + \frac{(3bd) \int \frac{(a+b \tanh^{-1}(c+dx))^2}{(e+fx)(1-c^2-2cdx-d^2x^2)} dx}{f} \\
&= -\frac{(a + b \tanh^{-1}(c + dx))^3}{f(e + fx)} + \frac{(3b) \text{Subst} \left(\int \frac{(a+b \tanh^{-1}(x))^2}{\left(\frac{de-cf}{d} + \frac{fx}{d}\right)(1-x^2)} dx, x, c + dx \right)}{f} \\
&= -\frac{(a + b \tanh^{-1}(c + dx))^3}{f(e + fx)} + \frac{(3b) \text{Subst} \left(\int \frac{d(a+b \tanh^{-1}(x))^2}{(de-cf+fx)(1-x^2)} dx, x, c + dx \right)}{f} \\
&= -\frac{(a + b \tanh^{-1}(c + dx))^3}{f(e + fx)} + \frac{(3bd) \text{Subst} \left(\int \frac{(a+b \tanh^{-1}(x))^2}{(de-cf+fx)(1-x^2)} dx, x, c + dx \right)}{f} \\
&= -\frac{(a + b \tanh^{-1}(c + dx))^3}{f(e + fx)} + \frac{(3bd) \text{Subst} \left(\int \left(-\frac{a^2}{(-1+x)(1+x)(de-cf+fx)} - \frac{2ab \tanh^{-1}(x)}{(-1+x)(1+x)} \right) dx, x, c + dx \right)}{f} \\
&= -\frac{(a + b \tanh^{-1}(c + dx))^3}{f(e + fx)} - \frac{(3a^2bd) \text{Subst} \left(\int \frac{1}{(-1+x)(1+x)(de-cf+fx)} dx, x, c + dx \right)}{f} \\
&= -\frac{(a + b \tanh^{-1}(c + dx))^3}{f(e + fx)} - \frac{(3a^2bd) \text{Subst} \left(\int \left(\frac{1}{2(de+f-cf)(-1+x)} + \frac{1}{2(-de+(1+c)f(1-x))} \right) dx, x, c + dx \right)}{f} \\
&= -\frac{(a + b \tanh^{-1}(c + dx))^3}{f(e + fx)} - \frac{3a^2bd \log(1 - c - dx)}{2f(de + f - cf)} + \frac{3a^2bd \log(1 + c + dx)}{2f(de - f - cf)} \\
&= -\frac{(a + b \tanh^{-1}(c + dx))^3}{f(e + fx)} + \frac{3ab^2d \tanh^{-1}(c + dx) \log\left(\frac{2}{1-c-dx}\right)}{f(de + f - cf)} + \frac{3b^3d \tanh^{-1}(c + dx)}{2f} \\
&= -\frac{(a + b \tanh^{-1}(c + dx))^3}{f(e + fx)} + \frac{3ab^2d \tanh^{-1}(c + dx) \log\left(\frac{2}{1-c-dx}\right)}{f(de + f - cf)} + \frac{3b^3d \tanh^{-1}(c + dx)}{2f} \\
&= -\frac{(a + b \tanh^{-1}(c + dx))^3}{f(e + fx)} + \frac{3ab^2d \tanh^{-1}(c + dx) \log\left(\frac{2}{1-c-dx}\right)}{f(de + f - cf)} + \frac{3b^3d \tanh^{-1}(c + dx)}{2f}
\end{aligned}$$

Mathematica [C] time = 23.21, size = 1946, normalized size = 1.79

result too large to display

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b*ArcTanh[c + d*x])^3/(e + f*x)^2,x]

[Out]
$$-(a^3/(f*(e + f*x))) - (3*a^2*b*ArcTanh[c + d*x])/(f*(e + f*x)) + (3*a^2*b*d*Log[1 - c - d*x])/(2*f*(-(d*e) - f + c*f)) - (3*a^2*b*d*Log[1 + c + d*x])/(2*f*(-(d*e) + f + c*f)) - (3*a^2*b*d*Log[e + f*x])/(d^2*e^2 - 2*c*d*e*f - f^2 + c^2*f^2) + (3*a*b^2*(1 - (c + d*x)^2)*((d*e - c*f)/Sqrt[1 - (c + d*x)^2] + (f*(c + d*x))/Sqrt[1 - (c + d*x)^2])^2*(-(ArcTanh[c + d*x]^2/(E^ArcTanh[(d*e - c*f)/f]*f*Sqrt[1 - (d*e - c*f)^2/f^2])) + ((c + d*x)*ArcTanh[c + d*x]^2)/(Sqrt[1 - (c + d*x)^2]*((d*e)/Sqrt[1 - (c + d*x)^2] - (c*f)/Sqrt[1 - (c + d*x)^2] + (f*(c + d*x))/Sqrt[1 - (c + d*x)^2])) + ((d*e - c*f)*(I*Pi*Log[1 + E^(2*ArcTanh[c + d*x])] - 2*ArcTanh[c + d*x]*Log[1 - E^(-2*(ArcTanh[(d*e - c*f)/f] + ArcTanh[c + d*x]))] - I*Pi*(ArcTanh[c + d*x] + Log[1/Sqrt[1 - (c + d*x)^2]]) - 2*ArcTanh[(d*e - c*f)/f]*(ArcTanh[c + d*x] + Log[1 - E^(-2*(ArcTanh[(d*e - c*f)/f] + ArcTanh[c + d*x]))] - Log[I*Sinh[ArcTanh[(d*e - c*f)/f] + ArcTanh[c + d*x]]]) + PolyLog[2, E^(-2*(ArcTanh[(d*e - c*f)/f] + ArcTanh[c + d*x]))])/(d^2*e^2 - 2*c*d*e*f + (-1 + c^2)*f^2))/(d*(d*e - c*f)*(e + f*x)^2) + (b^3*(1 - (c + d*x)^2)*((d*e - c*f)/Sqrt[1 - (c + d*x)^2] + (f*(c + d*x))/Sqrt[1 - (c + d*x)^2])^2*((d*(c + d*x)*ArcTanh[c + d*x]^3)/((d*e - c*f)*Sqrt[1 - (c + d*x)^2]*((d*e)/Sqrt[1 - (c + d*x)^2] - (c*f)/Sqrt[1 - (c + d*x)^2] + (f*(c + d*x))/Sqrt[1 - (c + d*x)^2])) - (d*(-3*d*e*ArcTanh[c + d*x]^3 + f*ArcTanh[c + d*x]^3 + 3*c*f*ArcTanh[c + d*x]^3 - (2*Sqrt[1 - c^2 - (d^2*e^2)/f^2 + (2*c*d*e)/f]*f*ArcTanh[c + d*x]^3)/E^ArcTanh[(d*e - c*f)/f] - (3*I)*d*e*Pi*ArcTanh[c + d*x]*Log[(1 + E^(2*ArcTanh[c + d*x]))]/(2*E^ArcTanh[c + d*x])) + (3*I)*c*f*Pi*ArcTanh[c + d*x]*Log[(1 + E^(2*ArcTanh[c + d*x]))]/(2*E^ArcTanh[c + d*x])) + 3*d*e*ArcTanh[c + d*x]^2*Log[1 - E^(ArcTanh[(d*e - c*f)/f] + ArcTanh[c + d*x])] - 3*c*f*ArcTanh[c + d*x]^2*Log[1 - E^(ArcTanh[(d*e - c*f)/f] + ArcTanh[c + d*x])] + 3*d*e*ArcTanh[c + d*x]^2*Log[1 + E^(ArcTanh[(d*e - c*f)/f] + ArcTanh[c + d*x])] - 3*c*f*ArcTanh[c + d*x]^2*Log[1 + E^(ArcTanh[(d*e - c*f)/f] + ArcTanh[c + d*x])] + 6*d*e*ArcTanh[(d*e - c*f)/f]*ArcTanh[c + d*x]*Log[(I/2)*E^(-ArcTanh[(d*e - c*f)/f] - ArcTanh[c + d*x])*(-1 + E^(2*(ArcTanh[(d*e - c*f)/f] + ArcTanh[c + d*x])))] - 6*c*f*ArcTanh[(d*e - c*f)/f]*ArcTanh[c + d*x]*Log[(I/2)*E^(-ArcTanh[(d*e - c*f)/f] - ArcTanh[c + d*x])*(-1 + E^(2*(ArcTanh[(d*e - c*f)/f] + ArcTanh[c + d*x])))] + 3*d*e*ArcTanh[c + d*x]^2*Log[(d*e*(1 + E^(2*ArcTanh[c + d*x])) - (1 + c - E^(2*ArcTanh[c + d*x])) + c*E^(2*ArcTanh[c + d*x]))*f]/(2*E^ArcTanh[c + d*x])] - 3*c*f*ArcTanh[c + d*x]^2*Log[(d*e*(1 + E^(2*ArcTanh[c + d*x])) - (1 + c - E^(2*ArcTanh[c + d*x])) + c*E^(2*ArcTanh[c + d*x]))*f]/(2*E^ArcTanh[c + d*x])] + (3*I)*d*e*Pi*ArcTanh[c + d*x]*Log[1/Sqrt[1 - (c + d*x)^2]] - (3*I)*c*f*Pi*ArcTanh[c + d*x]*Log[1/Sqrt[1 - (c + d*x)^2]]$$

$x^2] - 3d*e*ArcTanh[c + d*x]^2*Log[(d*e)/Sqrt[1 - (c + d*x)^2] - (c*f)/Sqrt[1 - (c + d*x)^2] + (f*(c + d*x))/Sqrt[1 - (c + d*x)^2]] + 3*c*f*ArcTanh[c + d*x]^2*Log[(d*e)/Sqrt[1 - (c + d*x)^2] - (c*f)/Sqrt[1 - (c + d*x)^2] + (f*(c + d*x))/Sqrt[1 - (c + d*x)^2]] - 6*d*e*ArcTanh[(d*e - c*f)/f]*ArcTanh[c + d*x]*Log[I*Sinh[ArcTanh[(d*e - c*f)/f] + ArcTanh[c + d*x]]] + 6*c*f*ArcTanh[(d*e - c*f)/f]*ArcTanh[c + d*x]*Log[I*Sinh[ArcTanh[(d*e - c*f)/f] + ArcTanh[c + d*x]]] + 6*(d*e - c*f)*ArcTanh[c + d*x]*PolyLog[2, -E^(ArcTanh[(d*e - c*f)/f] + ArcTanh[c + d*x])] + 6*(d*e - c*f)*ArcTanh[c + d*x]*PolyLog[2, E^(ArcTanh[(d*e - c*f)/f] + ArcTanh[c + d*x])] - 6*d*e*PolyLog[3, -E^(ArcTanh[(d*e - c*f)/f] + ArcTanh[c + d*x])] + 6*c*f*PolyLog[3, -E^(ArcTanh[(d*e - c*f)/f] + ArcTanh[c + d*x])] - 6*d*e*PolyLog[3, E^(ArcTanh[(d*e - c*f)/f] + ArcTanh[c + d*x])] + 6*c*f*PolyLog[3, E^(ArcTanh[(d*e - c*f)/f] + ArcTanh[c + d*x])])]/((d*e - c*f)*(d^2*e^2 - 2*c*d*e*f + (-1 + c^2)*f^2)))/(d^2*(e + f*x)^2)$

fricas [F] time = 0.90, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^3 \operatorname{artanh}(dx + c)^3 + 3ab^2 \operatorname{artanh}(dx + c)^2 + 3a^2b \operatorname{artanh}(dx + c) + a^3}{f^2x^2 + 2efx + e^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arctanh(d*x+c))^3/(f*x+e)^2,x, algorithm="fricas")

[Out] integral((b^3*arctanh(d*x + c)^3 + 3*a*b^2*arctanh(d*x + c)^2 + 3*a^2*b*arctanh(d*x + c) + a^3)/(f^2*x^2 + 2*e*f*x + e^2), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{artanh}(dx + c) + a)^3}{(fx + e)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arctanh(d*x+c))^3/(f*x+e)^2,x, algorithm="giac")

[Out] integrate((b*arctanh(d*x + c) + a)^3/(f*x + e)^2, x)

maple [C] time = 1.00, size = 5728, normalized size = 5.26

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*arctanh(d*x+c))^3/(f*x+e)^2,x)

[Out] result too large to display

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{3}{2} \left(d \left(\frac{\log(dx+c+1)}{def - (c+1)f^2} - \frac{\log(dx+c-1)}{def - (c-1)f^2} - \frac{2 \log(fx+e)}{d^2e^2 - 2cdef + (c^2-1)f^2} \right) - \frac{2 \operatorname{artanh}(dx+c)}{f^2x+ef} \right) a^2 b - \frac{a^3}{f^2x+ef} - \left(\frac{d^2}{f^2x+ef} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arctanh(d*x+c))^3/(f*x+e)^2,x, algorithm="maxima")

[Out] $\frac{3}{2} * (d * (\log(d*x + c + 1) / (d*e*f - (c + 1)*f^2) - \log(d*x + c - 1) / (d*e*f - (c - 1)*f^2) - 2 * \log(f*x + e) / (d^2*e^2 - 2*c*d*e*f + (c^2 - 1)*f^2)) - 2 * \operatorname{arctanh}(d*x + c) / (f^2*x + e*f)) * a^2 * b - a^3 / (f^2*x + e*f) - 1/8 * (((d^2*e*f - c*d*f^2 - d*f^2) * b^3 * x + (c*d*e*f - c^2*f^2 - d*e*f + f^2) * b^3) * \log(-d*x - c + 1)^3 + 3 * (2 * (d^2*e^2 - 2*c*d*e*f + c^2*f^2 - f^2) * a * b^2 - ((d^2*e*f - c*d*f^2 + d*f^2) * b^3 * x + (c*d*e*f - c^2*f^2 + d*e*f + f^2) * b^3) * \log(d*x + c + 1)) * \log(-d*x - c + 1)^2) / (d^2*e^3*f - 2*c*d*e^2*f^2 + c^2*e*f^3 - e*f^3 + (d^2*e^2*f^2 - 2*c*d*e*f^3 + c^2*f^4 - f^4) * x) - \operatorname{integrate}(-1/8 * (((d^2*e*f - c*d*f^2 - d*f^2) * b^3 * x + (c*d*e*f - c^2*f^2 - d*e*f + f^2) * b^3) * \log(d*x + c + 1)^3 + 6 * ((d^2*e*f - c*d*f^2 - d*f^2) * a * b^2 * x + (c*d*e*f - c^2*f^2 - d*e*f + f^2) * a * b^2) * \log(d*x + c + 1)^2 + 3 * (4 * (d^2*e*f - c*d*f^2 - d*f^2) * a * b^2 * x + 4 * (d^2*e^2 - c*d*e*f - d*e*f) * a * b^2 - ((d^2*e*f - c*d*f^2 - d*f^2) * b^3 * x + (c*d*e*f - c^2*f^2 - d*e*f + f^2) * b^3) * \log(d*x + c + 1)^2 - 2 * (b^3 * d^2 * f^2 * x^2 + 2 * (c*d*e*f - c^2*f^2 - d*e*f + f^2) * a * b^2 + (c*d*e*f + d*e*f) * b^3 + (2 * (d^2*e*f - c*d*f^2 - d*f^2) * a * b^2 + (d^2*e*f + c*d*f^2 + d*f^2) * b^3) * x) * \log(d*x + c + 1)) * \log(-d*x - c + 1)) / (c*d*e^3*f - c^2*e^2*f^2 - d*e^3*f + e^2*f^2 + (d^2*e*f^3 - c*d*f^4 - d*f^4) * x^3 + (2*d^2*e^2*f^2 - c*d*e*f^3 - c^2*f^4 - 3*d*e*f^3 + f^4) * x^2 + (d^2*e^3*f + c*d*e^2*f^2 - 2*c^2*e*f^3 - 3*d*e^2*f^2 + 2*e*f^3) * x), x)$

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + b \operatorname{atanh}(c + dx))^3}{(e + fx)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*atanh(c + d*x))^3/(e + f*x)^2,x)

[Out] int((a + b*atanh(c + d*x))^3/(e + f*x)^2, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{atanh}(c + dx))^3}{(e + fx)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(d*x+c))**3/(f*x+e)**2,x)
```

```
[Out] Integral((a + b*atanh(c + d*x))**3/(e + f*x)**2, x)
```

$$3.50 \quad \int (e + fx)^m (a + b \tanh^{-1}(c + dx))^3 dx$$

Optimal. Leaf size=23

$$\text{Int}\left((e + fx)^m (a + b \tanh^{-1}(c + dx))^3, x\right)$$

[Out] Unintegrable((f*x+e)^m*(a+b*arctanh(d*x+c))^3,x)

Rubi [A] time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (e + fx)^m (a + b \tanh^{-1}(c + dx))^3 dx$$

Verification is Not applicable to the result.

[In] Int[(e + f*x)^m*(a + b*ArcTanh[c + d*x])^3,x]

[Out] Defer[Subst][Defer[Int][((d*e - c*f)/d + (f*x)/d)^m*(a + b*ArcTanh[x])^3, x], x, c + d*x]/d

Rubi steps

$$\int (e + fx)^m (a + b \tanh^{-1}(c + dx))^3 dx = \frac{\text{Subst}\left(\int \left(\frac{de-cf}{d} + \frac{fx}{d}\right)^m (a + b \tanh^{-1}(x))^3 dx, x, c + dx\right)}{d}$$

Mathematica [A] time = 4.51, size = 0, normalized size = 0.00

$$\int (e + fx)^m (a + b \tanh^{-1}(c + dx))^3 dx$$

Verification is Not applicable to the result.

[In] Integrate[(e + f*x)^m*(a + b*ArcTanh[c + d*x])^3,x]

[Out] Integrate[(e + f*x)^m*(a + b*ArcTanh[c + d*x])^3, x]

fricas [A] time = 0.70, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b^3 \operatorname{artanh}(dx + c)^3 + 3ab^2 \operatorname{artanh}(dx + c)^2 + 3a^2b \operatorname{artanh}(dx + c) + a^3\right)(fx + e)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x+e)^m*(a+b*arctanh(d*x+c))^3,x, algorithm="fricas")

[Out] integral((b^3*arctanh(d*x + c)^3 + 3*a*b^2*arctanh(d*x + c)^2 + 3*a^2*b*arctanh(d*x + c) + a^3)*(f*x + e)^m, x)

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \operatorname{artanh}(dx + c) + a)^3 (fx + e)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x+e)^m*(a+b*arctanh(d*x+c))^3,x, algorithm="giac")

[Out] integrate((b*arctanh(d*x + c) + a)^3*(f*x + e)^m, x)

maple [A] time = 1.62, size = 0, normalized size = 0.00

$$\int (fx + e)^m (a + b \operatorname{arctanh}(dx + c))^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f*x+e)^m*(a+b*arctanh(d*x+c))^3,x)

[Out] int((f*x+e)^m*(a+b*arctanh(d*x+c))^3,x)

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{(b^3fx + b^3e)(fx + e)^m \log(-dx - c + 1)^3}{8f(m+1)} + \frac{(fx + e)^{m+1} a^3}{f(m+1)} + \int \frac{((b^3df(m+1)x + (cf(m+1) - f(m+1))b^3) \log(-dx - c + 1)^3 + \dots)}{\dots} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x+e)^m*(a+b*arctanh(d*x+c))^3,x, algorithm="maxima")

[Out] -1/8*(b^3*f*x + b^3*e)*(f*x + e)^m*log(-d*x - c + 1)^3/(f*(m + 1)) + (f*x + e)^(m + 1)*a^3/(f*(m + 1)) + integrate(1/8*((b^3*d*f*(m + 1)*x + (c*f*(m + 1) - f*(m + 1))*b^3)*log(d*x + c + 1)^3 + 6*(a*b^2*d*f*(m + 1)*x + (c*f*(m + 1) - f*(m + 1))*a*b^2)*log(d*x + c + 1)^2 + 3*(b^3*d*e + 2*(c*f*(m + 1) - f*(m + 1))*a*b^2 + (2*a*b^2*d*f*(m + 1) + b^3*d*f)*x + (b^3*d*f*(m + 1)*x + (c*f*(m + 1) - f*(m + 1))*b^3)*log(d*x + c + 1))*log(-d*x - c + 1)^2 + 12*(a^2*b*d*f*(m + 1)*x + (c*f*(m + 1) - f*(m + 1))*a^2*b)*log(d*x + c + 1) - 3*(4*a^2*b*d*f*(m + 1)*x + 4*(c*f*(m + 1) - f*(m + 1))*a^2*b + (b^3*d*f*(m + 1)*x + (c*f*(m + 1) - f*(m + 1))*b^3)*log(d*x + c + 1)^2 + 4*(a*b^2*d*f

```
*(m + 1)*x + (c*f*(m + 1) - f*(m + 1))*a*b^2)*log(d*x + c + 1))*log(-d*x -
c + 1))*(f*x + e)^m/(d*f*(m + 1)*x + c*f*(m + 1) - f*(m + 1)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int (e + f x)^m (a + b \operatorname{atanh}(c + d x))^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)^m*(a + b*atanh(c + d*x))^3,x)
```

```
[Out] int((e + f*x)^m*(a + b*atanh(c + d*x))^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**m*(a+b*atanh(d*x+c))**3,x)
```

```
[Out] Timed out
```

$$3.51 \quad \int (e + fx)^m (a + b \tanh^{-1}(c + dx))^2 dx$$

Optimal. Leaf size=23

$$\text{Int}\left((e + fx)^m (a + b \tanh^{-1}(c + dx))^2, x\right)$$

[Out] Unintegrable((f*x+e)^m*(a+b*arctanh(d*x+c))^2,x)

Rubi [A] time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (e + fx)^m (a + b \tanh^{-1}(c + dx))^2 dx$$

Verification is Not applicable to the result.

[In] Int[(e + f*x)^m*(a + b*ArcTanh[c + d*x])^2,x]

[Out] Defer[Subst][Defer[Int][((d*e - c*f)/d + (f*x)/d)^m*(a + b*ArcTanh[x])^2, x], x, c + d*x]/d

Rubi steps

$$\int (e + fx)^m (a + b \tanh^{-1}(c + dx))^2 dx = \frac{\text{Subst}\left(\int \left(\frac{de-cf}{d} + \frac{fx}{d}\right)^m (a + b \tanh^{-1}(x))^2 dx, x, c + dx\right)}{d}$$

Mathematica [A] time = 0.30, size = 0, normalized size = 0.00

$$\int (e + fx)^m (a + b \tanh^{-1}(c + dx))^2 dx$$

Verification is Not applicable to the result.

[In] Integrate[(e + f*x)^m*(a + b*ArcTanh[c + d*x])^2,x]

[Out] Integrate[(e + f*x)^m*(a + b*ArcTanh[c + d*x])^2, x]

fricas [A] time = 0.53, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b^2 \operatorname{artanh}(dx + c)^2 + 2ab \operatorname{artanh}(dx + c) + a^2\right)(fx + e)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x+e)^m*(a+b*arctanh(d*x+c))^2,x, algorithm="fricas")`

[Out] `integral((b^2*arctanh(d*x + c)^2 + 2*a*b*arctanh(d*x + c) + a^2)*(f*x + e)^m, x)`

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \operatorname{artanh}(dx + c) + a)^2 (fx + e)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x+e)^m*(a+b*arctanh(d*x+c))^2,x, algorithm="giac")`

[Out] `integrate((b*arctanh(d*x + c) + a)^2*(f*x + e)^m, x)`

maple [A] time = 1.40, size = 0, normalized size = 0.00

$$\int (fx + e)^m (a + b \operatorname{arctanh}(dx + c))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((f*x+e)^m*(a+b*arctanh(d*x+c))^2,x)`

[Out] `int((f*x+e)^m*(a+b*arctanh(d*x+c))^2,x)`

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{(b^2 fx + b^2 e)(fx + e)^m \log(-dx - c + 1)^2}{4 f(m + 1)} + \frac{(fx + e)^{m+1} a^2}{f(m + 1)} - \int \frac{((b^2 d f(m + 1)x + (c f(m + 1) - f(m + 1))b^2) \log$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x+e)^m*(a+b*arctanh(d*x+c))^2,x, algorithm="maxima")`

[Out] `1/4*(b^2*f*x + b^2*e)*(f*x + e)^m*log(-d*x - c + 1)^2/(f*(m + 1)) + (f*x + e)^(m + 1)*a^2/(f*(m + 1)) - integrate(-1/4*((b^2*d*f*(m + 1)*x + (c*f*(m + 1) - f*(m + 1))*b^2)*log(d*x + c + 1)^2 + 4*(a*b*d*f*(m + 1)*x + (c*f*(m + 1) - f*(m + 1))*a*b)*log(d*x + c + 1) - 2*(b^2*d*e + 2*(c*f*(m + 1) - f*(m + 1))*a*b + (2*a*b*d*f*(m + 1) + b^2*d*f)*x + (b^2*d*f*(m + 1)*x + (c*f*(m + 1) - f*(m + 1))*b^2)*log(d*x + c + 1))*log(-d*x - c + 1)*(f*x + e)^m/(d*f*(m + 1)*x + c*f*(m + 1) - f*(m + 1)), x)`

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int (e + fx)^m (a + b \operatorname{atanh}(c + dx))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)^m*(a + b*atanh(c + d*x))^2,x)
```

```
[Out] int((e + f*x)^m*(a + b*atanh(c + d*x))^2, x)
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**m*(a+b*atanh(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.52 $\int (e + fx)^m (a + b \tanh^{-1}(c + dx)) dx$

Optimal. Leaf size=162

$$\frac{(e + fx)^{m+1} (a + b \tanh^{-1}(c + dx))}{f(m+1)} + \frac{bd(e + fx)^{m+2} {}_2F_1\left(1, m+2; m+3; \frac{d(e+fx)}{de-cf-f}\right)}{2f(m+1)(m+2)(de - (c+1)f)} - \frac{bd(e + fx)^{m+2} {}_2F_1\left(1, m+2; m+3; \frac{d(e+fx)}{de-cf-f}\right)}{2f(m+1)(m+2)(-cf + d)}$$

[Out] (f*x+e)^(1+m)*(a+b*arctanh(d*x+c))/f/(1+m)+1/2*b*d*(f*x+e)^(2+m)*hypergeom([1, 2+m], [3+m], d*(f*x+e)/(-c*f+d*e-f))/f/(d*e-(1+c)*f)/(1+m)/(2+m)-1/2*b*d*(f*x+e)^(2+m)*hypergeom([1, 2+m], [3+m], d*(f*x+e)/(-c*f+d*e+f))/f/(-c*f+d*e+f)/(1+m)/(2+m)

Rubi [A] time = 0.26, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {6111, 5926, 712, 68}

$$\frac{(e + fx)^{m+1} (a + b \tanh^{-1}(c + dx))}{f(m+1)} + \frac{bd(e + fx)^{m+2} {}_2F_1\left(1, m+2; m+3; \frac{d(e+fx)}{de-cf-f}\right)}{2f(m+1)(m+2)(de - (c+1)f)} - \frac{bd(e + fx)^{m+2} {}_2F_1\left(1, m+2; m+3; \frac{d(e+fx)}{de-cf-f}\right)}{2f(m+1)(m+2)(-cf + d)}$$

Antiderivative was successfully verified.

[In] Int[(e + f*x)^m*(a + b*ArcTanh[c + d*x]), x]

[Out] ((e + f*x)^(1 + m)*(a + b*ArcTanh[c + d*x]))/(f*(1 + m)) + (b*d*(e + f*x)^(2 + m)*Hypergeometric2F1[1, 2 + m, 3 + m, (d*(e + f*x))/(d*e - f - c*f)])/((2*f*(d*e - (1 + c)*f)*(1 + m)*(2 + m)) - (b*d*(e + f*x)^(2 + m)*Hypergeometric2F1[1, 2 + m, 3 + m, (d*(e + f*x))/(d*e + f - c*f)])/((2*f*(d*e + f - c*f)*(1 + m)*(2 + m)))

Rule 68

Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] := Simp[((b*c - a*d)^(n+1)*(a + b*x)^(m+1)*Hypergeometric2F1[-n, m+1, m+2, -((d*(a + b*x))/(b*c - a*d))])/((b^(n+1)*(m+1)), x) /; FreeQ[{a, b, c, d, m}, x] && NeQ[b*c - a*d, 0] && !IntegerQ[m] && IntegerQ[n]

Rule 712

Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] := Int[ExpandIntegrand[(d + e*x)^m, 1/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] && !IntegerQ[m]

Rule 5926

```
Int[((a_.) + ArcTanh[(c_.)*(x_.)]*(b_.))*((d_.) + (e_.)*(x_.))^(q_.), x_Symbol]
:> Simp[((d + e*x)^(q + 1)*(a + b*ArcTanh[c*x]))/(e*(q + 1)), x] - Dist[(b*c)/(e*(q + 1)), Int[(d + e*x)^(q + 1)/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[q, -1]
```

Rule 6111

```
Int[((a_.) + ArcTanh[(c_.) + (d_.)*(x_.)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_.))^(m_.), x_Symbol]
:> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*ArcTanh[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned} \int (e + fx)^m (a + b \tanh^{-1}(c + dx)) dx &= \frac{\text{Subst}\left(\int \left(\frac{de-cf}{d} + \frac{fx}{d}\right)^m (a + b \tanh^{-1}(x)) dx, x, c + dx\right)}{d} \\ &= \frac{(e + fx)^{1+m} (a + b \tanh^{-1}(c + dx))}{f(1 + m)} - \frac{b \text{Subst}\left(\int \frac{\left(\frac{de-cf}{d} + \frac{fx}{d}\right)^{1+m}}{1-x^2} dx, x, c\right)}{f(1 + m)} \\ &= \frac{(e + fx)^{1+m} (a + b \tanh^{-1}(c + dx))}{f(1 + m)} - \frac{b \text{Subst}\left(\int \left(\frac{\left(\frac{de-cf}{d} + \frac{fx}{d}\right)^{1+m}}{2(1-x)} + \frac{\left(\frac{de-cf}{d} + \frac{fx}{d}\right)^{1+m}}{2(1+x)}\right) dx, x, c\right)}{f(1 + m)} \\ &= \frac{(e + fx)^{1+m} (a + b \tanh^{-1}(c + dx))}{f(1 + m)} - \frac{b \text{Subst}\left(\int \frac{\left(\frac{de-cf}{d} + \frac{fx}{d}\right)^{1+m}}{1-x} dx, x, c\right)}{2f(1 + m)} \\ &= \frac{(e + fx)^{1+m} (a + b \tanh^{-1}(c + dx))}{f(1 + m)} + \frac{bd(e + fx)^{2+m} {}_2F_1\left(1, 2 + m; 3 + m; \frac{de - (1 + c)f}{(e + fx)^2}\right)}{2f(de - (1 + c)f)(1 + m)} \end{aligned}$$

Mathematica [F] time = 0.08, size = 0, normalized size = 0.00

$$\int (e + fx)^m (a + b \tanh^{-1}(c + dx)) dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(e + f*x)^m*(a + b*ArcTanh[c + d*x]), x]
```

[Out] Integrate[(e + f*x)^m*(a + b*ArcTanh[c + d*x]), x]

fricas [F] time = 0.70, size = 0, normalized size = 0.00

$$\text{integral}\left((b \operatorname{artanh}(dx + c) + a)(fx + e)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x+e)^m*(a+b*arctanh(d*x+c)),x, algorithm="fricas")

[Out] integral((b*arctanh(d*x + c) + a)*(f*x + e)^m, x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \operatorname{artanh}(dx + c) + a)(fx + e)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x+e)^m*(a+b*arctanh(d*x+c)),x, algorithm="giac")

[Out] integrate((b*arctanh(d*x + c) + a)*(f*x + e)^m, x)

maple [F] time = 1.42, size = 0, normalized size = 0.00

$$\int (fx + e)^m (a + b \operatorname{arctanh}(dx + c)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f*x+e)^m*(a+b*arctanh(d*x+c)),x)

[Out] int((f*x+e)^m*(a+b*arctanh(d*x+c)),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{2} b \left(\frac{(fx + e)(fx + e)^m \log(-dx - c + 1)}{f(m + 1)} - \int \frac{(dfx + de + (df(m + 1)x + cf(m + 1) - f(m + 1)) \log(dx + c + 1))}{df(m + 1)x + cf(m + 1) - f(m + 1)} dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x+e)^m*(a+b*arctanh(d*x+c)),x, algorithm="maxima")

[Out] -1/2*b*((f*x + e)*(f*x + e)^m*log(-d*x - c + 1)/(f*(m + 1)) - integrate((d*f*x + d*e + (d*f*(m + 1)*x + c*f*(m + 1) - f*(m + 1))*log(d*x + c + 1))*(f*x + e)^m/(d*f*(m + 1)*x + c*f*(m + 1) - f*(m + 1)), x)) + (f*x + e)^(m + 1)*a/(f*(m + 1))

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (e + f x)^m (a + b \operatorname{atanh}(c + d x)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e + f*x)^m*(a + b*atanh(c + d*x)), x)`

[Out] `int((e + f*x)^m*(a + b*atanh(c + d*x)), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \operatorname{atanh}(c + d x)) (e + f x)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x+e)**m*(a+b*atanh(d*x+c)), x)`

[Out] `Integral((a + b*atanh(c + d*x))*(e + f*x)**m, x)`

$$3.53 \quad \int \frac{\tanh^{-1}(a+bx)}{c+dx^3} dx$$

Optimal. Leaf size=780

$$\frac{\operatorname{Li}_2\left(\frac{\sqrt[3]{d}(-a-bx+1)}{\sqrt[3]{d}(1-a)+b\sqrt[3]{c}}\right)}{6c^{2/3}\sqrt[3]{d}} - \frac{(-1)^{2/3}\operatorname{Li}_2\left(-\frac{\sqrt[3]{-1}\sqrt[3]{d}(-a-bx+1)}{b\sqrt[3]{c}-\sqrt[3]{-1}(1-a)\sqrt[3]{d}}\right)}{6c^{2/3}\sqrt[3]{d}} + \frac{\sqrt[3]{-1}\operatorname{Li}_2\left(\frac{(-1)^{2/3}\sqrt[3]{d}(-a-bx+1)}{(-1)^{2/3}\sqrt[3]{d}(1-a)+b\sqrt[3]{c}}\right)}{6c^{2/3}\sqrt[3]{d}} + \frac{\operatorname{Li}_2\left(-\frac{\sqrt[3]{d}(a+bx+1)}{b\sqrt[3]{c}-(a+1)\sqrt[3]{d}}\right)}{6c^{2/3}\sqrt[3]{d}} + \frac{(-1)^{2/3}}{6c^{2/3}\sqrt[3]{d}}$$

[Out] $-1/6*\ln(-b*x-a+1)*\ln(b*(c^{(1/3)}+d^{(1/3)}*x)/(b*c^{(1/3)}+(1-a)*d^{(1/3)}))/c^{(2/3)}/d^{(1/3)}+1/6*\ln(b*x+a+1)*\ln(b*(c^{(1/3)}+d^{(1/3)}*x)/(b*c^{(1/3)}-(1+a)*d^{(1/3)}))/c^{(2/3)}/d^{(1/3)}-1/6*(-1)^{(2/3)}*\ln(-b*x-a+1)*\ln(b*(c^{(1/3)}-(-1)^{(1/3)}*d^{(1/3)}*x)/(b*c^{(1/3)}-(-1)^{(1/3)}*(1-a)*d^{(1/3)}))/c^{(2/3)}/d^{(1/3)}+1/6*(-1)^{(2/3)}*\ln(b*x+a+1)*\ln(b*(c^{(1/3)}-(-1)^{(1/3)}*d^{(1/3)}*x)/(b*c^{(1/3)}+(-1)^{(1/3)}*(1+a)*d^{(1/3)}))/c^{(2/3)}/d^{(1/3)}+1/6*(-1)^{(1/3)}*\ln(-b*x-a+1)*\ln(b*(c^{(1/3)}+(-1)^{(2/3)}*d^{(1/3)}*x)/(b*c^{(1/3)}+(-1)^{(2/3)}*(1-a)*d^{(1/3)}))/c^{(2/3)}/d^{(1/3)}-1/6*(-1)^{(1/3)}*\ln(b*x+a+1)*\ln(b*(c^{(1/3)}+(-1)^{(2/3)}*d^{(1/3)}*x)/(b*c^{(1/3)}-(-1)^{(2/3)}*(1+a)*d^{(1/3)}))/c^{(2/3)}/d^{(1/3)}-1/6*polylog(2,d^{(1/3)}*(-b*x-a+1)/(b*c^{(1/3)}+(1-a)*d^{(1/3)}))/c^{(2/3)}/d^{(1/3)}-1/6*(-1)^{(2/3)}*polylog(2,-(-1)^{(1/3)}*d^{(1/3)}*(-b*x-a+1)/(b*c^{(1/3)}-(-1)^{(1/3)}*(1-a)*d^{(1/3)}))/c^{(2/3)}/d^{(1/3)}+1/6*(-1)^{(1/3)}*polylog(2,(-1)^{(2/3)}*d^{(1/3)}*(-b*x-a+1)/(b*c^{(1/3)}+(-1)^{(2/3)}*(1-a)*d^{(1/3)}))/c^{(2/3)}/d^{(1/3)}+1/6*polylog(2,-d^{(1/3)}*(b*x+a+1)/(b*c^{(1/3)}-(1+a)*d^{(1/3)}))/c^{(2/3)}/d^{(1/3)}+1/6*(-1)^{(2/3)}*polylog(2,(-1)^{(1/3)}*d^{(1/3)}*(b*x+a+1)/(b*c^{(1/3)}+(-1)^{(1/3)}*(1+a)*d^{(1/3)}))/c^{(2/3)}/d^{(1/3)}-1/6*(-1)^{(1/3)}*polylog(2,-(-1)^{(2/3)}*d^{(1/3)}*(b*x+a+1)/(b*c^{(1/3)}-(-1)^{(2/3)}*(1+a)*d^{(1/3)}))/c^{(2/3)}/d^{(1/3)}$

Rubi [A] time = 1.40, antiderivative size = 780, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {6115, 2409, 2394, 2393, 2391}

$$\frac{\operatorname{PolyLog}\left(2,\frac{\sqrt[3]{d}(-a-bx+1)}{(1-a)\sqrt[3]{d}+b\sqrt[3]{c}}\right)}{6c^{2/3}\sqrt[3]{d}} - \frac{(-1)^{2/3}\operatorname{PolyLog}\left(2,-\frac{\sqrt[3]{-1}\sqrt[3]{d}(-a-bx+1)}{b\sqrt[3]{c}-\sqrt[3]{-1}(1-a)\sqrt[3]{d}}\right)}{6c^{2/3}\sqrt[3]{d}} + \frac{\sqrt[3]{-1}\operatorname{PolyLog}\left(2,\frac{(-1)^{2/3}\sqrt[3]{d}(-a-bx+1)}{(-1)^{2/3}(1-a)\sqrt[3]{d}+b\sqrt[3]{c}}\right)}{6c^{2/3}\sqrt[3]{d}} + \frac{\operatorname{PolyLog}\left(2,-\frac{\sqrt[3]{d}(a+bx+1)}{b\sqrt[3]{c}-(a+1)\sqrt[3]{d}}\right)}{6c^{2/3}\sqrt[3]{d}} + \frac{(-1)^{2/3}}{6c^{2/3}\sqrt[3]{d}}$$

Antiderivative was successfully verified.

[In] Int[ArcTanh[a + b*x]/(c + d*x^3), x]

[Out] $-(\operatorname{Log}[1-a-b*x]*\operatorname{Log}[(b*(c^{(1/3)}+d^{(1/3)}*x))/(b*c^{(1/3)}+(1-a)*d^{(1/3)})])/(6*c^{(2/3)}*d^{(1/3)})+(\operatorname{Log}[1+a+b*x]*\operatorname{Log}[(b*(c^{(1/3)}+d^{(1/3)}*x))/(b*c^{(1/3)}-(1+a)*d^{(1/3)})])/(6*c^{(2/3)}*d^{(1/3)})-((-1)^{(2/3)}*\operatorname{Log}[1-a-b*x]*\operatorname{Log}[(b*(c^{(1/3)}-(-1)^{(1/3)}*d^{(1/3)}*x))/(b*c^{(1/3)}-(-1)^{(1/3)}*(1-a)*d^{(1/3)})])/(6*c^{(2/3)}*d^{(1/3)})+((-1)^{(2/3)}*\operatorname{Log}[1+a+b*x]*\operatorname{Log}[(b$

$$\begin{aligned} & * (c^{1/3} - (-1)^{1/3} d^{1/3} x) / (b c^{1/3} + (-1)^{1/3} (1 + a) d^{1/3}) \\ &] / (6 c^{2/3} d^{1/3}) + ((-1)^{1/3} \text{Log}[1 - a - b x] \text{Log}[(b c^{1/3} + (-1)^{2/3} d^{1/3} x) / (b c^{1/3} + (-1)^{2/3} (1 - a) d^{1/3})]) / (6 c^{2/3} d^{1/3}) \\ & - ((-1)^{1/3} \text{Log}[1 + a + b x] \text{Log}[(b c^{1/3} + (-1)^{2/3} d^{1/3} x) / (b c^{1/3} - (-1)^{2/3} (1 + a) d^{1/3})]) / (6 c^{2/3} d^{1/3}) - \text{PolyLog}[2, (d^{1/3} (1 - a - b x)) / (b c^{1/3} + (1 - a) d^{1/3})] / (6 c^{2/3} d^{1/3}) \\ & - ((-1)^{2/3} \text{PolyLog}[2, -(((-1)^{1/3} d^{1/3} (1 - a - b x)) / (b c^{1/3} - (-1)^{1/3} (1 - a) d^{1/3}))]) / (6 c^{2/3} d^{1/3}) + ((-1)^{1/3} \text{PolyLog}[2, ((-1)^{2/3} d^{1/3} (1 - a - b x)) / (b c^{1/3} + (-1)^{2/3} (1 - a) d^{1/3})]) / (6 c^{2/3} d^{1/3}) \\ & + \text{PolyLog}[2, -((d^{1/3} (1 + a + b x)) / (b c^{1/3} - (1 + a) d^{1/3}))] / (6 c^{2/3} d^{1/3}) + ((-1)^{2/3} \text{PolyLog}[2, ((-1)^{1/3} d^{1/3} (1 + a + b x)) / (b c^{1/3} + (-1)^{1/3} (1 + a) d^{1/3})]) / (6 c^{2/3} d^{1/3}) \\ & - ((-1)^{1/3} \text{PolyLog}[2, -(((-1)^{2/3} d^{1/3} (1 + a + b x)) / (b c^{1/3} - (-1)^{2/3} (1 + a) d^{1/3}))]) / (6 c^{2/3} d^{1/3}) \end{aligned}$$
Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] := -Simp[PolyLog[2, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_Symbol] := Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]*(b_.))/((f_.) + (g_.)*(x_)), x_Symbol] := Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x)^n])/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2409

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]*(b_.))^(p_.)*((f_.) + (g_.)*(x_)^(r_))^(q_.), x_Symbol] := Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && IntegerQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

Rule 6115

```
Int[ArcTanh[(c_) + (d_.)*(x_)]/((e_) + (f_.)*(x_)^(n_.)), x_Symbol] := Dist[1/2, Int[Log[1 + c + d*x]/(e + f*x^n), x], x] - Dist[1/2, Int[Log[1 - c -
```

d*x]/(e + f*x^n), x], x] /; FreeQ[{c, d, e, f}, x] && RationalQ[n]

Rubi steps

$$\begin{aligned}
 \int \frac{\tanh^{-1}(a + bx)}{c + dx^3} dx &= -\left(\frac{1}{2} \int \frac{\log(1 - a - bx)}{c + dx^3} dx\right) + \frac{1}{2} \int \frac{\log(1 + a + bx)}{c + dx^3} dx \\
 &= -\left(\frac{1}{2} \int \left(-\frac{\log(1 - a - bx)}{3c^{2/3}(-\sqrt[3]{c} - \sqrt[3]{d}x)} - \frac{\log(1 - a - bx)}{3c^{2/3}(-\sqrt[3]{c} + \sqrt[3]{-1}\sqrt[3]{d}x)} - \frac{\log(1 - a - bx)}{3c^{2/3}(-\sqrt[3]{c} - (-1)^{2/3}\sqrt[3]{d}x)} \right) dx \right) \\
 &= \frac{\int \frac{\log(1-a-bx)}{-\sqrt[3]{c}-\sqrt[3]{d}x} dx}{6c^{2/3}} + \frac{\int \frac{\log(1-a-bx)}{-\sqrt[3]{c}+\sqrt[3]{-1}\sqrt[3]{d}x} dx}{6c^{2/3}} + \frac{\int \frac{\log(1-a-bx)}{-\sqrt[3]{c}-(-1)^{2/3}\sqrt[3]{d}x} dx}{6c^{2/3}} - \frac{\int \frac{\log(1+a+bx)}{-\sqrt[3]{c}-\sqrt[3]{d}x} dx}{6c^{2/3}} - \frac{\int \frac{\log(1+a+bx)}{-\sqrt[3]{c}+\sqrt[3]{-1}\sqrt[3]{d}x} dx}{6c^{2/3}} - \frac{\int \frac{\log(1+a+bx)}{-\sqrt[3]{c}-(-1)^{2/3}\sqrt[3]{d}x} dx}{6c^{2/3}} \\
 &= -\frac{\log(1 - a - bx) \log\left(\frac{b(\sqrt[3]{c} + \sqrt[3]{d}x)}{b\sqrt[3]{c} + (1-a)\sqrt[3]{d}}\right)}{6c^{2/3}\sqrt[3]{d}} + \frac{\log(1 + a + bx) \log\left(\frac{b(\sqrt[3]{c} + \sqrt[3]{d}x)}{b\sqrt[3]{c} - (1+a)\sqrt[3]{d}}\right)}{6c^{2/3}\sqrt[3]{d}} - \frac{(-1)^{2/3} \log(1 - a - bx) \log\left(\frac{b(\sqrt[3]{c} + \sqrt[3]{d}x)}{b\sqrt[3]{c} - (-1+a)\sqrt[3]{d}}\right)}{6c^{2/3}\sqrt[3]{d}} \\
 &= -\frac{\log(1 - a - bx) \log\left(\frac{b(\sqrt[3]{c} + \sqrt[3]{d}x)}{b\sqrt[3]{c} + (1-a)\sqrt[3]{d}}\right)}{6c^{2/3}\sqrt[3]{d}} + \frac{\log(1 + a + bx) \log\left(\frac{b(\sqrt[3]{c} + \sqrt[3]{d}x)}{b\sqrt[3]{c} - (1+a)\sqrt[3]{d}}\right)}{6c^{2/3}\sqrt[3]{d}} - \frac{(-1)^{2/3} \log(1 - a - bx) \log\left(\frac{b(\sqrt[3]{c} + \sqrt[3]{d}x)}{b\sqrt[3]{c} - (-1+a)\sqrt[3]{d}}\right)}{6c^{2/3}\sqrt[3]{d}} \\
 &= -\frac{\log(1 - a - bx) \log\left(\frac{b(\sqrt[3]{c} + \sqrt[3]{d}x)}{b\sqrt[3]{c} + (1-a)\sqrt[3]{d}}\right)}{6c^{2/3}\sqrt[3]{d}} + \frac{\log(1 + a + bx) \log\left(\frac{b(\sqrt[3]{c} + \sqrt[3]{d}x)}{b\sqrt[3]{c} - (1+a)\sqrt[3]{d}}\right)}{6c^{2/3}\sqrt[3]{d}} - \frac{(-1)^{2/3} \log(1 - a - bx) \log\left(\frac{b(\sqrt[3]{c} + \sqrt[3]{d}x)}{b\sqrt[3]{c} - (-1+a)\sqrt[3]{d}}\right)}{6c^{2/3}\sqrt[3]{d}}
 \end{aligned}$$

Mathematica [A] time = 0.76, size = 623, normalized size = 0.80

$$-\text{Li}_2\left(-\frac{\sqrt[3]{d}(a+bx-1)}{b\sqrt[3]{c}-(a-1)\sqrt[3]{d}}\right) - (-1)^{2/3}\text{Li}_2\left(\frac{\sqrt[3]{-1}\sqrt[3]{d}(a+bx-1)}{\sqrt[3]{-1}\sqrt[3]{d}(a-1)+b\sqrt[3]{c}}\right) + \sqrt[3]{-1}\text{Li}_2\left(\frac{(-1)^{2/3}\sqrt[3]{d}(a+bx-1)}{(-1)^{2/3}(a-1)\sqrt[3]{d}-b\sqrt[3]{c}}\right) + \text{Li}_2\left(-\frac{\sqrt[3]{d}(a+bx+1)}{b\sqrt[3]{c}-(a+1)\sqrt[3]{d}}\right) + (-1)^{2/3}\text{Li}_2\left(\frac{\sqrt[3]{-1}\sqrt[3]{d}(a+bx+1)}{\sqrt[3]{-1}\sqrt[3]{d}(a+1)+b\sqrt[3]{c}}\right) + \sqrt[3]{-1}\text{Li}_2\left(\frac{(-1)^{2/3}\sqrt[3]{d}(a+bx+1)}{(-1)^{2/3}(a+1)\sqrt[3]{d}-b\sqrt[3]{c}}\right)$$

Antiderivative was successfully verified.

[In] Integrate[ArcTanh[a + b*x]/(c + d*x^3), x]

[Out] $(-\text{Log}[1 - a - b*x]*\text{Log}[(b*(c^{1/3} + d^{1/3}*x))/(b*c^{1/3} - (-1 + a)*d^{1/3})]) + \text{Log}[1 + a + b*x]*\text{Log}[(b*(c^{1/3} + d^{1/3}*x))/(b*c^{1/3} - (1 + a)*d^{1/3})] - (-1)^{2/3}*\text{Log}[1 - a - b*x]*\text{Log}[(b*(c^{1/3} - (-1)^{1/3}*d^{1/3}*x))/(b*c^{1/3} + (-1)^{1/3}*(-1 + a)*d^{1/3})] + (-1)^{2/3}*\text{Log}[1 + a + b*x]*\text{Log}[(b*(c^{1/3} - (-1)^{1/3}*d^{1/3}*x))/(b*c^{1/3} + (-1)^{1/3}*(1 + a)*d^{1/3})]$

$$+ a)d^{1/3})] + (-1)^{1/3} \text{Log}[1 - a - b*x] \text{Log}[(b*(c^{1/3} + (-1)^{2/3}*d^{1/3}*x))/(b*c^{1/3} - (-1)^{2/3}*(-1 + a)*d^{1/3})] - (-1)^{1/3} \text{Log}[1 + a + b*x] \text{Log}[(b*(c^{1/3} + (-1)^{2/3}*d^{1/3}*x))/(b*c^{1/3} - (-1)^{2/3}*(1 + a)*d^{1/3})] - \text{PolyLog}[2, -((d^{1/3}*(-1 + a + b*x))/(b*c^{1/3} - (-1 + a)*d^{1/3}))] - (-1)^{2/3} \text{PolyLog}[2, ((-1)^{1/3}*d^{1/3}*(-1 + a + b*x))/(b*c^{1/3} + (-1)^{1/3}*(-1 + a)*d^{1/3})] + (-1)^{1/3} \text{PolyLog}[2, ((-1)^{2/3}*d^{1/3}*(-1 + a + b*x))/(-b*c^{1/3}) + (-1)^{2/3}*(-1 + a)*d^{1/3})] + \text{PolyLog}[2, -((d^{1/3}*(1 + a + b*x))/(b*c^{1/3} - (1 + a)*d^{1/3}))] + (-1)^{2/3} \text{PolyLog}[2, ((-1)^{1/3}*d^{1/3}*(1 + a + b*x))/(b*c^{1/3} + (-1)^{1/3}*(1 + a)*d^{1/3})] - (-1)^{1/3} \text{PolyLog}[2, ((-1)^{2/3}*d^{1/3}*(1 + a + b*x))/(-b*c^{1/3}) + (-1)^{2/3}*(1 + a)*d^{1/3})]/(6*c^{2/3}*d^{1/3})$$

fricas [F] time = 0.55, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\text{artanh}(bx+a)}{dx^3+c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctanh(b*x+a)/(d*x^3+c), x, algorithm="fricas")

[Out] integral(arctanh(b*x + a)/(d*x^3 + c), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

sage0x

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctanh(b*x+a)/(d*x^3+c), x, algorithm="giac")

[Out] sage0*x

maple [C] time = 0.91, size = 587, normalized size = 0.75

$$2b^2 \left(\sum_{R1=\text{RootOf}((a^3d-cb^3-3a^2d+3ad-d)_Z^6+(3a^3d-3cb^3-3a^2d-3ad+3d)_Z^4+(3a^3d-3cb^3+3a^2d-3ad-3d)_Z^2+a^3d-cb^3+3a^2d+3ad+d)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arctanh(b*x+a)/(d*x^3+c), x)

```
[Out] -2/3*b^2*sum(1/(_R1^4*a^3*d-_R1^4*b^3*c-3*_R1^4*a^2*d+3*_R1^4*a*d+2*_R1^2*a^3*d-2*_R1^2*b^3*c-_R1^4*d-2*_R1^2*a^2*d-2*_R1^2*a*d+a^3*d-b^3*c+2*_R1^2*d+a^2*d-a*d-d)*(arctanh(b*x+a)*ln((_R1-(b*x+a+1)/(1-(b*x+a)^2)^(1/2))/_R1)+dilog((_R1-(b*x+a+1)/(1-(b*x+a)^2)^(1/2))/_R1)),_R1=RootOf((a^3*d-b^3*c-3*a^2*d+3*a*d-d)*_Z^6+(3*a^3*d-3*b^3*c-3*a^2*d-3*a*d+3*d)*_Z^4+(3*a^3*d-3*b^3*c+3*a^2*d-3*a*d-3*d)*_Z^2+a^3*d-c*b^3+3*a^2*d+3*a*d+d))-2/3*b^2*sum(_R1^2/(_R1^4*a^3*d-_R1^4*b^3*c-3*_R1^4*a^2*d+3*_R1^4*a*d+2*_R1^2*a^3*d-2*_R1^2*b^3*c-_R1^4*d-2*_R1^2*a^2*d-2*_R1^2*a*d+a^3*d-b^3*c+2*_R1^2*d+a^2*d-a*d-d)*(arctanh(b*x+a)*ln((_R1-(b*x+a+1)/(1-(b*x+a)^2)^(1/2))/_R1)+dilog((_R1-(b*x+a+1)/(1-(b*x+a)^2)^(1/2))/_R1)),_R1=RootOf((a^3*d-b^3*c-3*a^2*d+3*a*d-d)*_Z^6+(3*a^3*d-3*b^3*c-3*a^2*d-3*a*d+3*d)*_Z^4+(3*a^3*d-3*b^3*c+3*a^2*d-3*a*d-3*d)*_Z^2+a^3*d-c*b^3+3*a^2*d+3*a*d+d))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{artanh}(bx+a)}{dx^3+c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(b*x+a)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate(arctanh(b*x + a)/(d*x^3 + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{atanh}(a+bx)}{dx^3+c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a + b*x)/(c + d*x^3),x)
```

```
[Out] int(atanh(a + b*x)/(c + d*x^3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(b*x+a)/(d*x**3+c),x)
```

```
[Out] Timed out
```

$$3.54 \quad \int \frac{\tanh^{-1}(a+bx)}{c+dx^2} dx$$

Optimal. Leaf size=481

$$\frac{\operatorname{Li}_2\left(-\frac{\sqrt{d}(-a-bx+1)}{b\sqrt{-c}-(1-a)\sqrt{d}}\right)}{4\sqrt{-c}\sqrt{d}} + \frac{\operatorname{Li}_2\left(\frac{\sqrt{d}(-a-bx+1)}{\sqrt{d}(1-a)+b\sqrt{-c}}\right)}{4\sqrt{-c}\sqrt{d}} - \frac{\operatorname{Li}_2\left(-\frac{\sqrt{d}(a+bx+1)}{b\sqrt{-c}-(a+1)\sqrt{d}}\right)}{4\sqrt{-c}\sqrt{d}} + \frac{\operatorname{Li}_2\left(\frac{\sqrt{d}(a+bx+1)}{\sqrt{d}(a+1)+b\sqrt{-c}}\right)}{4\sqrt{-c}\sqrt{d}} - \frac{\log(-a-bx+1)\log\left(\frac{\sqrt{d}(-a-bx+1)}{b\sqrt{-c}-(1-a)\sqrt{d}}\right)}{4\sqrt{-c}\sqrt{d}}$$

[Out] $-1/4*\ln(-b*x-a+1)*\ln(b*((-c)^{(1/2)}-x*d^{(1/2)})/(b*(-c)^{(1/2)}-(1-a)*d^{(1/2)}))$
 $/(-c)^{(1/2)}/d^{(1/2)}+1/4*\ln(b*x+a+1)*\ln(b*((-c)^{(1/2)}-x*d^{(1/2)})/(b*(-c)^{(1/2)}$
 $+ (1+a)*d^{(1/2)})))/(-c)^{(1/2)}/d^{(1/2)}+1/4*\ln(-b*x-a+1)*\ln(b*((-c)^{(1/2)}+x*d$
 $^{(1/2)})/(b*(-c)^{(1/2)}+(1-a)*d^{(1/2)})))/(-c)^{(1/2)}/d^{(1/2)}-1/4*\ln(b*x+a+1)*\ln$
 $(b*((-c)^{(1/2)}+x*d^{(1/2)})/(b*(-c)^{(1/2)}-(1+a)*d^{(1/2)})))/(-c)^{(1/2)}/d^{(1/2)}$
 $-1/4*\operatorname{polylog}(2, -(b*x-a+1)*d^{(1/2)}/(b*(-c)^{(1/2)}-(1-a)*d^{(1/2)})))/(-c)^{(1/2)}/$
 $d^{(1/2)}+1/4*\operatorname{polylog}(2, (b*x-a+1)*d^{(1/2)}/(b*(-c)^{(1/2)}+(1-a)*d^{(1/2)})))/(-c)$
 $^{(1/2)}/d^{(1/2)}-1/4*\operatorname{polylog}(2, -(b*x+a+1)*d^{(1/2)}/(b*(-c)^{(1/2)}-(1+a)*d^{(1/2)}$
 $)))/(-c)^{(1/2)}/d^{(1/2)}+1/4*\operatorname{polylog}(2, (b*x+a+1)*d^{(1/2)}/(b*(-c)^{(1/2)}+(1+a)*d$
 $^{(1/2)})))/(-c)^{(1/2)}/d^{(1/2)}$

Rubi [A] time = 0.61, antiderivative size = 481, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {6115, 2409, 2394, 2393, 2391}

$$\frac{\operatorname{PolyLog}\left(2, -\frac{\sqrt{d}(-a-bx+1)}{b\sqrt{-c}-(1-a)\sqrt{d}}\right)}{4\sqrt{-c}\sqrt{d}} + \frac{\operatorname{PolyLog}\left(2, \frac{\sqrt{d}(-a-bx+1)}{(1-a)\sqrt{d}+b\sqrt{-c}}\right)}{4\sqrt{-c}\sqrt{d}} - \frac{\operatorname{PolyLog}\left(2, -\frac{\sqrt{d}(a+bx+1)}{b\sqrt{-c}-(a+1)\sqrt{d}}\right)}{4\sqrt{-c}\sqrt{d}} + \frac{\operatorname{PolyLog}\left(2, \frac{\sqrt{d}(a+bx+1)}{(a+1)\sqrt{d}+b\sqrt{-c}}\right)}{4\sqrt{-c}\sqrt{d}}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\operatorname{ArcTanh}[a + b*x]/(c + d*x^2), x]$

[Out] $-(\operatorname{Log}[1 - a - b*x]*\operatorname{Log}[(b*(\operatorname{Sqrt}[-c] - \operatorname{Sqrt}[d]*x))/(b*\operatorname{Sqrt}[-c] - (1 - a)*\operatorname{Sqrt}[d])])/(4*\operatorname{Sqrt}[-c]*\operatorname{Sqrt}[d]) + (\operatorname{Log}[1 + a + b*x]*\operatorname{Log}[(b*(\operatorname{Sqrt}[-c] - \operatorname{Sqrt}[d]*x))/(b*\operatorname{Sqrt}[-c] + (1 + a)*\operatorname{Sqrt}[d])])/(4*\operatorname{Sqrt}[-c]*\operatorname{Sqrt}[d]) + (\operatorname{Log}[1 - a - b*x]*\operatorname{Log}[(b*(\operatorname{Sqrt}[-c] + \operatorname{Sqrt}[d]*x))/(b*\operatorname{Sqrt}[-c] + (1 - a)*\operatorname{Sqrt}[d])])/(4*\operatorname{Sqrt}[-c]*\operatorname{Sqrt}[d]) - (\operatorname{Log}[1 + a + b*x]*\operatorname{Log}[(b*(\operatorname{Sqrt}[-c] + \operatorname{Sqrt}[d]*x))/(b*\operatorname{Sqrt}[-c] - (1 + a)*\operatorname{Sqrt}[d])])/(4*\operatorname{Sqrt}[-c]*\operatorname{Sqrt}[d]) - \operatorname{PolyLog}[2, -((\operatorname{Sqrt}[d]*(1 - a - b*x))/(b*\operatorname{Sqrt}[-c] - (1 - a)*\operatorname{Sqrt}[d]))]/(4*\operatorname{Sqrt}[-c]*\operatorname{Sqrt}[d]) + \operatorname{PolyLog}[2, (\operatorname{Sqrt}[d]*(1 - a - b*x))/(b*\operatorname{Sqrt}[-c] + (1 - a)*\operatorname{Sqrt}[d])]/(4*\operatorname{Sqrt}[-c]*\operatorname{Sqrt}[d]) - \operatorname{PolyLog}[2, -((\operatorname{Sqrt}[d]*(1 + a + b*x))/(b*\operatorname{Sqrt}[-c] - (1 + a)*\operatorname{Sqrt}[d]))]/(4*\operatorname{Sqrt}[-c]*\operatorname{Sqrt}[d]) + \operatorname{PolyLog}[2, (\operatorname{Sqrt}[d]*(1 + a + b*x))/(b*\operatorname{Sqrt}[-c] + (1 + a)*\operatorname{Sqrt}[d])]/(4*\operatorname{Sqrt}[-c]*\operatorname{Sqrt}[d])$

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] := -Simp[PolyLog[2, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_Symbol] := Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]*(b_.))/((f_.) + (g_.)*(x_)), x_Symbol] := Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x)^n])/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2409

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]*(b_.))^(p_.)*((f_.) + (g_.)*(x_)^(r_))^(q_.), x_Symbol] := Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && IntegerQ[p, 0] && IntegerQ[q] && (IntegerQ[r] || (IntegerQ[r] && NeQ[r, 1]))
```

Rule 6115

```
Int[ArcTanh[(c_) + (d_.)*(x_)]/((e_) + (f_.)*(x_)^(n_.)), x_Symbol] := Dist[1/2, Int[Log[1 + c + d*x]/(e + f*x^n), x], x] - Dist[1/2, Int[Log[1 - c - d*x]/(e + f*x^n), x], x] /; FreeQ[{c, d, e, f}, x] && RationalQ[n]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tanh^{-1}(a+bx)}{c+dx^2} dx &= -\left(\frac{1}{2} \int \frac{\log(1-a-bx)}{c+dx^2} dx\right) + \frac{1}{2} \int \frac{\log(1+a+bx)}{c+dx^2} dx \\
&= -\left(\frac{1}{2} \int \left(\frac{\sqrt{-c} \log(1-a-bx)}{2c(\sqrt{-c}-\sqrt{d}x)} + \frac{\sqrt{-c} \log(1-a-bx)}{2c(\sqrt{-c}+\sqrt{d}x)}\right) dx\right) + \frac{1}{2} \int \left(\frac{\sqrt{-c} \log(1+a+bx)}{2c(\sqrt{-c}-\sqrt{d}x)} + \frac{\sqrt{-c} \log(1+a+bx)}{2c(\sqrt{-c}+\sqrt{d}x)}\right) dx \\
&= \frac{\int \frac{\log(1-a-bx)}{\sqrt{-c}-\sqrt{d}x} dx}{4\sqrt{-c}} + \frac{\int \frac{\log(1-a-bx)}{\sqrt{-c}+\sqrt{d}x} dx}{4\sqrt{-c}} - \frac{\int \frac{\log(1+a+bx)}{\sqrt{-c}-\sqrt{d}x} dx}{4\sqrt{-c}} - \frac{\int \frac{\log(1+a+bx)}{\sqrt{-c}+\sqrt{d}x} dx}{4\sqrt{-c}} \\
&= -\frac{\log(1-a-bx) \log\left(\frac{b(\sqrt{-c}-\sqrt{d}x)}{b\sqrt{-c}-(1-a)\sqrt{d}}\right)}{4\sqrt{-c}\sqrt{d}} + \frac{\log(1+a+bx) \log\left(\frac{b(\sqrt{-c}-\sqrt{d}x)}{b\sqrt{-c}+(1+a)\sqrt{d}}\right)}{4\sqrt{-c}\sqrt{d}} + \frac{\log(1-a-bx) \log\left(\frac{b(\sqrt{-c}+\sqrt{d}x)}{b\sqrt{-c}-(1-a)\sqrt{d}}\right)}{4\sqrt{-c}\sqrt{d}} \\
&= -\frac{\log(1-a-bx) \log\left(\frac{b(\sqrt{-c}-\sqrt{d}x)}{b\sqrt{-c}-(1-a)\sqrt{d}}\right)}{4\sqrt{-c}\sqrt{d}} + \frac{\log(1+a+bx) \log\left(\frac{b(\sqrt{-c}-\sqrt{d}x)}{b\sqrt{-c}+(1+a)\sqrt{d}}\right)}{4\sqrt{-c}\sqrt{d}} + \frac{\log(1-a-bx) \log\left(\frac{b(\sqrt{-c}+\sqrt{d}x)}{b\sqrt{-c}-(1-a)\sqrt{d}}\right)}{4\sqrt{-c}\sqrt{d}} \\
&= -\frac{\log(1-a-bx) \log\left(\frac{b(\sqrt{-c}-\sqrt{d}x)}{b\sqrt{-c}-(1-a)\sqrt{d}}\right)}{4\sqrt{-c}\sqrt{d}} + \frac{\log(1+a+bx) \log\left(\frac{b(\sqrt{-c}-\sqrt{d}x)}{b\sqrt{-c}+(1+a)\sqrt{d}}\right)}{4\sqrt{-c}\sqrt{d}} + \frac{\log(1-a-bx) \log\left(\frac{b(\sqrt{-c}+\sqrt{d}x)}{b\sqrt{-c}-(1-a)\sqrt{d}}\right)}{4\sqrt{-c}\sqrt{d}}
\end{aligned}$$

Mathematica [A] time = 0.31, size = 365, normalized size = 0.76

$$\text{Li}_2\left(-\frac{\sqrt{d}(a+bx-1)}{b\sqrt{-c}-(a-1)\sqrt{d}}\right) - \text{Li}_2\left(\frac{\sqrt{d}(a+bx-1)}{\sqrt{d}(a-1)+b\sqrt{-c}}\right) - \text{Li}_2\left(-\frac{\sqrt{d}(a+bx+1)}{b\sqrt{-c}-(a+1)\sqrt{d}}\right) + \text{Li}_2\left(\frac{\sqrt{d}(a+bx+1)}{\sqrt{d}(a+1)+b\sqrt{-c}}\right) - \log(-a-bx+1) \log\left(\frac{b(\sqrt{-c}-\sqrt{d}x)}{b\sqrt{-c}-(1-a)\sqrt{d}}\right)$$

Antiderivative was successfully verified.

[In] Integrate[ArcTanh[a + b*x]/(c + d*x^2), x]

[Out] $(-\text{Log}[1-a-b*x]*\text{Log}[(b*(\text{Sqrt}[-c]-\text{Sqrt}[d]*x))/(b*\text{Sqrt}[-c]+(-1+a)*\text{Sqrt}[d]]) + \text{Log}[1+a+b*x]*\text{Log}[(b*(\text{Sqrt}[-c]-\text{Sqrt}[d]*x))/(b*\text{Sqrt}[-c]+(1+a)*\text{Sqrt}[d]]) + \text{Log}[1-a-b*x]*\text{Log}[(b*(\text{Sqrt}[-c]+\text{Sqrt}[d]*x))/(b*\text{Sqrt}[-c]-(-1+a)*\text{Sqrt}[d]]) - \text{Log}[1+a+b*x]*\text{Log}[(b*(\text{Sqrt}[-c]+\text{Sqrt}[d]*x))/(b*\text{Sqrt}[-c]-(1+a)*\text{Sqrt}[d]]) + \text{PolyLog}[2, -((\text{Sqrt}[d]*(-1+a+b*x))/(b*\text{Sqrt}[-c]-(-1+a)*\text{Sqrt}[d]))] - \text{PolyLog}[2, (\text{Sqrt}[d]*(-1+a+b*x))/(b*\text{Sqrt}[-c]+(-1+a)*\text{Sqrt}[d])] - \text{PolyLog}[2, -((\text{Sqrt}[d]*(1+a+b*x))/(b*\text{Sqrt}[-c]-(1+a)*\text{Sqrt}[d]))] + \text{PolyLog}[2, (\text{Sqrt}[d]*(1+a+b*x))/(b*\text{Sqrt}[-c]+(1+a)*\text{Sqrt}[d])])/(4*\text{Sqrt}[-c]*\text{Sqrt}[d])$

fricas [F] time = 0.52, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\text{artanh}(bx+a)}{dx^2+c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctanh(b*x+a)/(d*x^2+c),x, algorithm="fricas")

[Out] integral(arctanh(b*x + a)/(d*x^2 + c), x)

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctanh(b*x+a)/(d*x^2+c),x, algorithm="giac")

[Out] Timed out

maple [B] time = 0.74, size = 1300, normalized size = 2.70

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arctanh(b*x+a)/(d*x^2+c),x)

[Out] $\frac{1}{2} \frac{1}{b} (-b^2 c d)^{1/2} / c \ln(1 - (a^2 d + b^2 c - 2 a d + d) (b x + a + 1)^2 / (1 - (b x + a)^2)) / (-a^2 d - b^2 c - 2 (-b^2 c d)^{1/2} + d) * \text{arctanh}(b x + a) * a^2 / (a^2 d + b^2 c + 2 (-b^2 c d)^{1/2} - d) + \frac{1}{2} \frac{1}{b} (-b^2 c d)^{1/2} / d \ln(1 - (a^2 d + b^2 c - 2 a d + d) (b x + a + 1)^2 / (1 - (b x + a)^2)) / (-a^2 d - b^2 c - 2 (-b^2 c d)^{1/2} + d) * \text{arctanh}(b x + a) / (a^2 d + b^2 c + 2 (-b^2 c d)^{1/2} - d) - \frac{1}{2} \frac{1}{b} (-b^2 c d)^{1/2} / c * \text{arctanh}(b x + a)^2 * a^2 / (a^2 d + b^2 c + 2 (-b^2 c d)^{1/2} - d) - \frac{1}{2} \frac{1}{b} (-b^2 c d)^{1/2} / d * \text{arctanh}(b x + a)^2 / (a^2 d + b^2 c + 2 (-b^2 c d)^{1/2} - d) + \frac{1}{4} \frac{1}{b} (-b^2 c d)^{1/2} / c * \text{polylog}(2, (a^2 d + b^2 c - 2 a d + d) (b x + a + 1)^2 / (1 - (b x + a)^2)) / (-a^2 d - b^2 c - 2 (-b^2 c d)^{1/2} + d) * a^2 / (a^2 d + b^2 c + 2 (-b^2 c d)^{1/2} - d) + \frac{1}{4} \frac{1}{b} (-b^2 c d)^{1/2} / d * \text{polylog}(2, (a^2 d + b^2 c - 2 a d + d) (b x + a + 1)^2 / (1 - (b x + a)^2)) / (-a^2 d - b^2 c - 2 (-b^2 c d)^{1/2} + d) / (a^2 d + b^2 c + 2 (-b^2 c d)^{1/2} - d) - b / (a^2 d + b^2 c + 2 (-b^2 c d)^{1/2} - d) * \ln(1 - (a^2 d + b^2 c - 2 a d + d) (b x + a + 1)^2 / (1 - (b x + a)^2)) / (-a^2 d - b^2 c - 2 (-b^2 c d)^{1/2} + d) * \text{arctanh}(b x + a) + b / (a^2 d + b^2 c + 2 (-b^2 c d)^{1/2} - d) * \text{arctanh}(b x + a)^2 - \frac{1}{2} \frac{1}{b} (-b^2 c d)^{1/2} / c \ln(1 - (a^2 d + b^2 c - 2 a d + d) (b x + a + 1)^2 / (1 - (b x + a)^2)) / (-a^2 d - b^2 c - 2 (-b^2 c d)^{1/2} + d) * \text{arctanh}(b x + a) / (a^2 d + b^2 c + 2 (-b^2 c d)^{1/2} - d) + \frac{1}{2} \frac{1}{b} (-b^2 c d)^{1/2} / c * \text{arctanh}(b x + a)^2 / (a^2 d + b^2 c + 2 (-b^2 c d)^{1/2} - d) - \frac{1}{2} \frac{1}{b} (-b^2 c d)^{1/2} / (a^2 d + b^2 c + 2 (-b^2 c d)^{1/2} - d) * \text{polylog}(2, (a^2 d + b^2 c - 2 a d + d) (b x + a + 1)^2 / (1 - (b x + a)^2)) / (-a^2 d - b^2 c - 2 (-b^2 c d)^{1/2} + d)$

$$-b^2c-2*(-b^2c*d)^{(1/2)+d)-1/4/b*(-b^2c*d)^{(1/2)}/c*\text{polylog}(2, (a^2*d+b^2*c-2*a*d+d)*(b*x+a+1)^2/(1-(b*x+a)^2)/(-a^2*d-b^2c-2*(-b^2c*d)^{(1/2)+d}))/ (a^2*d+b^2c+2*(-b^2c*d)^{(1/2)-d}-1/2/b*(-b^2c*d)^{(1/2)}/c/d*\text{arctanh}(b*x+a)*\ln(1-(a^2*d+b^2c-2*a*d+d)*(b*x+a+1)^2/(1-(b*x+a)^2)/(-a^2*d-b^2c+2*(-b^2c*d)^{(1/2)+d}))+1/2/b*(-b^2c*d)^{(1/2)}/c/d*\text{arctanh}(b*x+a)^2-1/4/b*(-b^2c*d)^{(1/2)}/c/d*\text{polylog}(2, (a^2*d+b^2c-2*a*d+d)*(b*x+a+1)^2/(1-(b*x+a)^2)/(-a^2*d-b^2c+2*(-b^2c*d)^{(1/2)+d}))$$

maxima [C] time = 0.59, size = 589, normalized size = 1.22

$$\frac{\arctan\left(\frac{dx}{\sqrt{cd}}\right) \operatorname{artanh}(bx+a)}{\sqrt{cd}} + \frac{\left(\arctan\left(\frac{(b^2x+(a+1)b)\sqrt{c}\sqrt{d}}{b^2c+(a^2+2a+1)d}, \frac{(a+1)bdx+(a^2+2a+1)d}{b^2c+(a^2+2a+1)d}\right) - \arctan\left(\frac{(b^2x+(a-1)b)\sqrt{c}\sqrt{d}}{b^2c+(a^2-2a+1)d}, \frac{(a-1)bdx+(a^2-2a+1)d}{b^2c+(a^2-2a+1)d}\right)\right)}{\sqrt{cd}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctanh(b*x+a)/(d*x^2+c),x, algorithm="maxima")

[Out] arctan(d*x/sqrt(c*d))*arctanh(b*x + a)/sqrt(c*d) + 1/4*((arctan2((b^2*x + (a + 1)*b)*sqrt(c)*sqrt(d)/(b^2*c + (a^2 + 2*a + 1)*d), ((a + 1)*b*d*x + (a^2 + 2*a + 1)*d)/(b^2*c + (a^2 + 2*a + 1)*d)) - arctan2((b^2*x + (a - 1)*b)*sqrt(c)*sqrt(d)/(b^2*c + (a^2 - 2*a + 1)*d), ((a - 1)*b*d*x + (a^2 - 2*a + 1)*d)/(b^2*c + (a^2 - 2*a + 1)*d)))*log(d*x^2 + c) - arctan(sqrt(d)*x/sqrt(c))*log((b^2*d*x^2 + 2*(a + 1)*b*d*x + (a^2 + 2*a + 1)*d)/(b^2*c + (a^2 + 2*a + 1)*d)) + arctan(sqrt(d)*x/sqrt(c))*log((b^2*d*x^2 + 2*(a - 1)*b*d*x + (a^2 - 2*a + 1)*d)/(b^2*c + (a^2 - 2*a + 1)*d)) - I*dilog(((a + 1)*b*d*x + b^2*c - (I*b^2*x + (-I*a - I)*b)*sqrt(c)*sqrt(d))/(b^2*c + (2*I*a + 2*I)*b*sqrt(c)*sqrt(d) - (a^2 + 2*a + 1)*d)) + I*dilog(((a + 1)*b*d*x + b^2*c + (I*b^2*x + (-I*a - I)*b)*sqrt(c)*sqrt(d))/(b^2*c - (2*I*a + 2*I)*b*sqrt(c)*sqrt(d) - (a^2 + 2*a + 1)*d)) + I*dilog(((a - 1)*b*d*x + b^2*c - (I*b^2*x + (-I*a + I)*b)*sqrt(c)*sqrt(d))/(b^2*c + (2*I*a - 2*I)*b*sqrt(c)*sqrt(d) - (a^2 - 2*a + 1)*d)) - I*dilog(((a - 1)*b*d*x + b^2*c + (I*b^2*x + (-I*a + I)*b)*sqrt(c)*sqrt(d))/(b^2*c - (2*I*a - 2*I)*b*sqrt(c)*sqrt(d) - (a^2 - 2*a + 1)*d)))/sqrt(c*d)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{atanh}(a + bx)}{dx^2 + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(atanh(a + b*x)/(c + d*x^2),x)

[Out] int(atanh(a + b*x)/(c + d*x^2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(atanh(b*x+a)/(d*x**2+c),x)

[Out] Timed out

$$3.55 \quad \int \frac{\tanh^{-1}(a+bx)}{c+dx} dx$$

Optimal. Leaf size=120

$$\frac{\operatorname{Li}_2\left(1 - \frac{2b(c+dx)}{(bc-ad+d)(a+bx+1)}\right)}{2d} + \frac{\tanh^{-1}(a+bx) \log\left(\frac{2b(c+dx)}{(a+bx+1)(-ad+bc+d)}\right)}{d} + \frac{\operatorname{Li}_2\left(1 - \frac{2}{a+bx+1}\right)}{2d} - \frac{\log\left(\frac{2}{a+bx+1}\right) \tanh^{-1}(a+bx)}{d}$$

[Out] $-\operatorname{arctanh}(b*x+a)*\ln(2/(b*x+a+1))/d+\operatorname{arctanh}(b*x+a)*\ln(2*b*(d*x+c)/(-a*d+b*c+d)/(b*x+a+1))/d+1/2*\operatorname{polylog}(2,1-2/(b*x+a+1))/d-1/2*\operatorname{polylog}(2,1-2*b*(d*x+c)/(-a*d+b*c+d)/(b*x+a+1))/d$

Rubi [A] time = 0.13, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {6111, 5920, 2402, 2315, 2447}

$$-\frac{\operatorname{PolyLog}\left(2,1 - \frac{2b(c+dx)}{(a+bx+1)(-ad+bc+d)}\right)}{2d} + \frac{\operatorname{PolyLog}\left(2,1 - \frac{2}{a+bx+1}\right)}{2d} + \frac{\tanh^{-1}(a+bx) \log\left(\frac{2b(c+dx)}{(a+bx+1)(-ad+bc+d)}\right)}{d} - \frac{\log\left(\frac{2}{a+bx+1}\right) \tanh^{-1}(a+bx)}{d}$$

Antiderivative was successfully verified.

[In] `Int[ArcTanh[a + b*x]/(c + d*x), x]`

[Out] $-\left(\left(\operatorname{ArcTanh}[a + b*x]*\operatorname{Log}\left[\frac{2}{1 + a + b*x}\right]\right)/d\right) + \left(\operatorname{ArcTanh}[a + b*x]*\operatorname{Log}\left[\frac{2*b*(c + d*x)}{(b*c + d - a*d)*(1 + a + b*x)}\right]\right)/d + \operatorname{PolyLog}\left[2, 1 - \frac{2}{1 + a + b*x}\right]/(2*d) - \operatorname{PolyLog}\left[2, 1 - \frac{2*b*(c + d*x)}{(b*c + d - a*d)*(1 + a + b*x)}\right]/(2*d)$

Rule 2315

`Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] := -Simp[PolyLog[2, 1 - c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]`

Rule 2402

`Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] := -Dist[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]`

Rule 2447

`Int[Log[u]*(Pq_)^(m_.), x_Symbol] := With[{C = FullSimplify[(Pq^m*(1 - u))/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x] /; IntegerQ[m] && PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,`

x][[2]], Expon[Pq, x]]

Rule 5920

```
Int[((a_.) + ArcTanh[(c_.)*(x_.)]*(b_.))/((d_.) + (e_.)*(x_.)), x_Symbol] :> -
Simp[((a + b*ArcTanh[c*x])*Log[2/(1 + c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x))
/((c*d + e)*(1 + c*x))]/(1 - c^2*x^2), x], x] + Simp[((a + b*ArcTanh[c*x])*
Log[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))]/e, x)) /; FreeQ[{a, b, c, d, e}
, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rule 6111

```
Int[((a_.) + ArcTanh[(c_.) + (d_.)*(x_.)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_.))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcTanh[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGt
Q[p, 0]
```

Rubi steps

$$\int \frac{\tanh^{-1}(a + bx)}{c + dx} dx = \frac{\text{Subst}\left(\int \frac{\tanh^{-1}(x)}{\frac{bc-ad}{b} + \frac{dx}{b}} dx, x, a + bx\right)}{b}$$

$$= -\frac{\tanh^{-1}(a + bx) \log\left(\frac{2}{1+a+bx}\right)}{d} + \frac{\tanh^{-1}(a + bx) \log\left(\frac{2b(c+dx)}{(bc+d-ad)(1+a+bx)}\right)}{d} + \frac{\text{Subst}\left(\int \frac{\log\left(\frac{2}{1-x^2}\right)}{1-x^2} dx, x, a + bx\right)}{2d}$$

$$= -\frac{\tanh^{-1}(a + bx) \log\left(\frac{2}{1+a+bx}\right)}{d} + \frac{\tanh^{-1}(a + bx) \log\left(\frac{2b(c+dx)}{(bc+d-ad)(1+a+bx)}\right)}{d} - \frac{\text{Li}_2\left(1 - \frac{2b(c+dx)}{(bc+d-ad)(1+a+bx)}\right)}{2d}$$

$$= -\frac{\tanh^{-1}(a + bx) \log\left(\frac{2}{1+a+bx}\right)}{d} + \frac{\tanh^{-1}(a + bx) \log\left(\frac{2b(c+dx)}{(bc+d-ad)(1+a+bx)}\right)}{d} + \frac{\text{Li}_2\left(1 - \frac{2}{1+a+bx}\right)}{2d}$$

Mathematica [A] time = 0.01, size = 138, normalized size = 1.15

$$-\frac{\text{Li}_2\left(-\frac{d(-a-bx+1)}{-bc+ad-d}\right)}{2d} + \frac{\text{Li}_2\left(\frac{d(a+bx+1)}{-bc+ad+d}\right)}{2d} - \frac{\log(-a - bx + 1) \log\left(-\frac{b(c+dx)}{-(1-a)d-bc}\right)}{2d} + \frac{\log(a + bx + 1) \log\left(\frac{b(c+dx)}{bc-(a+1)d}\right)}{2d}$$

Warning: Unable to verify antiderivative.

[In] Integrate[ArcTanh[a + b*x]/(c + d*x), x]

[Out] $-1/2*(\text{Log}[1 - a - b*x]*\text{Log}[-((b*(c + d*x))/(-(b*c) - (1 - a)*d))])/d + (\text{Log}[1 + a + b*x]*\text{Log}[(b*(c + d*x))/(b*c - (1 + a)*d)])/(2*d) - \text{PolyLog}[2, -((d*(1 - a - b*x))/(-(b*c) - d + a*d))]/(2*d) + \text{PolyLog}[2, (d*(1 + a + b*x))/(-(b*c) + d + a*d)]/(2*d)$

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\text{artanh}(bx + a)}{dx + c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctanh(b*x+a)/(d*x+c), x, algorithm="fricas")

[Out] integral(arctanh(b*x + a)/(d*x + c), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{artanh}(bx + a)}{dx + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctanh(b*x+a)/(d*x+c), x, algorithm="giac")

[Out] integrate(arctanh(b*x + a)/(d*x + c), x)

maple [A] time = 0.12, size = 176, normalized size = 1.47

$$\frac{\ln(d(bx + a) - ad + bc) \text{arctanh}(bx + a)}{d} + \frac{\ln(d(bx + a) - ad + bc) \ln\left(\frac{d(bx+a)-d}{ad-bc-d}\right)}{2d} + \frac{\text{dilog}\left(\frac{d(bx+a)-d}{ad-bc-d}\right)}{2d} - \frac{\ln\left(\frac{d(bx+a)+d}{ad-bc+d}\right)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arctanh(b*x+a)/(d*x+c), x)

[Out] $\ln(d*(b*x+a)-a*d+b*c)/d*\text{arctanh}(b*x+a)+1/2/d*\ln(d*(b*x+a)-a*d+b*c)*\ln((d*(b*x+a)-d)/(a*d-b*c-d))+1/2/d*\text{dilog}((d*(b*x+a)-d)/(a*d-b*c-d))-1/2/d*\ln((d*(b*x+a)+d)/(a*d-b*c+d))*\ln(d*(b*x+a)-a*d+b*c)-1/2/d*\text{dilog}((d*(b*x+a)+d)/(a*d-b*c+d))$

maxima [A] time = 0.33, size = 192, normalized size = 1.60

$$-\frac{1}{2}b \left(\frac{\log(bx + a - 1) \log\left(\frac{bdx+ad-d}{bc-ad+d} + 1\right) + \text{Li}_2\left(-\frac{bdx+ad-d}{bc-ad+d}\right)}{bd} - \frac{\log(bx + a + 1) \log\left(\frac{bdx+ad+d}{bc-ad-d} + 1\right) + \text{Li}_2\left(-\frac{bdx+ad+d}{bc-ad-d}\right)}{bd} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctanh(b*x+a)/(d*x+c),x, algorithm="maxima")

[Out] $-1/2*b*((\log(b*x + a - 1)*\log((b*d*x + a*d - d)/(b*c - a*d + d) + 1) + \operatorname{dilog}(- (b*d*x + a*d - d)/(b*c - a*d + d)))/(b*d) - (\log(b*x + a + 1)*\log((b*d*x + a*d + d)/(b*c - a*d - d) + 1) + \operatorname{dilog}(- (b*d*x + a*d + d)/(b*c - a*d - d)))/(b*d)) - 1/2*b*(\log(b*x + a + 1)/b - \log(b*x + a - 1)/b)*\log(d*x + c)/d + \operatorname{arctanh}(b*x + a)*\log(d*x + c)/d$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{atanh}(a + bx)}{c + dx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(atanh(a + b*x)/(c + d*x),x)

[Out] int(atanh(a + b*x)/(c + d*x), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}(a + bx)}{c + dx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(atanh(b*x+a)/(d*x+c),x)

[Out] Integral(atanh(a + b*x)/(c + d*x), x)

$$3.56 \quad \int \frac{\tanh^{-1}(a+bx)}{c+\frac{d}{x}} dx$$

Optimal. Leaf size=186

$$\frac{d\text{Li}_2\left(\frac{c(-a-bx+1)}{-ac+c+bd}\right)}{2c^2} - \frac{d\text{Li}_2\left(\frac{c(a+bx+1)}{ac+c-bd}\right)}{2c^2} + \frac{d \log(-a-bx+1) \log\left(\frac{b(cx+d)}{-ac+bd+c}\right)}{2c^2} - \frac{d \log(a+bx+1) \log\left(-\frac{b(cx+d)}{ac-bd+c}\right)}{2c^2} + \frac{(-a-bx+1) \log(-a-bx+1)}{2c^2}$$

[Out] $1/2*(-b*x-a+1)*\ln(-b*x-a+1)/b/c+1/2*(b*x+a+1)*\ln(b*x+a+1)/b/c-1/2*d*\ln(b*x+a+1)*\ln(-b*(c*x+d)/(a*c-b*d+c))/c^2+1/2*d*\ln(-b*x-a+1)*\ln(b*(c*x+d)/(-a*c+b*d+c))/c^2+1/2*d*\text{polylog}(2,c*(-b*x-a+1)/(-a*c+b*d+c))/c^2-1/2*d*\text{polylog}(2,c*(b*x+a+1)/(a*c-b*d+c))/c^2$

Rubi [A] time = 0.24, antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {6115, 2409, 2389, 2295, 2394, 2393, 2391}

$$\frac{d\text{PolyLog}\left(2, \frac{c(-a-bx+1)}{-ac+bd+c}\right)}{2c^2} - \frac{d\text{PolyLog}\left(2, \frac{c(a+bx+1)}{ac-bd+c}\right)}{2c^2} + \frac{d \log(-a-bx+1) \log\left(\frac{b(cx+d)}{-ac+bd+c}\right)}{2c^2} - \frac{d \log(a+bx+1) \log\left(-\frac{b(cx+d)}{ac-bd+c}\right)}{2c^2} + \frac{(-a-bx+1) \log(-a-bx+1)}{2c^2}$$

Antiderivative was successfully verified.

[In] Int[ArcTanh[a + b*x]/(c + d/x), x]

[Out] $((1-a-b*x)*\text{Log}[1-a-b*x])/(2*b*c) + ((1+a+b*x)*\text{Log}[1+a+b*x])/(2*b*c) - (d*\text{Log}[1+a+b*x]*\text{Log}[-((b*(d+c*x))/(c+a*c-b*d))])/(2*c^2) + (d*\text{Log}[1-a-b*x]*\text{Log}[(b*(d+c*x))/(c-a*c+b*d)])/(2*c^2) + (d*\text{PolyLog}[2, (c*(1-a-b*x))/(c-a*c+b*d)])/(2*c^2) - (d*\text{PolyLog}[2, (c*(1+a+b*x))/(c+a*c-b*d)])/(2*c^2)$

Rule 2295

Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] := Simp[x*Log[c*x^n], x] - Simp[n*x, x] /; FreeQ[{c, n}, x]

Rule 2389

Int[((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_)^(n_.))]*(b_.))^(p_.), x_Symbol] := Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, n, p}, x]

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] := -Simp[PolyLog[2, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_Symbol] := Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]*(b_.))/((f_.) + (g_.)*(x_)), x_Symbol] := Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x)^n])/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2409

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]*(b_.))^(p_.)*((f_.) + (g_.)*(x_)^(r_))^(q_.), x_Symbol] := Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && IntegerQ[p, 0] && IntegerQ[q] && (IntegerQ[r] && NeQ[r, 1])
```

Rule 6115

```
Int[ArcTanh[(c_) + (d_.)*(x_)]/((e_) + (f_.)*(x_)^(n_.)), x_Symbol] := Dist[1/2, Int[Log[1 + c + d*x]/(e + f*x^n), x], x] - Dist[1/2, Int[Log[1 - c - d*x]/(e + f*x^n), x], x] /; FreeQ[{c, d, e, f}, x] && RationalQ[n]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tanh^{-1}(a+bx)}{c+\frac{d}{x}} dx &= -\left(\frac{1}{2} \int \frac{\log(1-a-bx)}{c+\frac{d}{x}} dx\right) + \frac{1}{2} \int \frac{\log(1+a+bx)}{c+\frac{d}{x}} dx \\
&= -\left(\frac{1}{2} \int \left(\frac{\log(1-a-bx)}{c} - \frac{d \log(1-a-bx)}{c(d+cx)}\right) dx\right) + \frac{1}{2} \int \left(\frac{\log(1+a+bx)}{c} - \frac{d \log(1+a+bx)}{c(d+cx)}\right) dx \\
&= -\frac{\int \log(1-a-bx) dx}{2c} + \frac{\int \log(1+a+bx) dx}{2c} + \frac{d \int \frac{\log(1-a-bx)}{d+cx} dx}{2c} - \frac{d \int \frac{\log(1+a+bx)}{d+cx} dx}{2c} \\
&= -\frac{d \log(1+a+bx) \log\left(-\frac{b(d+cx)}{c+ac-bd}\right)}{2c^2} + \frac{d \log(1-a-bx) \log\left(\frac{b(d+cx)}{c-ac+bd}\right)}{2c^2} + \frac{\text{Subst}\left(\int \log(x) dx, x, \frac{b(d+cx)}{c+ac-bd}\right)}{2b} \\
&= \frac{(1-a-bx) \log(1-a-bx)}{2bc} + \frac{(1+a+bx) \log(1+a+bx)}{2bc} - \frac{d \log(1+a+bx) \log\left(-\frac{b}{c}\right)}{2c^2} \\
&= \frac{(1-a-bx) \log(1-a-bx)}{2bc} + \frac{(1+a+bx) \log(1+a+bx)}{2bc} - \frac{d \log(1+a+bx) \log\left(-\frac{b}{c}\right)}{2c^2}
\end{aligned}$$

Mathematica [C] time = 4.15, size = 759, normalized size = 4.08

$$bcd\sqrt{-a^2 + \frac{2abd}{c} - \frac{b^2d^2}{c^2} + 1} \tanh^{-1}(a+bx)^2 e^{\tanh^{-1}\left(a-\frac{bd}{c}\right)} - 2a^2c^2 \tanh^{-1}(a+bx) + 2b^2d^2 \tanh^{-1}\left(a-\frac{bd}{c}\right) \log\left(1 - \frac{bd}{c}\right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[ArcTanh[a + b*x]/(c + d/x), x]

[Out] $(-2*a^2*c^2*ArcTanh[a + b*x] + 2*a*b*c*d*ArcTanh[a + b*x] + I*a*b*c*d*Pi*ArcTanh[a + b*x] - I*b^2*d^2*Pi*ArcTanh[a + b*x] - 2*a*b*c^2*x*ArcTanh[a + b*x] + 2*b^2*c*d*x*ArcTanh[a + b*x] - 2*a*b*c*d*ArcTanh[a - (b*d)/c]*ArcTanh[a + b*x] + 2*b^2*d^2*ArcTanh[a - (b*d)/c]*ArcTanh[a + b*x] - b*c*d*ArcTanh[a + b*x]^2 - a*b*c*d*ArcTanh[a + b*x]^2 + b^2*d^2*ArcTanh[a + b*x]^2 + b*c*d*Sqrt[1 - a^2 + (2*a*b*d)/c - (b^2*d^2)/c^2]*E^ArcTanh[a - (b*d)/c]*ArcTanh[a + b*x]^2 - 2*a*b*c*d*ArcTanh[a - (b*d)/c]*Log[1 - E^(2*(ArcTanh[a - (b*d)/c] - ArcTanh[a + b*x]))] + 2*b^2*d^2*ArcTanh[a - (b*d)/c]*Log[1 - E^(2*(ArcTanh[a - (b*d)/c] - ArcTanh[a + b*x]))] + 2*a*b*c*d*ArcTanh[a + b*x]*Log[1 - E^(2*(ArcTanh[a - (b*d)/c] - ArcTanh[a + b*x]))] - 2*b^2*d^2*ArcTanh[a + b*x]*Log[1 - E^(2*(ArcTanh[a - (b*d)/c] - ArcTanh[a + b*x]))] - 2*a*b*c*d*ArcTanh[a + b*x]*Log[1 + E^(-2*ArcTanh[a + b*x])] + 2*b^2*d^2*ArcTanh[a + b*x]*Log[1 + E^(-2*ArcTanh[a + b*x])] - I*a*b*c*d*Pi*Log[1 + E^(2*ArcTanh[a + b*x])])$

$a + b*x]] + I*b^2*d^2*Pi*Log[1 + E^(2*ArcTanh[a + b*x])] + 2*a*c^2*Log[1/Sqrt[1 - (a + b*x)^2]] - 2*b*c*d*Log[1/Sqrt[1 - (a + b*x)^2]] + I*a*b*c*d*Pi*Log[1/Sqrt[1 - (a + b*x)^2]] - I*b^2*d^2*Pi*Log[1/Sqrt[1 - (a + b*x)^2]] + 2*a*b*c*d*ArcTanh[a - (b*d)/c]*Log[(-I)*Sinh[ArcTanh[a - (b*d)/c] - ArcTanh[a + b*x]]] - 2*b^2*d^2*ArcTanh[a - (b*d)/c]*Log[(-I)*Sinh[ArcTanh[a - (b*d)/c] - ArcTanh[a + b*x]]] + b*d*(-(a*c) + b*d)*PolyLog[2, E^(2*(ArcTanh[a - (b*d)/c] - ArcTanh[a + b*x]))] + b*d*(a*c - b*d)*PolyLog[2, -E^(-2*ArcTanh[a + b*x])]]/(2*b*c^2*(-(a*c) + b*d))$

fricas [F] time = 0.53, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x \operatorname{artanh}(bx + a)}{cx + d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctanh(b*x+a)/(c+d/x),x, algorithm="fricas")

[Out] integral(x*arctanh(b*x + a)/(c*x + d), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{artanh}(bx + a)}{c + \frac{d}{x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctanh(b*x+a)/(c+d/x),x, algorithm="giac")

[Out] integrate(arctanh(b*x + a)/(c + d/x), x)

maple [A] time = 0.12, size = 297, normalized size = 1.60

$$\frac{\operatorname{arctanh}(bx + a)x}{c} + \frac{\operatorname{arctanh}(bx + a)a}{bc} - \frac{\operatorname{arctanh}(bx + a)d \ln(c(bx + a) - ac + bd)}{c^2} + \frac{d \ln(c(bx + a) - ac + bd)}{2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arctanh(b*x+a)/(c+d/x),x)

[Out] arctanh(b*x+a)/c*x+1/b*arctanh(b*x+a)/c*a-arctanh(b*x+a)/c^2*d*ln(c*(b*x+a)-a*c+b*d)+1/2/c^2*d*ln(c*(b*x+a)-a*c+b*d)*ln((c*(b*x+a)+c)/(a*c-b*d+c))+1/2/c^2*d*dilog((c*(b*x+a)+c)/(a*c-b*d+c))-1/2/c^2*d*ln(c*(b*x+a)-a*c+b*d)*ln((c*(b*x+a)-c)/(a*c-b*d-c))-1/2/c^2*d*dilog((c*(b*x+a)-c)/(a*c-b*d-c))+1/2/b/c*ln(a^2*c^2-2*a*b*c*d+b^2*d^2+2*(c*(b*x+a)-a*c+b*d)*a*c-2*(c*(b*x+a)-a*c+b*d)*b*d+(c*(b*x+a)-a*c+b*d)^2-c^2)

maxima [A] time = 0.32, size = 192, normalized size = 1.03

$$\frac{1}{2} b \left(\frac{\left(\log(cx + d) \log\left(\frac{bcx+bd}{ac-bd+c} + 1\right) + \text{Li}_2\left(-\frac{bcx+bd}{ac-bd+c}\right) \right) d}{bc^2} - \frac{\left(\log(cx + d) \log\left(\frac{bcx+bd}{ac-bd-c} + 1\right) + \text{Li}_2\left(-\frac{bcx+bd}{ac-bd-c}\right) \right) d}{bc^2} + \dots \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctanh(b*x+a)/(c+d/x),x, algorithm="maxima")

[Out] 1/2*b*((log(c*x + d)*log((b*c*x + b*d)/(a*c - b*d + c) + 1) + dilog(-(b*c*x + b*d)/(a*c - b*d + c)))*d/(b*c^2) - (log(c*x + d)*log((b*c*x + b*d)/(a*c - b*d - c) + 1) + dilog(-(b*c*x + b*d)/(a*c - b*d - c)))*d/(b*c^2) + (a + 1)*log(b*x + a + 1)/(b^2*c) - (a - 1)*log(b*x + a - 1)/(b^2*c)) + (x/c - d*log(c*x + d)/c^2)*arctanh(b*x + a)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{atanh}(a + bx)}{c + \frac{d}{x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(atanh(a + b*x)/(c + d/x),x)

[Out] int(atanh(a + b*x)/(c + d/x), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x \operatorname{atanh}(a + bx)}{cx + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(atanh(b*x+a)/(c+d/x),x)

[Out] Integral(x*atanh(a + b*x)/(c*x + d), x)

$$3.57 \quad \int \frac{\tanh^{-1}(a+bx)}{c + \frac{d}{x^2}} dx$$

Optimal. Leaf size=545

$$\frac{\sqrt{d} \operatorname{Li}_2\left(\frac{\sqrt{-c}(-a-bx+1)}{-\sqrt{-c}a+\sqrt{-c}-b\sqrt{d}}\right)}{4(-c)^{3/2}} - \frac{\sqrt{d} \operatorname{Li}_2\left(\frac{\sqrt{-c}(-a-bx+1)}{\sqrt{-c}(1-a)+b\sqrt{d}}\right)}{4(-c)^{3/2}} + \frac{\sqrt{d} \operatorname{Li}_2\left(\frac{\sqrt{-c}(a+bx+1)}{(a+1)\sqrt{-c}-b\sqrt{d}}\right)}{4(-c)^{3/2}} - \frac{\sqrt{d} \operatorname{Li}_2\left(\frac{\sqrt{-c}(a+bx+1)}{\sqrt{-c}(a+1)+b\sqrt{d}}\right)}{4(-c)^{3/2}} + \frac{\sqrt{d} \log(-a - \dots)}{4(-c)^{3/2}}$$

[Out] $\frac{1}{2}(-bx-a+1) \ln(-bx-a+1)/b/c + \frac{1}{2}(bx+a+1) \ln(bx+a+1)/b/c + \frac{1}{4} \ln(-bx-a+1) \ln(-b(-x(-c)^{1/2}+d^{1/2})/((1-a)(-c)^{1/2}-bd^{1/2}))/d^{1/2}/(-c)^{3/2} + \frac{1}{4} \ln(bx+a+1) \ln(-b(x(-c)^{1/2}+d^{1/2})/((1+a)(-c)^{1/2}-bd^{1/2}))/d^{1/2}/(-c)^{3/2} - \frac{1}{4} \ln(-bx-a+1) \ln(b(x(-c)^{1/2}+d^{1/2})/((1-a)(-c)^{1/2}+bd^{1/2}))/d^{1/2}/(-c)^{3/2} - \frac{1}{4} \ln(bx+a+1) \ln(b(-x(-c)^{1/2}+d^{1/2})/((1+a)(-c)^{1/2}+bd^{1/2}))/d^{1/2}/(-c)^{3/2} + \frac{1}{4} \operatorname{polylog}(2, (-bx-a+1)(-c)^{1/2}/((-c)^{1/2}-a(-c)^{1/2}-bd^{1/2}))/d^{1/2}/(-c)^{3/2} + \frac{1}{4} \operatorname{polylog}(2, (bx+a+1)(-c)^{1/2}/((1+a)(-c)^{1/2}-bd^{1/2}))/d^{1/2}/(-c)^{3/2} - \frac{1}{4} \operatorname{polylog}(2, (-bx-a+1)(-c)^{1/2}/((1-a)(-c)^{1/2}+bd^{1/2}))/d^{1/2}/(-c)^{3/2} - \frac{1}{4} \operatorname{polylog}(2, (bx+a+1)(-c)^{1/2}/((1+a)(-c)^{1/2}+bd^{1/2}))/d^{1/2}/(-c)^{3/2}$

Rubi [A] time = 0.91, antiderivative size = 545, normalized size of antiderivative = 1.00, number of steps used = 25, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {6115, 2409, 2389, 2295, 2394, 2393, 2391}

$$\frac{\sqrt{d} \operatorname{PolyLog}\left(2, \frac{\sqrt{-c}(-a-bx+1)}{a(-\sqrt{-c})-b\sqrt{d}+\sqrt{-c}}\right)}{4(-c)^{3/2}} - \frac{\sqrt{d} \operatorname{PolyLog}\left(2, \frac{\sqrt{-c}(-a-bx+1)}{(1-a)\sqrt{-c}+b\sqrt{d}}\right)}{4(-c)^{3/2}} + \frac{\sqrt{d} \operatorname{PolyLog}\left(2, \frac{\sqrt{-c}(a+bx+1)}{(a+1)\sqrt{-c}-b\sqrt{d}}\right)}{4(-c)^{3/2}} - \frac{\sqrt{d} \operatorname{PolyLog}\left(2, \frac{\sqrt{-c}(a+bx+1)}{\sqrt{-c}(a+1)+b\sqrt{d}}\right)}{4(-c)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[ArcTanh[a + b*x]/(c + d/x^2), x]

[Out] $((1-a-bx) \operatorname{Log}[1-a-bx])/(2bc) + ((1+a+bx) \operatorname{Log}[1+a+bx])/(2bc) + (\operatorname{Sqrt}[d] \operatorname{Log}[1-a-bx] \operatorname{Log}[-(b(\operatorname{Sqrt}[d]-\operatorname{Sqrt}[-c]x))]/((1-a)\operatorname{Sqrt}[-c]-b\operatorname{Sqrt}[d]))/(4(-c)^{3/2}) - (\operatorname{Sqrt}[d] \operatorname{Log}[1+a+bx] \operatorname{Log}[(b(\operatorname{Sqrt}[d]-\operatorname{Sqrt}[-c]x))]/((1+a)\operatorname{Sqrt}[-c]+b\operatorname{Sqrt}[d]))/(4(-c)^{3/2}) + (\operatorname{Sqrt}[d] \operatorname{Log}[1+a+bx] \operatorname{Log}[-(b(\operatorname{Sqrt}[d]+\operatorname{Sqrt}[-c]x))]/((1+a)\operatorname{Sqrt}[-c]-b\operatorname{Sqrt}[d]))/(4(-c)^{3/2}) - (\operatorname{Sqrt}[d] \operatorname{Log}[1-a-bx] \operatorname{Log}[(b(\operatorname{Sqrt}[d]+\operatorname{Sqrt}[-c]x))]/((1-a)\operatorname{Sqrt}[-c]+b\operatorname{Sqrt}[d]))/(4(-c)^{3/2}) + (\operatorname{Sqrt}[d] \operatorname{PolyLog}[2, (\operatorname{Sqrt}[-c](1-a-bx))/(\operatorname{Sqrt}[-c]-a\operatorname{Sqrt}[-c]-b\operatorname{Sqrt}[d])])/(4(-c)^{3/2}) - (\operatorname{Sqrt}[d] \operatorname{PolyLog}[2, (\operatorname{Sqrt}[-c](1-a-bx))/((1-a)\operatorname{Sqrt}[-c]+b\operatorname{Sqrt}[d])])/(4(-c)^{3/2}) + (\operatorname{Sqrt}[d] \operatorname{PolyLog}[2, (\operatorname{Sqrt}[-c](1+a+bx))/((1+a)\operatorname{Sqrt}[-c]-b\operatorname{Sqrt}[d])])/(4(-c)^{3/2}) - (\operatorname{Sqrt}[d] \operatorname{PolyLo$

$g[2, (\text{Sqrt}[-c]*(1 + a + b*x))/((1 + a)*\text{Sqrt}[-c] + b*\text{Sqrt}[d])]/(4*(-c)^{(3/2)})$

Rule 2295

$\text{Int}[\text{Log}[(c_)*(x_)^{(n_)}], x_Symbol] \rightarrow \text{Simp}[x*\text{Log}[c*x^n], x] - \text{Simp}[n*x, x] /; \text{FreeQ}\{c, n\}, x]$

Rule 2389

$\text{Int}[(a_ + \text{Log}[(c_)*((d_ + (e_)*(x_))^{(n_)})*(b_)]^{(p_)}), x_Symbol] :> \text{Dist}[1/e, \text{Subst}[\text{Int}[(a + b*\text{Log}[c*x^n])^p, x], x, d + e*x], x] /; \text{FreeQ}\{a, b, c, d, e, n, p\}, x]$

Rule 2391

$\text{Int}[\text{Log}[(c_)*((d_ + (e_)*(x_))^{(n_)})]/(x_), x_Symbol] \rightarrow -\text{Simp}[\text{PolyLog}[2, -(c*e*x^n)]/n, x] /; \text{FreeQ}\{c, d, e, n\}, x \&\& \text{EqQ}[c*d, 1]$

Rule 2393

$\text{Int}[(a_ + \text{Log}[(c_)*((d_ + (e_)*(x_)))]*(b_))/((f_ + (g_)*(x_))), x_Symbol] \rightarrow \text{Dist}[1/g, \text{Subst}[\text{Int}[(a + b*\text{Log}[1 + (c*e*x)/g])/x, x], x, f + g*x], x] /; \text{FreeQ}\{a, b, c, d, e, f, g\}, x \&\& \text{NeQ}[e*f - d*g, 0] \&\& \text{EqQ}[g + c*(e*f - d*g), 0]$

Rule 2394

$\text{Int}[(a_ + \text{Log}[(c_)*((d_ + (e_)*(x_))^{(n_)})*(b_))/((f_ + (g_)*(x_)))]*(b_)/((f_ + (g_)*(x_))), x_Symbol] \rightarrow \text{Simp}[(\text{Log}[(e*(f + g*x))/(e*f - d*g)]*(a + b*\text{Log}[c*(d + e*x)^n])/g, x] - \text{Dist}[(b*e*n)/g, \text{Int}[\text{Log}[(e*(f + g*x))/(e*f - d*g)]/(d + e*x), x], x] /; \text{FreeQ}\{a, b, c, d, e, f, g, n\}, x \&\& \text{NeQ}[e*f - d*g, 0]$

Rule 2409

$\text{Int}[(a_ + \text{Log}[(c_)*((d_ + (e_)*(x_))^{(n_)})*(b_)]^{(p_)}*(f_ + (g_)*(x_)^{(r_))^{(q_)}), x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*\text{Log}[c*(d + e*x)^n])^p, (f + g*x^r)^q, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, g, n, r\}, x \&\& \text{IntegerQ}[p, 0] \&\& \text{IntegerQ}[q] \&\& (\text{GtQ}[q, 0] \mid\mid (\text{IntegerQ}[r] \&\& \text{NeQ}[r, 1]))]$

Rule 6115

$\text{Int}[\text{ArcTanh}[(c_ + (d_)*(x_)]/((e_ + (f_)*(x_))^{(n_)}), x_Symbol] \rightarrow \text{Dist}[1/2, \text{Int}[\text{Log}[1 + c + d*x]/(e + f*x^n), x], x] - \text{Dist}[1/2, \text{Int}[\text{Log}[1 - c - d*x]/(e + f*x^n), x], x] /; \text{FreeQ}\{c, d, e, f\}, x \&\& \text{RationalQ}[n]$

Rubi steps

$$\begin{aligned}
\int \frac{\tanh^{-1}(a+bx)}{c+\frac{d}{x^2}} dx &= -\left(\frac{1}{2} \int \frac{\log(1-a-bx)}{c+\frac{d}{x^2}} dx\right) + \frac{1}{2} \int \frac{\log(1+a+bx)}{c+\frac{d}{x^2}} dx \\
&= -\left(\frac{1}{2} \int \left(\frac{\log(1-a-bx)}{c} - \frac{d \log(1-a-bx)}{c(d+cx^2)}\right) dx\right) + \frac{1}{2} \int \left(\frac{\log(1+a+bx)}{c} - \frac{d \log(1+a+bx)}{c(d+cx^2)}\right) dx \\
&= -\frac{\int \log(1-a-bx) dx}{2c} + \frac{\int \log(1+a+bx) dx}{2c} + \frac{d \int \frac{\log(1-a-bx)}{d+cx^2} dx}{2c} - \frac{d \int \frac{\log(1+a+bx)}{d+cx^2} dx}{2c} \\
&= \frac{\text{Subst}\left(\int \log(x) dx, x, 1-a-bx\right)}{2bc} + \frac{\text{Subst}\left(\int \log(x) dx, x, 1+a+bx\right)}{2bc} + \frac{d \int \left(\frac{\log(1-a-bx)}{2\sqrt{d}(\sqrt{d}-\sqrt{-cx})}\right) dx}{4c} \\
&= \frac{(1-a-bx) \log(1-a-bx)}{2bc} + \frac{(1+a+bx) \log(1+a+bx)}{2bc} + \frac{\sqrt{d} \int \frac{\log(1-a-bx)}{\sqrt{d}-\sqrt{-cx}} dx}{4c} + \frac{\sqrt{d} \int \frac{\log(1+a+bx)}{\sqrt{d}+\sqrt{-cx}} dx}{4c} \\
&= \frac{(1-a-bx) \log(1-a-bx)}{2bc} + \frac{(1+a+bx) \log(1+a+bx)}{2bc} + \frac{\sqrt{d} \log(1-a-bx) \log\left(-\frac{\sqrt{d}-\sqrt{-cx}}{\sqrt{d}+\sqrt{-cx}}\right)}{4(-c)^{3/2}} \\
&= \frac{(1-a-bx) \log(1-a-bx)}{2bc} + \frac{(1+a+bx) \log(1+a+bx)}{2bc} + \frac{\sqrt{d} \log(1-a-bx) \log\left(-\frac{\sqrt{d}-\sqrt{-cx}}{\sqrt{d}+\sqrt{-cx}}\right)}{4(-c)^{3/2}} \\
&= \frac{(1-a-bx) \log(1-a-bx)}{2bc} + \frac{(1+a+bx) \log(1+a+bx)}{2bc} + \frac{\sqrt{d} \log(1-a-bx) \log\left(-\frac{\sqrt{d}-\sqrt{-cx}}{\sqrt{d}+\sqrt{-cx}}\right)}{4(-c)^{3/2}}
\end{aligned}$$

Mathematica [C] time = 25.32, size = 1456, normalized size = 2.67

result too large to display

Warning: Unable to verify antiderivative.

[In] Integrate[ArcTanh[a + b*x]/(c + d/x^2), x]

[Out] ((a + b*x)*ArcTanh[a + b*x] - Log[1/Sqrt[1 - (a + b*x)^2]])/(b*c) + (Sqrt[d] * ((2*I)*Sqrt[c]*ArcTan[(-1 + a)*Sqrt[c]]/(b*Sqrt[d]))*ArcTan[(Sqrt[c]*x)/Sqrt[d]] - (2*I)*a^2*Sqrt[c]*ArcTan[(-1 + a)*Sqrt[c]]/(b*Sqrt[d]))*ArcTan[(Sqrt[c]*x)/Sqrt[d]] - (2*I)*Sqrt[c]*ArcTan[(1 + a)*Sqrt[c]]/(b*Sqrt[d]))*ArcTan[(Sqrt[c]*x)/Sqrt[d]] + (2*I)*a^2*Sqrt[c]*ArcTan[(1 + a)*Sqrt[c]]/(b

$$\begin{aligned} & * \text{Sqrt}[d]] * \text{ArcTan}[(\text{Sqrt}[c]*x)/\text{Sqrt}[d]] - 2*b*\text{Sqrt}[d]*\text{ArcTan}[(\text{Sqrt}[c]*x)/\text{Sqrt}[d]]^2 + (b*\text{Sqrt}[d]*\text{Sqrt}[(1+a)^2*c + b^2*d]/(b^2*d)] * \text{ArcTan}[(\text{Sqrt}[c]*x)/\text{Sqrt}[d]]^2 / E^{(I*\text{ArcTan}[(1+a)*\text{Sqrt}[c])/(b*\text{Sqrt}[d])]} + (a*b*\text{Sqrt}[d]*\text{Sqrt}[(1+a)^2*c + b^2*d]/(b^2*d)] * \text{ArcTan}[(\text{Sqrt}[c]*x)/\text{Sqrt}[d]]^2 / E^{(I*\text{ArcTan}[(1+a)*\text{Sqrt}[c])/(b*\text{Sqrt}[d])]} + (b*\text{Sqrt}[d]*\text{Sqrt}[(1+a)^2*c + b^2*d]/(b^2*d)] * \text{ArcTan}[(\text{Sqrt}[c]*x)/\text{Sqrt}[d]]^2 / E^{(I*\text{ArcTan}[(1+a)*\text{Sqrt}[c])/(b*\text{Sqrt}[d])]} - (a*b*\text{Sqrt}[d]*\text{Sqrt}[(1+a)^2*c + b^2*d]/(b^2*d)] * \text{ArcTan}[(\text{Sqrt}[c]*x)/\text{Sqrt}[d]]^2 / E^{(I*\text{ArcTan}[(1+a)*\text{Sqrt}[c])/(b*\text{Sqrt}[d])]} - 4*(-1+a^2)*\text{Sqrt}[c]*\text{ArcTan}[(\text{Sqrt}[c]*x)/\text{Sqrt}[d]] * \text{ArcTanh}[a + b*x] + 2*\text{Sqrt}[c]*\text{ArcTan}[(1+a)*\text{Sqrt}[c]/(b*\text{Sqrt}[d])] * \text{Log}[1 - E^{(-2*I)*(\text{ArcTan}[(1+a)*\text{Sqrt}[c])/(b*\text{Sqrt}[d])]} + \text{ArcTan}[(\text{Sqrt}[c]*x)/\text{Sqrt}[d])]} + \text{ArcTan}[(\text{Sqrt}[c]*x)/\text{Sqrt}[d])] - 2*a^2*\text{Sqrt}[c]*\text{ArcTan}[(1+a)*\text{Sqrt}[c]/(b*\text{Sqrt}[d])] * \text{Log}[1 - E^{(-2*I)*(\text{ArcTan}[(1+a)*\text{Sqrt}[c])/(b*\text{Sqrt}[d])]} + \text{ArcTan}[(\text{Sqrt}[c]*x)/\text{Sqrt}[d])]} + \text{ArcTan}[(\text{Sqrt}[c]*x)/\text{Sqrt}[d])] - 2*a^2*\text{Sqrt}[c]*\text{ArcTan}[(\text{Sqrt}[c]*x)/\text{Sqrt}[d]] * \text{Log}[1 - E^{(-2*I)*(\text{ArcTan}[(1+a)*\text{Sqrt}[c])/(b*\text{Sqrt}[d])]} + \text{ArcTan}[(\text{Sqrt}[c]*x)/\text{Sqrt}[d])]} + \text{ArcTan}[(\text{Sqrt}[c]*x)/\text{Sqrt}[d])] - 2*a^2*\text{Sqrt}[c]*\text{ArcTan}[(1+a)*\text{Sqrt}[c]/(b*\text{Sqrt}[d])] * \text{Log}[1 - E^{(-2*I)*(\text{ArcTan}[(1+a)*\text{Sqrt}[c])/(b*\text{Sqrt}[d])]} + \text{ArcTan}[(\text{Sqrt}[c]*x)/\text{Sqrt}[d])]} + \text{ArcTan}[(\text{Sqrt}[c]*x)/\text{Sqrt}[d])] + 2*a^2*\text{Sqrt}[c]*\text{ArcTan}[(1+a)*\text{Sqrt}[c]/(b*\text{Sqrt}[d])] * \text{Log}[1 - E^{(-2*I)*(\text{ArcTan}[(1+a)*\text{Sqrt}[c])/(b*\text{Sqrt}[d])]} + \text{ArcTan}[(\text{Sqrt}[c]*x)/\text{Sqrt}[d])]} + \text{ArcTan}[(\text{Sqrt}[c]*x)/\text{Sqrt}[d])] - 2*\text{Sqrt}[c]*\text{ArcTan}[(1+a)*\text{Sqrt}[c]/(b*\text{Sqrt}[d])] * \text{Log}[1 - E^{(-2*I)*(\text{ArcTan}[(1+a)*\text{Sqrt}[c])/(b*\text{Sqrt}[d])]} + \text{ArcTan}[(\text{Sqrt}[c]*x)/\text{Sqrt}[d])]} + \text{ArcTan}[(\text{Sqrt}[c]*x)/\text{Sqrt}[d])] + 2*a^2*\text{Sqrt}[c]*\text{ArcTan}[(1+a)*\text{Sqrt}[c]/(b*\text{Sqrt}[d])] * \text{Log}[1 - E^{(-2*I)*(\text{ArcTan}[(1+a)*\text{Sqrt}[c])/(b*\text{Sqrt}[d])]} + \text{ArcTan}[(\text{Sqrt}[c]*x)/\text{Sqrt}[d])]} + \text{ArcTan}[(\text{Sqrt}[c]*x)/\text{Sqrt}[d])] - 2*\text{Sqrt}[c]*\text{ArcTan}[(1+a)*\text{Sqrt}[c]/(b*\text{Sqrt}[d])] * \text{Log}[-\text{Sin}[\text{ArcTan}[(1+a)*\text{Sqrt}[c]/(b*\text{Sqrt}[d])] + \text{ArcTan}[(\text{Sqrt}[c]*x)/\text{Sqrt}[d]]] + 2*a^2*\text{Sqrt}[c]*\text{ArcTan}[(1+a)*\text{Sqrt}[c]/(b*\text{Sqrt}[d])] * \text{Log}[-\text{Sin}[\text{ArcTan}[(1+a)*\text{Sqrt}[c]/(b*\text{Sqrt}[d])] + \text{ArcTan}[(\text{Sqrt}[c]*x)/\text{Sqrt}[d]]] + 2*\text{Sqrt}[c]*\text{ArcTan}[(1+a)*\text{Sqrt}[c]/(b*\text{Sqrt}[d])] * \text{Log}[-\text{Sin}[\text{ArcTan}[(1+a)*\text{Sqrt}[c]/(b*\text{Sqrt}[d])] + \text{ArcTan}[(\text{Sqrt}[c]*x)/\text{Sqrt}[d]]] - 2*a^2*\text{Sqrt}[c]*\text{ArcTan}[(1+a)*\text{Sqrt}[c]/(b*\text{Sqrt}[d])] * \text{Log}[-\text{Sin}[\text{ArcTan}[(1+a)*\text{Sqrt}[c]/(b*\text{Sqrt}[d])] + \text{ArcTan}[(\text{Sqrt}[c]*x)/\text{Sqrt}[d]]] + \text{ArcTan}[(\text{Sqrt}[c]*x)/\text{Sqrt}[d]]] - I*(-1+a^2)*\text{Sqrt}[c]*\text{PolyLog}[2, E^{(-2*I)*(\text{ArcTan}[(1+a)*\text{Sqrt}[c])/(b*\text{Sqrt}[d])]} + \text{ArcTan}[(\text{Sqrt}[c]*x)/\text{Sqrt}[d])]}] + I*(-1+a^2)*\text{Sqrt}[c]*\text{PolyLog}[2, E^{(-2*I)*(\text{ArcTan}[(1+a)*\text{Sqrt}[c])/(b*\text{Sqrt}[d])]} + \text{ArcTan}[(\text{Sqrt}[c]*x)/\text{Sqrt}[d])]}] + \text{ArcTan}[(\text{Sqrt}[c]*x)/\text{Sqrt}[d]])) / (4*(-1+a^2)*c^2)
\end{aligned}$$

fricas [F] time = 1.12, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^2 \operatorname{artanh}(bx+a)}{cx^2+d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctanh(b*x+a)/(c+d/x^2),x, algorithm="fricas")

[Out] integral(x^2*arctanh(b*x + a)/(c*x^2 + d), x)

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctanh(b*x+a)/(c+d/x^2),x, algorithm="giac")

[Out] Timed out

maple [C] time = 1.81, size = 20505, normalized size = 37.62

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arctanh(b*x+a)/(c+d/x^2),x)

[Out] result too large to display

maxima [C] time = 0.55, size = 647, normalized size = 1.19

$$-\left(\frac{d \arctan\left(\frac{cx}{\sqrt{cd}}\right)}{\sqrt{cd}c} - \frac{x}{c}\right) \operatorname{arctanh}(bx+a) + \frac{2(a+1)c \log(bx+a+1) - 2(a-1)c \log(bx+a-1) + \left(b \arctan\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)\right)}{\dots}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctanh(b*x+a)/(c+d/x^2),x, algorithm="maxima")

[Out] $-(d \arctan(c*x/\sqrt{c*d})/(\sqrt{c*d}*c) - x/c)*\operatorname{arctanh}(b*x + a) + 1/4*(2*(a + 1)*c*\log(b*x + a + 1) - 2*(a - 1)*c*\log(b*x + a - 1) + (b*\arctan(\sqrt{c}*x/\sqrt{d}))*\log((b^2*c*x^2 + 2*(a + 1)*b*c*x + (a^2 + 2*a + 1)*c)/(b^2*d + (a^2 + 2*a + 1)*c)) - b*\arctan(\sqrt{c}*x/\sqrt{d})*\log((b^2*c*x^2 + 2*(a - 1)*b*c*x + (a^2 - 2*a + 1)*c)/(b^2*d + (a^2 - 2*a + 1)*c)) + I*b*\operatorname{dilog}(((a + 1)*b*c*x + b^2*d - (I*b^2*x + (-I*a - I)*b)*\sqrt{c}*\sqrt{d})/((2*I*a + 2*I)*b*\sqrt{c}*\sqrt{d} + b^2*d - (a^2 + 2*a + 1)*c)) - I*b*\operatorname{dilog}(-((a + 1)*b*c*x + b^2*d + (I*b^2*x + (-I*a - I)*b)*\sqrt{c}*\sqrt{d})/((2*I*a + 2*I)*b*\sqrt{c}*\sqrt{d} - b^2*d + (a^2 + 2*a + 1)*c)) - I*b*\operatorname{dilog}(((a - 1)*b*c*x + b^2*d - (I*b^2*x + (-I*a + I)*b)*\sqrt{c}*\sqrt{d})/((2*I*a - 2*I)*b*\sqrt{c}*\sqrt{d} + b^2*d - (a^2 - 2*a + 1)*c)) + I*b*\operatorname{dilog}(-((a - 1)*b*c*x + b^2*d + (I*b^2*x + (-I*a + I)*b)*\sqrt{c}*\sqrt{d})/((2*I*a - 2*I)*b*\sqrt{c}*\sqrt{d} - b^2*d + (a^2 - 2*a + 1)*c)) - (b*\arctan2((b^2*x + (a + 1)*b)*\sqrt{c}*\sqrt{d})/(b^2*d + (a^2 + 2*a + 1)*c), ((a + 1)*b*c*x + (a^2 + 2*a + 1)*c)/(b^2*d + (a^2 + 2*a + 1)*c)) - b*\arctan2((b^2*x + (a - 1)*b)*\sqrt{c}*\sqrt{d})/(b^2*d$

+ (a² - 2*a + 1)*c), ((a - 1)*b*c*x + (a² - 2*a + 1)*c)/(b²*d + (a² - 2*a + 1)*c))) * log(c*x² + d) * sqrt(c) * sqrt(d) / (b*c²)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{atanh}(a + b x)}{c + \frac{d}{x^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(atanh(a + b*x)/(c + d/x^2), x)

[Out] int(atanh(a + b*x)/(c + d/x^2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(atanh(b*x+a)/(c+d/x**2), x)

[Out] Timed out

$$3.58 \quad \int \frac{\tanh^{-1}(a+bx)}{c+\frac{d}{x^3}} dx$$

Optimal. Leaf size=832

$$\frac{(-a-bx+1)\log(-a-bx+1)}{2bc} + \frac{\sqrt[3]{d} \log\left(\frac{b(\sqrt[3]{c}x+\sqrt[3]{d})}{\sqrt[3]{c}(1-a)+b\sqrt[3]{d}}\right) \log(-a-bx+1)}{6c^{4/3}} + \frac{(-1)^{2/3} \sqrt[3]{d} \log\left(-\frac{b(\sqrt[3]{d}-\sqrt[3]{-1}\sqrt[3]{c}x)}{\sqrt[3]{-1}(1-a)\sqrt[3]{c}-b\sqrt[3]{d}}\right) \log(-a-bx+1)}{6c^{4/3}}$$

[Out] $\frac{1}{2}*(-b*x-a+1)*\ln(-b*x-a+1)/b/c + \frac{1}{2}*(b*x+a+1)*\ln(b*x+a+1)/b/c - \frac{1}{6}*d^{(1/3)}*1$
 $n(b*x+a+1)*\ln(-b*(d^{(1/3)}+c^{(1/3)}*x)/((1+a)*c^{(1/3)}-b*d^{(1/3)}))/c^{(4/3)} + \frac{1}{6}$
 $*d^{(1/3)}*\ln(-b*x-a+1)*\ln(b*(d^{(1/3)}+c^{(1/3)}*x)/((1-a)*c^{(1/3)}+b*d^{(1/3)}))/c$
 $^{(4/3)} + \frac{1}{6}*(-1)^{(2/3)}*d^{(1/3)}*\ln(-b*x-a+1)*\ln(-b*(d^{(1/3)}-(-1)^{(1/3)}*c^{(1/3)}$
 $)x)/((-1)^{(1/3)}*(1-a)*c^{(1/3)}-b*d^{(1/3)}))/c^{(4/3)} - \frac{1}{6}*(-1)^{(2/3)}*d^{(1/3)}*1$
 $n(b*x+a+1)*\ln(b*(d^{(1/3)}-(-1)^{(1/3)}*c^{(1/3)}*x)/((-1)^{(1/3)}*(1+a)*c^{(1/3)}+b*$
 $d^{(1/3)}))/c^{(4/3)} + \frac{1}{6}*(-1)^{(1/3)}*d^{(1/3)}*\ln(b*x+a+1)*\ln(-b*(d^{(1/3)}+(-1)^{(2/3)}$
 $*c^{(1/3)}*x)/((-1)^{(2/3)}*(1+a)*c^{(1/3)}-b*d^{(1/3)}))/c^{(4/3)} - \frac{1}{6}*(-1)^{(1/3)}$
 $*d^{(1/3)}*\ln(-b*x-a+1)*\ln(b*(d^{(1/3)}+(-1)^{(2/3)}*c^{(1/3)}*x)/((-1)^{(2/3)}*(1-a)$
 $*c^{(1/3)}+b*d^{(1/3)}))/c^{(4/3)} + \frac{1}{6}*(-1)^{(2/3)}*d^{(1/3)}*\text{polylog}(2, (-1)^{(1/3)}*c^{(1/3)}$
 $*(-b*x-a+1)/((-1)^{(1/3)}*(1-a)*c^{(1/3)}-b*d^{(1/3)}))/c^{(4/3)} + \frac{1}{6}*d^{(1/3)}*$
 $\text{polylog}(2, c^{(1/3)}*(-b*x-a+1)/((1-a)*c^{(1/3)}+b*d^{(1/3)}))/c^{(4/3)} - \frac{1}{6}*(-1)^{(1/3)}$
 $*d^{(1/3)}*\text{polylog}(2, (-1)^{(2/3)}*c^{(1/3)}*(-b*x-a+1)/((-1)^{(2/3)}*(1-a)*c^{(1/3)}$
 $+b*d^{(1/3)}))/c^{(4/3)} - \frac{1}{6}*d^{(1/3)}*\text{polylog}(2, c^{(1/3)}*(b*x+a+1)/((1+a)*c^{(1/3)}$
 $-b*d^{(1/3)}))/c^{(4/3)} + \frac{1}{6}*(-1)^{(1/3)}*d^{(1/3)}*\text{polylog}(2, (-1)^{(2/3)}*c^{(1/3)}*$
 $(b*x+a+1)/((-1)^{(2/3)}*(1+a)*c^{(1/3)}-b*d^{(1/3)}))/c^{(4/3)} - \frac{1}{6}*(-1)^{(2/3)}*d^{(1/3)}$
 $*\text{polylog}(2, (-1)^{(1/3)}*c^{(1/3)}*(b*x+a+1)/((-1)^{(1/3)}*(1+a)*c^{(1/3)}+b*d^{(1/3)}))/c^{(4/3)}$

Rubi [A] time = 1.43, antiderivative size = 832, normalized size of antiderivative = 1.00, number of steps used = 31, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {6115, 2409, 2389, 2295, 2394, 2393, 2391}

$$\frac{(-a-bx+1)\log(-a-bx+1)}{2bc} + \frac{\sqrt[3]{d} \log\left(\frac{b(\sqrt[3]{c}x+\sqrt[3]{d})}{\sqrt[3]{c}(1-a)+b\sqrt[3]{d}}\right) \log(-a-bx+1)}{6c^{4/3}} + \frac{(-1)^{2/3} \sqrt[3]{d} \log\left(-\frac{b(\sqrt[3]{d}-\sqrt[3]{-1}\sqrt[3]{c}x)}{\sqrt[3]{-1}(1-a)\sqrt[3]{c}-b\sqrt[3]{d}}\right) \log(-a-bx+1)}{6c^{4/3}}$$

Antiderivative was successfully verified.

[In] Int[ArcTanh[a + b*x]/(c + d/x^3), x]

[Out] $((1-a-b*x)*\text{Log}[1-a-b*x])/(2*b*c) + ((1+a+b*x)*\text{Log}[1+a+b*x])/(2*b*c) - (d^{(1/3)}*\text{Log}[1+a+b*x]*\text{Log}[-(b*(d^{(1/3)}+c^{(1/3)}*x))/((1+a)*c^{(1/3)}-b*d^{(1/3)})])/(6*c^{(4/3)}) + (d^{(1/3)}*\text{Log}[1-a-b*x]*\text{Log}[(b*($

$$\begin{aligned} & d^{1/3} + c^{1/3}x) / ((1 - a)c^{1/3} + b d^{1/3}) / (6c^{4/3}) + ((-1)^{2/3} d^{1/3} \text{Log}[1 - a - b x] \text{Log}[-((b(d^{1/3}) - (-1)^{1/3} c^{1/3} x)) / ((-1)^{1/3} (1 - a) c^{1/3} - b d^{1/3})]) / (6c^{4/3}) - ((-1)^{2/3} d^{1/3} \text{Log}[1 + a + b x] \text{Log}[(b(d^{1/3}) - (-1)^{1/3} c^{1/3} x)) / ((-1)^{1/3} (1 + a) c^{1/3} + b d^{1/3})]) / (6c^{4/3}) + ((-1)^{1/3} d^{1/3} \text{Log}[1 + a + b x] \text{Log}[-((b(d^{1/3}) + (-1)^{2/3} c^{1/3} x)) / ((-1)^{2/3} (1 + a) c^{1/3} - b d^{1/3})]) / (6c^{4/3}) - ((-1)^{1/3} d^{1/3} \text{Log}[1 - a - b x] \text{Log}[(b(d^{1/3}) + (-1)^{2/3} c^{1/3} x)) / ((-1)^{2/3} (1 - a) c^{1/3} + b d^{1/3})]) / (6c^{4/3}) + ((-1)^{2/3} d^{1/3} \text{PolyLog}[2, ((-1)^{1/3} c^{1/3} (1 - a - b x)) / ((-1)^{1/3} (1 - a) c^{1/3} - b d^{1/3})]) / (6c^{4/3}) + (d^{1/3} \text{PolyLog}[2, (c^{1/3} (1 - a - b x)) / ((1 - a) c^{1/3} + b d^{1/3})]) / (6c^{4/3}) - ((-1)^{1/3} d^{1/3} \text{PolyLog}[2, ((-1)^{2/3} c^{1/3} (1 - a - b x)) / ((-1)^{2/3} (1 - a) c^{1/3} + b d^{1/3})]) / (6c^{4/3}) - (d^{1/3} \text{PolyLog}[2, (c^{1/3} (1 + a + b x)) / ((1 + a) c^{1/3} - b d^{1/3})]) / (6c^{4/3}) + ((-1)^{1/3} d^{1/3} \text{PolyLog}[2, ((-1)^{2/3} c^{1/3} (1 + a + b x)) / ((-1)^{2/3} (1 + a) c^{1/3} - b d^{1/3})]) / (6c^{4/3}) - ((-1)^{2/3} d^{1/3} \text{PolyLog}[2, ((-1)^{1/3} c^{1/3} (1 + a + b x)) / ((-1)^{1/3} (1 + a) c^{1/3} + b d^{1/3})]) / (6c^{4/3}) \end{aligned}$$
Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] := Simp[x*Log[c*x^n], x] - Simp[n*x, x]
/; FreeQ[{c, n}, x]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a,
b, c, d, e, n, p}, x]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] := -Simp[PolyLog[2,
-(c*e*x^n)/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] := Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x],
x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] := Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
```

$\int \frac{(b*e*x)^n}{g} dx - \text{Dist}[\frac{b*e*x}{g}, \int \frac{\text{Log}[(e*(f + g*x))/(e*f - d*g)]}{(d + e*x)}, x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, g, n\}, x] \&\& \text{NeQ}[e*f - d*g, 0]$

Rule 2409

$\text{Int}[(a + \text{Log}[c*(d + e*x)]*(b))^p * (f + g*x)^r]^q, x_Symbol] :> \text{Int}[\text{ExpandIntegrand}[(a + b*\text{Log}[c*(d + e*x)^n])^p, (f + g*x^r)^q, x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, g, n, r\}, x] \&\& \text{IntegerQ}[q] \&\& (\text{GtQ}[p, 0] \&\& \text{IntegerQ}[r] \&\& \text{NeQ}[r, 1])$

Rule 6115

$\text{Int}[\text{ArcTanh}[(c + d*x)/(e + f*x^n)], x_Symbol] :> \text{Dist}[1/2, \int \frac{\text{Log}[1 + c + d*x]}{e + f*x^n} dx, x] - \text{Dist}[1/2, \int \frac{\text{Log}[1 - c - d*x]}{e + f*x^n} dx, x] /; \text{FreeQ}[\{c, d, e, f\}, x] \&\& \text{RationalQ}[n]$

Rubi steps

$$\begin{aligned}
\int \frac{\tanh^{-1}(a+bx)}{c+\frac{d}{x^3}} dx &= -\left(\frac{1}{2} \int \frac{\log(1-a-bx)}{c+\frac{d}{x^3}} dx\right) + \frac{1}{2} \int \frac{\log(1+a+bx)}{c+\frac{d}{x^3}} dx \\
&= -\left(\frac{1}{2} \int \left(\frac{\log(1-a-bx)}{c} - \frac{d \log(1-a-bx)}{c(d+cx^3)}\right) dx\right) + \frac{1}{2} \int \left(\frac{\log(1+a+bx)}{c} - \frac{d \log(1+a+bx)}{c(d+cx^3)}\right) dx \\
&= -\frac{\int \log(1-a-bx) dx}{2c} + \frac{\int \log(1+a+bx) dx}{2c} + \frac{d \int \frac{\log(1-a-bx)}{d+cx^3} dx}{2c} - \frac{d \int \frac{\log(1+a+bx)}{d+cx^3} dx}{2c} \\
&= \frac{\text{Subst}(\int \log(x) dx, x, 1-a-bx)}{2bc} + \frac{\text{Subst}(\int \log(x) dx, x, 1+a+bx)}{2bc} + \frac{d \int \left(-\frac{\log(1-a-bx)}{3d^{2/3}(-\sqrt[3]{d}-\sqrt[3]{c}x)}\right) dx}{6c} \\
&= \frac{(1-a-bx) \log(1-a-bx)}{2bc} + \frac{(1+a+bx) \log(1+a+bx)}{2bc} - \frac{\sqrt[3]{d} \int \frac{\log(1-a-bx)}{-\sqrt[3]{d}-\sqrt[3]{c}x} dx}{6c} - \frac{\sqrt[3]{d} \int \frac{\log(1+a+bx)}{\sqrt[3]{d}+\sqrt[3]{c}x} dx}{6c} \\
&= \frac{(1-a-bx) \log(1-a-bx)}{2bc} + \frac{(1+a+bx) \log(1+a+bx)}{2bc} - \frac{\sqrt[3]{d} \log(1+a+bx) \log\left(-\frac{\sqrt[3]{d}-\sqrt[3]{c}x}{\sqrt[3]{d}+\sqrt[3]{c}x}\right)}{6c^{4/3}} \\
&= \frac{(1-a-bx) \log(1-a-bx)}{2bc} + \frac{(1+a+bx) \log(1+a+bx)}{2bc} - \frac{\sqrt[3]{d} \log(1+a+bx) \log\left(-\frac{\sqrt[3]{d}-\sqrt[3]{c}x}{\sqrt[3]{d}+\sqrt[3]{c}x}\right)}{6c^{4/3}} \\
&= \frac{(1-a-bx) \log(1-a-bx)}{2bc} + \frac{(1+a+bx) \log(1+a+bx)}{2bc} - \frac{\sqrt[3]{d} \log(1+a+bx) \log\left(-\frac{\sqrt[3]{d}-\sqrt[3]{c}x}{\sqrt[3]{d}+\sqrt[3]{c}x}\right)}{6c^{4/3}}
\end{aligned}$$

Mathematica [C] time = 8.06, size = 917, normalized size = 1.10

$$d\text{RootSum} \left[c\#1^3 a^3 + 3c\#1^2 a^3 + ca^3 + 3c\#1 a^3 - 3c\#1^3 a^2 - 3c\#1^2 a^2 + 3ca^2 + 3c\#1 a^2 + 3c\#1^3 a - 3c\#1^2 a + 3ca \right]$$

Warning: Unable to verify antiderivative.

[In] Integrate[ArcTanh[a + b*x]/(c + d/x^3), x]

[Out] -1/6*(-6*(a + b*x)*ArcTanh[a + b*x] + 6*Log[1/Sqrt[1 - (a + b*x)^2]] + b^3*d*RootSum[c + 3*a*c + 3*a^2*c + a^3*c - b^3*d - 3*c*#1 - 3*a*c*#1 + 3*a^2*c

```

*#1 + 3*a^3*c*#1 - 3*b^3*d*#1 + 3*c*#1^2 - 3*a*c*#1^2 - 3*a^2*c*#1^2 + 3*a^
3*c*#1^2 - 3*b^3*d*#1^2 - c*#1^3 + 3*a*c*#1^3 - 3*a^2*c*#1^3 + a^3*c*#1^3 -
b^3*d*#1^3 & , ((-I)*Pi*ArcTanh[a + b*x] - 2*ArcTanh[a + b*x]^2 - 2*ArcTan
h[a + b*x]*ArcTanh[(1 - #1)/(1 + #1)] + I*Pi*Log[1 + E^(2*ArcTanh[a + b*x])
] - 2*ArcTanh[a + b*x]*Log[1 - E^(-2*(ArcTanh[a + b*x] + ArcTanh[(1 - #1)/(
1 + #1])))] - 2*ArcTanh[(1 - #1)/(1 + #1)]*Log[1 - E^(-2*(ArcTanh[a + b*x]
+ ArcTanh[(1 - #1)/(1 + #1])))] - I*Pi*Log[1/Sqrt[1 - (a + b*x)^2]] + 2*Arc
Tanh[(1 - #1)/(1 + #1)]*Log[I*Sinh[ArcTanh[a + b*x] + ArcTanh[(1 - #1)/(1 +
#1)]]] + PolyLog[2, E^(-2*(ArcTanh[a + b*x] + ArcTanh[(1 - #1)/(1 + #1])))]
] - 2*ArcTanh[a + b*x]^2*#1 + I*Pi*ArcTanh[a + b*x]*#1^2 + 2*ArcTanh[a + b*
x]*ArcTanh[(1 - #1)/(1 + #1)]*#1^2 - I*Pi*Log[1 + E^(2*ArcTanh[a + b*x])]*#
1^2 + 2*ArcTanh[a + b*x]*Log[1 - E^(-2*(ArcTanh[a + b*x] + ArcTanh[(1 - #1)
/(1 + #1])))]*#1^2 + 2*ArcTanh[(1 - #1)/(1 + #1)]*Log[1 - E^(-2*(ArcTanh[a
+ b*x] + ArcTanh[(1 - #1)/(1 + #1])))]*#1^2 + I*Pi*Log[1/Sqrt[1 - (a + b*x)
^2]]*#1^2 - 2*ArcTanh[(1 - #1)/(1 + #1)]*Log[I*Sinh[ArcTanh[a + b*x] + ArcT
anh[(1 - #1)/(1 + #1)]]*#1^2 - PolyLog[2, E^(-2*(ArcTanh[a + b*x] + ArcTan
h[(1 - #1)/(1 + #1])))]*#1^2 + (2*ArcTanh[a + b*x]^2*Sqrt[#1/(1 + #1)^2])/E
^ArcTanh[(1 - #1)/(1 + #1)] + (4*ArcTanh[a + b*x]^2*#1*Sqrt[#1/(1 + #1)^2])
/E^ArcTanh[(1 - #1)/(1 + #1)] + (2*ArcTanh[a + b*x]^2*#1^2*Sqrt[#1/(1 + #1)
^2])/E^ArcTanh[(1 - #1)/(1 + #1)]/(a*c + 2*a^2*c + a^3*c - b^3*d - 2*a*c*#
1 + 2*a^3*c*#1 - 2*b^3*d*#1 + a*c*#1^2 - 2*a^2*c*#1^2 + a^3*c*#1^2 - b^3*d*
#1^2) & ])/(b*c)

```

fricas [F] time = 0.56, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^3 \operatorname{artanh}(bx+a)}{cx^3+d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(b*x+a)/(c+d/x^3),x, algorithm="fricas")
```

```
[Out] integral(x^3*arctanh(b*x + a)/(c*x^3 + d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

sage0x

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(b*x+a)/(c+d/x^3),x, algorithm="giac")
```

```
[Out] sage0*x
```


maple [C] time = 0.89, size = 650, normalized size = 0.78

$$\frac{\operatorname{arctanh}(bx+a)x}{c} + \frac{\operatorname{arctanh}(bx+a)a}{bc} + \frac{2b^2d \left(\sum_{R1=\text{RootOf}((a^3c-db^3-3a^2c+3ac-c)_Z^6+(3a^3c-3db^3-3a^2c-3ac+3c)_Z^4+(3a^3c-3db^3-3a^2c-3ac+3c)_Z^2+a^3c-db^3+3a^2c+3ac+c)} \right)}{c^2d \sum_{R1=\text{RootOf}((a^3c-db^3-3a^2c+3ac-c)_Z^6+(3a^3c-3db^3-3a^2c-3ac+3c)_Z^4+(3a^3c-3db^3-3a^2c-3ac+3c)_Z^2+a^3c-db^3+3a^2c+3ac+c)} \left(\frac{1}{R1^4 a^3 c - R1^4 b^3 d - 3 R1^4 a^2 c + 3 R1^4 a^3 c + 2 R1^2 a^3 c - 2 R1^2 b^3 d - R1^4 c - 2 R1^2 a^2 c - 2 R1^2 a^3 c + a^3 c - b^3 d + 2 R1^2 c + a^2 c - a^3 c - c} \right) \left(\operatorname{arctanh}(bx+a) \ln\left(\frac{R1 - (bx+a+1)}{(1 - (bx+a)^2)^{1/2}}\right) / R1 + \operatorname{dilog}\left(\frac{R1 - (bx+a+1)}{(1 - (bx+a)^2)^{1/2}}\right) / R1\right)},$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arctanh(b*x+a)/(c+d/x^3),x)

[Out] arctanh(b*x+a)/c*x+1/b*arctanh(b*x+a)/c*a+2/3*b^2/c*d*sum(1/(_R1^4*a^3*c-_R1^4*b^3*d-3*_R1^4*a^2*c+3*_R1^4*a^3*c+2*_R1^2*a^3*c-2*_R1^2*b^3*d-_R1^4*c-2*_R1^2*a^2*c-2*_R1^2*a^3*c+a^3*c-b^3*d+2*_R1^2*c+a^2*c-a^3*c-c)*(arctanh(b*x+a)*ln((R1-(b*x+a+1))/(1-(b*x+a)^2)^(1/2)))/R1)+dilog((R1-(b*x+a+1))/(1-(b*x+a)^2)^(1/2))/R1),_R1=RootOf((a^3*c-b^3*d-3*a^2*c+3*a^3*c-c)*_Z^6+(3*a^3*c-3*b^3*d-3*a^2*c-3*a^3*c+3*c)*_Z^4+(3*a^3*c-3*b^3*d+3*a^2*c-3*a^3*c-3*c)*_Z^2+a^3*c-d*b^3+3*a^2*c+3*a^3*c+c))+2/3*b^2/c*d*sum(_R1^2/(_R1^4*a^3*c-_R1^4*b^3*d-3*_R1^4*a^2*c+3*_R1^4*a^3*c+2*_R1^2*a^3*c-2*_R1^2*b^3*d-_R1^4*c-2*_R1^2*a^2*c-2*_R1^2*a^3*c+a^3*c-b^3*d+2*_R1^2*c+a^2*c-a^3*c-c)*(arctanh(b*x+a)*ln((R1-(b*x+a+1))/(1-(b*x+a)^2)^(1/2)))/R1)+dilog((R1-(b*x+a+1))/(1-(b*x+a)^2)^(1/2))/R1),_R1=RootOf((a^3*c-b^3*d-3*a^2*c+3*a^3*c-c)*_Z^6+(3*a^3*c-3*b^3*d-3*a^2*c-3*a^3*c+3*c)*_Z^4+(3*a^3*c-3*b^3*d+3*a^2*c-3*a^3*c-3*c)*_Z^2+a^3*c-d*b^3+3*a^2*c+3*a^3*c+c))+1/2/b/c*ln(b*x+a-1)+1/2/b/c*ln(b*x+a+1)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{artanh}(bx+a)}{c + \frac{d}{x^3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctanh(b*x+a)/(c+d/x^3),x, algorithm="maxima")

[Out] integrate(arctanh(b*x + a)/(c + d/x^3), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{atanh}(a+bx)}{c + \frac{d}{x^3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a + b*x)/(c + d/x^3),x)
```

```
[Out] int(atanh(a + b*x)/(c + d/x^3), x)
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(b*x+a)/(c+d/x**3),x)
```

```
[Out] Timed out
```

$$3.59 \quad \int \frac{\tanh^{-1}(a+bx)}{c+d\sqrt{x}} dx$$

Optimal. Leaf size=585

$$\frac{c\operatorname{Li}_2\left(\frac{\sqrt{b}(c+d\sqrt{x})}{\sqrt{b}c-\sqrt{-a-1}d}\right)}{d^2} + \frac{c\operatorname{Li}_2\left(\frac{\sqrt{b}(c+d\sqrt{x})}{\sqrt{b}c+\sqrt{-a-1}d}\right)}{d^2} - \frac{c\operatorname{Li}_2\left(\frac{\sqrt{b}(c+d\sqrt{x})}{\sqrt{b}c-\sqrt{1-ad}}\right)}{d^2} - \frac{c\operatorname{Li}_2\left(\frac{\sqrt{b}(c+d\sqrt{x})}{\sqrt{b}c+\sqrt{1-ad}}\right)}{d^2} + \frac{c\log(c+d\sqrt{x})\log\left(\frac{d(\sqrt{-a-1}-\sqrt{-a-1}d)}{\sqrt{-a-1}d}\right)}{d^2}$$

[Out] $c*\ln(-b*x-a+1)*\ln(c+d*x^{(1/2)})/d^2-c*\ln(b*x+a+1)*\ln(c+d*x^{(1/2)})/d^2+c*\ln(c+d*x^{(1/2)})*\ln(d*((-1-a)^{(1/2)}-b^{(1/2)}*x^{(1/2)})/(d*(-1-a)^{(1/2)}+c*b^{(1/2)}))/d^2-c*\ln(c+d*x^{(1/2)})*\ln(d*((1-a)^{(1/2)}-b^{(1/2)}*x^{(1/2)})/(d*(1-a)^{(1/2)}+c*b^{(1/2)}))/d^2+c*\ln(c+d*x^{(1/2)})*\ln(-d*((-1-a)^{(1/2)}+b^{(1/2)}*x^{(1/2)})/(-d*(-1-a)^{(1/2)}+c*b^{(1/2)}))/d^2-c*\ln(c+d*x^{(1/2)})*\ln(-d*((1-a)^{(1/2)}+b^{(1/2)}*x^{(1/2)})/(-d*(1-a)^{(1/2)}+c*b^{(1/2)}))/d^2+c*\operatorname{polylog}(2,b^{(1/2)}*(c+d*x^{(1/2)})/(-d*(-1-a)^{(1/2)}+c*b^{(1/2)}))/d^2+c*\operatorname{polylog}(2,b^{(1/2)}*(c+d*x^{(1/2)})/(d*(-1-a)^{(1/2)}+c*b^{(1/2)}))/d^2-c*\operatorname{polylog}(2,b^{(1/2)}*(c+d*x^{(1/2)})/(-d*(1-a)^{(1/2)}+c*b^{(1/2)}))/d^2-c*\operatorname{polylog}(2,b^{(1/2)}*(c+d*x^{(1/2)})/(d*(1-a)^{(1/2)}+c*b^{(1/2)}))/d^2-2*\operatorname{arctanh}(b^{(1/2)}*x^{(1/2)/(1-a)^{(1/2)})*(1-a)^{(1/2)}/d/b^{(1/2)}+2*\operatorname{arctan}(b^{(1/2)}*x^{(1/2)/(1+a)^{(1/2)})*(1+a)^{(1/2)}/d/b^{(1/2)}-\ln(-b*x-a+1)*x^{(1/2)}/d+\ln(b*x+a+1)*x^{(1/2)}/d$

Rubi [A] time = 1.01, antiderivative size = 585, normalized size of antiderivative = 1.00, number of steps used = 31, number of rules used = 13, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.722$, Rules used = {6115, 2408, 2466, 2448, 321, 205, 2462, 260, 2416, 2394, 2393, 2391, 208}

$$\frac{c\operatorname{PolyLog}\left(2,\frac{\sqrt{b}(c+d\sqrt{x})}{\sqrt{b}c-\sqrt{-a-1}d}\right)}{d^2} + \frac{c\operatorname{PolyLog}\left(2,\frac{\sqrt{b}(c+d\sqrt{x})}{\sqrt{-a-1}d+\sqrt{b}c}\right)}{d^2} - \frac{c\operatorname{PolyLog}\left(2,\frac{\sqrt{b}(c+d\sqrt{x})}{\sqrt{b}c-\sqrt{1-ad}}\right)}{d^2} - \frac{c\operatorname{PolyLog}\left(2,\frac{\sqrt{b}(c+d\sqrt{x})}{\sqrt{1-ad}+\sqrt{b}c}\right)}{d^2}$$

Antiderivative was successfully verified.

[In] Int[ArcTanh[a + b*x]/(c + d*Sqrt[x]), x]

[Out] $(2*\operatorname{Sqrt}[1+a]*\operatorname{ArcTan}[(\operatorname{Sqrt}[b]*\operatorname{Sqrt}[x])/(\operatorname{Sqrt}[1+a])])/(\operatorname{Sqrt}[b]*d) - (2*\operatorname{Sqrt}[1-a]*\operatorname{ArcTanh}[(\operatorname{Sqrt}[b]*\operatorname{Sqrt}[x])/(\operatorname{Sqrt}[1-a])])/(\operatorname{Sqrt}[b]*d) + (c*\operatorname{Log}[(d*(\operatorname{Sqrt}[-1-a] - \operatorname{Sqrt}[b]*\operatorname{Sqrt}[x]))/(\operatorname{Sqrt}[b]*c + \operatorname{Sqrt}[-1-a]*d)]*\operatorname{Log}[c + d*\operatorname{Sqrt}[x]])/d^2 - (c*\operatorname{Log}[(d*(\operatorname{Sqrt}[1-a] - \operatorname{Sqrt}[b]*\operatorname{Sqrt}[x]))/(\operatorname{Sqrt}[b]*c + \operatorname{Sqrt}[1-a]*d)]*\operatorname{Log}[c + d*\operatorname{Sqrt}[x]])/d^2 + (c*\operatorname{Log}[-((d*(\operatorname{Sqrt}[-1-a] + \operatorname{Sqrt}[b]*\operatorname{Sqrt}[x]))/(\operatorname{Sqrt}[b]*c - \operatorname{Sqrt}[-1-a]*d))]*\operatorname{Log}[c + d*\operatorname{Sqrt}[x]])/d^2 - (c*\operatorname{Log}[-((d*(\operatorname{Sqrt}[1-a] + \operatorname{Sqrt}[b]*\operatorname{Sqrt}[x]))/(\operatorname{Sqrt}[b]*c - \operatorname{Sqrt}[1-a]*d))]*\operatorname{Log}[c + d*\operatorname{Sqrt}[x]])/d^2 - (\operatorname{Sqrt}[x]*\operatorname{Log}[1-a-b*x])/d + (c*\operatorname{Log}[c + d*\operatorname{Sqrt}[x]]*\operatorname{Log}[1-a-b*x])/d^2 + (\operatorname{Sqrt}[x]*\operatorname{Log}[1+a+b*x])/d - (c*\operatorname{Log}[c + d*\operatorname{Sqrt}[x]]*\operatorname{Log}[1$

$$+ a + b*x))/d^2 + (c*PolyLog[2, (Sqrt[b]*(c + d*Sqrt[x]))/(Sqrt[b]*c - Sqrt[-1 - a]*d)))/d^2 + (c*PolyLog[2, (Sqrt[b]*(c + d*Sqrt[x]))/(Sqrt[b]*c + Sqrt[-1 - a]*d)))/d^2 - (c*PolyLog[2, (Sqrt[b]*(c + d*Sqrt[x]))/(Sqrt[b]*c - Sqrt[1 - a]*d)))/d^2 - (c*PolyLog[2, (Sqrt[b]*(c + d*Sqrt[x]))/(Sqrt[b]*c + Sqrt[1 - a]*d)))/d^2$$

Rule 205

$$\text{Int}[\frac{(a + b*x^2)^{-1}}{x}, x_Symbol] \rightarrow \text{Simp}[\frac{\text{Rt}[a/b, 2]*\text{ArcTan}[x/\text{Rt}[a/b, 2]]}{a}, x] \text{ ; FreeQ}\{a, b, x\} \ \&\& \ \text{PosQ}[a/b]$$

Rule 208

$$\text{Int}[\frac{(a + b*x^2)^{-1}}{x}, x_Symbol] \rightarrow \text{Simp}[\frac{\text{Rt}[-(a/b), 2]*\text{ArcTanh}[x/\text{Rt}[-(a/b), 2]]}{a}, x] \text{ ; FreeQ}\{a, b, x\} \ \&\& \ \text{NegQ}[a/b]$$

Rule 260

$$\text{Int}[\frac{x^{m_1}}{(a + b*x^{n_1})}, x_Symbol] \rightarrow \text{Simp}[\frac{\text{Log}[\text{RemoveContent}[a + b*x^n, x]]}{(b*n)}, x] \text{ ; FreeQ}\{a, b, m, n, x\} \ \&\& \ \text{EqQ}[m, n - 1]$$

Rule 321

$$\text{Int}[\frac{(c*x)^{m_1}*(a + b*x^{n_1})^{p_1}}{x}, x_Symbol] \rightarrow \text{Simp}[\frac{c^{n-1}*(c*x)^{m-n+1}*(a + b*x^n)^{p+1}}{b*(m + n*p + 1)}, x] - \text{Dist}[\frac{a*c^{n-1}*(m-n+1)}{b*(m + n*p + 1)}, \text{Int}[\frac{(c*x)^{m-n}*(a + b*x^n)^p}{x}, x] \text{ ; FreeQ}\{a, b, c, p, x\} \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ \text{GtQ}[m, n - 1] \ \&\& \ \text{NeQ}[m + n*p + 1, 0] \ \&\& \ \text{IntBinomialQ}[a, b, c, n, m, p, x]$$

Rule 2391

$$\text{Int}[\frac{\text{Log}[(c*x)^{(d_1)} + (e*x)^{(n_1)}]}{x}, x_Symbol] \rightarrow -\text{Simp}[\frac{\text{PolyLog}[2, -(c*e*x^n)]}{n}, x] \text{ ; FreeQ}\{c, d, e, n, x\} \ \&\& \ \text{EqQ}[c*d, 1]$$

Rule 2393

$$\text{Int}[\frac{(a + \text{Log}[(c*x)^{(d_1)} + (e*x)^{(n_1)}])*(b*x)}{(f + g*x)}, x_Symbol] \rightarrow \text{Dist}[\frac{1}{g}, \text{Subst}[\text{Int}[\frac{a + b*\text{Log}[1 + (c*e*x)/g]}{x}, x], x, f + g*x], x] \text{ ; FreeQ}\{a, b, c, d, e, f, g, x\} \ \&\& \ \text{NeQ}[e*f - d*g, 0] \ \&\& \ \text{EqQ}[g + c*(e*f - d*g), 0]$$

Rule 2394

$$\text{Int}[\frac{(a + \text{Log}[(c*x)^{(d_1)} + (e*x)^{(n_1)}])*(b*x)}{(f + g*x)}, x_Symbol] \rightarrow \text{Simp}[\frac{\text{Log}[(e*(f + g*x))/(e*f - d*g)]*(a + b*\text{Log}[c*(d + e*x)^n])}{g}, x] - \text{Dist}[\frac{b*e*n}{g}, \text{Int}[\frac{\text{Log}[(e*(f + g*x))/(e*f - d*g)]}{(d + e*x)}$$

, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]

Rule 2408

Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]*(b_.))^(p_.)*((f_.) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] := With[{k = Denominator[r]}, Dist[k, Subst[Int[x^(k - 1)*(f + g*x^(k*r))^q*(a + b*Log[c*(d + e*x^k)^n])^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && FractionQ[r] && IntegerQ[p, 0]

Rule 2416

Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]*(b_.))^(p_.)*((h_.)*(x_)^(m_.)*((f_.) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] := Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]

Rule 2448

Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))^(p_.)], x_Symbol] := Simp[x*Log[c*(d + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d, e, n, p}, x]

Rule 2462

Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))^(p_.)]*(b_.))/((f_.) + (g_.)*(x_.)), x_Symbol] := Simp[(Log[f + g*x]*(a + b*Log[c*(d + e*x^n)^p])/g, x] - Dist[(b*e*n*p)/g, Int[(x^(n - 1)*Log[f + g*x])/(d + e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]

Rule 2466

Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))^(p_.)]*(b_.))^(q_.)*(x_)^(m_.)*((f_.) + (g_.)*(x_)^(r_.)), x_Symbol] := Int[ExpandIntegrand[(a + b*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && IntegerQ[m] && IntegerQ[r]

Rule 6115

Int[ArcTanh[(c_) + (d_.)*(x_)]/((e_) + (f_.)*(x_)^(n_.)), x_Symbol] := Dist[1/2, Int[Log[1 + c + d*x]/(e + f*x^n), x], x] - Dist[1/2, Int[Log[1 - c - d*x]/(e + f*x^n), x], x] /; FreeQ[{c, d, e, f}, x] && RationalQ[n]

Rubi steps

$$\begin{aligned}
\int \frac{\tanh^{-1}(a+bx)}{c+d\sqrt{x}} dx &= -\left(\frac{1}{2} \int \frac{\log(1-a-bx)}{c+d\sqrt{x}} dx\right) + \frac{1}{2} \int \frac{\log(1+a+bx)}{c+d\sqrt{x}} dx \\
&= -\text{Subst}\left(\int \frac{x \log(1-a-bx^2)}{c+dx} dx, x, \sqrt{x}\right) + \text{Subst}\left(\int \frac{x \log(1+a+bx^2)}{c+dx} dx, x, \sqrt{x}\right) \\
&= -\text{Subst}\left(\int \left(\frac{\log(1-a-bx^2)}{d} - \frac{c \log(1-a-bx^2)}{d(c+dx)}\right) dx, x, \sqrt{x}\right) + \text{Subst}\left(\int \left(\frac{\log(1+a+bx^2)}{d} - \frac{c \log(1+a+bx^2)}{d(c+dx)}\right) dx, x, \sqrt{x}\right) \\
&= -\frac{\text{Subst}\left(\int \log(1-a-bx^2) dx, x, \sqrt{x}\right)}{d} + \frac{\text{Subst}\left(\int \log(1+a+bx^2) dx, x, \sqrt{x}\right)}{d} + \frac{c \text{Subst}\left(\int \frac{\log(1-a-bx^2)}{c+dx} dx, x, \sqrt{x}\right)}{d} \\
&= -\frac{\sqrt{x} \log(1-a-bx)}{d} + \frac{c \log(c+d\sqrt{x}) \log(1-a-bx)}{d^2} + \frac{\sqrt{x} \log(1+a+bx)}{d} - \frac{c \log(c+d\sqrt{x}) \log(1+a+bx)}{d^2} \\
&= -\frac{\sqrt{x} \log(1-a-bx)}{d} + \frac{c \log(c+d\sqrt{x}) \log(1-a-bx)}{d^2} + \frac{\sqrt{x} \log(1+a+bx)}{d} - \frac{c \log(c+d\sqrt{x}) \log(1+a+bx)}{d^2} \\
&= \frac{2\sqrt{1+a} \tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{1+a}}\right)}{\sqrt{b}d} - \frac{2\sqrt{1-a} \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{1-a}}\right)}{\sqrt{b}d} - \frac{\sqrt{x} \log(1-a-bx)}{d} + \frac{c \log(c+d\sqrt{x}) \log\left(\frac{d(\sqrt{1-a}-\sqrt{b}\sqrt{x})}{\sqrt{b}c+\sqrt{1-a}d}\right)}{d^2} \\
&= \frac{2\sqrt{1+a} \tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{1+a}}\right)}{\sqrt{b}d} - \frac{2\sqrt{1-a} \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{1-a}}\right)}{\sqrt{b}d} + \frac{c \log\left(\frac{d(\sqrt{1-a}-\sqrt{b}\sqrt{x})}{\sqrt{b}c+\sqrt{1-a}d}\right) \log(c+d\sqrt{x})}{d^2} \\
&= \frac{2\sqrt{1+a} \tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{1+a}}\right)}{\sqrt{b}d} - \frac{2\sqrt{1-a} \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{1-a}}\right)}{\sqrt{b}d} + \frac{c \log\left(\frac{d(\sqrt{1-a}-\sqrt{b}\sqrt{x})}{\sqrt{b}c+\sqrt{1-a}d}\right) \log(c+d\sqrt{x})}{d^2} \\
&= \frac{2\sqrt{1+a} \tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{1+a}}\right)}{\sqrt{b}d} - \frac{2\sqrt{1-a} \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{1-a}}\right)}{\sqrt{b}d} + \frac{c \log\left(\frac{d(\sqrt{1-a}-\sqrt{b}\sqrt{x})}{\sqrt{b}c+\sqrt{1-a}d}\right) \log(c+d\sqrt{x})}{d^2}
\end{aligned}$$

Mathematica [A] time = 0.54, size = 549, normalized size = 0.94

$$\frac{c \text{Li}_2\left(\frac{\sqrt{b}(c+d\sqrt{x})}{\sqrt{b}c-\sqrt{1-a}d}\right) + c \text{Li}_2\left(\frac{\sqrt{b}(c+d\sqrt{x})}{\sqrt{b}c+\sqrt{1-a}d}\right) - c \text{Li}_2\left(\frac{\sqrt{b}(c+d\sqrt{x})}{\sqrt{b}c-\sqrt{1-a}d}\right) - c \text{Li}_2\left(\frac{\sqrt{b}(c+d\sqrt{x})}{\sqrt{b}c+\sqrt{1-a}d}\right) + c \log(c+d\sqrt{x}) \log\left(\frac{d(\sqrt{1-a}-\sqrt{b}\sqrt{x})}{\sqrt{b}c+\sqrt{1-a}d}\right)}{d^2}$$

Antiderivative was successfully verified.

[In] Integrate[ArcTanh[a + b*x]/(c + d*Sqrt[x]),x]

[Out]
$$\left(\frac{2\sqrt{1+a}d\operatorname{ArcTan}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{1+a}}\right)/\sqrt{b} - (2\sqrt{1-a}d\operatorname{ArcTanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{1-a}}\right)/\sqrt{b} + c\operatorname{Log}\left[\frac{d(\sqrt{-1-a} - \sqrt{b}\sqrt{x})}{\sqrt{b}c + \sqrt{-1-a}d}\right]\operatorname{Log}[c + d\sqrt{x}] - c\operatorname{Log}\left[\frac{d(\sqrt{1-a} - \sqrt{b}\sqrt{x})}{\sqrt{b}c + \sqrt{1-a}d}\right]\operatorname{Log}[c + d\sqrt{x}] + c\operatorname{Log}\left[\frac{d(\sqrt{-1-a} + \sqrt{b}\sqrt{x})}{-(\sqrt{b}c + \sqrt{-1-a}d)}\right]\operatorname{Log}[c + d\sqrt{x}] - c\operatorname{Log}\left[\frac{d(\sqrt{1-a} + \sqrt{b}\sqrt{x})}{-(\sqrt{b}c + \sqrt{1-a}d)}\right]\operatorname{Log}[c + d\sqrt{x}] - d\sqrt{x}\operatorname{Log}[1 - a - b*x] + c\operatorname{Log}[c + d\sqrt{x}]\operatorname{Log}[1 - a - b*x] + d\sqrt{x}\operatorname{Log}[1 + a + b*x] - c\operatorname{Log}[c + d\sqrt{x}]\operatorname{Log}[1 + a + b*x] + c\operatorname{PolyLog}[2, (\sqrt{b}(c + d\sqrt{x}))]/(\sqrt{b}c - \sqrt{-1-a}d)] + c\operatorname{PolyLog}[2, (\sqrt{b}(c + d\sqrt{x}))]/(\sqrt{b}c + \sqrt{-1-a}d)] - c\operatorname{PolyLog}[2, (\sqrt{b}(c + d\sqrt{x}))]/(\sqrt{b}c - \sqrt{1-a}d)] - c\operatorname{PolyLog}[2, (\sqrt{b}(c + d\sqrt{x}))]/(\sqrt{b}c + \sqrt{1-a}d)] \right) / d^2$$

fricas [F] time = 0.51, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{d\sqrt{x}\operatorname{artanh}(bx+a) - c\operatorname{artanh}(bx+a)}{d^2x - c^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctanh(b*x+a)/(c+d*x^(1/2)),x, algorithm="fricas")

[Out]
$$\operatorname{integral}\left(\frac{d\sqrt{x}\operatorname{arctanh}(bx+a) - c\operatorname{arctanh}(bx+a)}{d^2x - c^2}, x\right)$$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{artanh}(bx+a)}{d\sqrt{x} + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctanh(b*x+a)/(c+d*x^(1/2)),x, algorithm="giac")

[Out]
$$\operatorname{integrate}\left(\frac{\operatorname{arctanh}(bx+a)}{d\sqrt{x} + c}, x\right)$$

maple [A] time = 0.10, size = 738, normalized size = 1.26

$$\frac{2\operatorname{arctanh}(bx+a)\sqrt{x}}{d} - \frac{2\operatorname{arctanh}(bx+a)c\ln(c+d\sqrt{x})}{d^2} + \frac{c\ln(c+d\sqrt{x})\ln\left(\frac{-b(c+d\sqrt{x})+bc+\sqrt{-ad^2b-bd^2}}{bc+\sqrt{-ad^2b-bd^2}}\right)}{d^2} + \frac{c\ln\left(\frac{-b(c+d\sqrt{x})+bc+\sqrt{-ad^2b-bd^2}}{bc+\sqrt{-ad^2b-bd^2}}\right)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arctanh(b*x+a)/(c+d*x^(1/2)),x)

[Out] $2*\arctanh(b*x+a)/d*x^{(1/2)}-2*\arctanh(b*x+a)*c/d^2*\ln(c+d*x^{(1/2)})+1/d^2*c*\ln(c+d*x^{(1/2)})*\ln((-b*(c+d*x^{(1/2)})+b*c+(-a*b*d^2-b*d^2)^{(1/2)})/(b*c+(-a*b*d^2-b*d^2)^{(1/2)}))+1/d^2*c*\ln(c+d*x^{(1/2)})*\ln((b*(c+d*x^{(1/2)})-b*c+(-a*b*d^2-b*d^2)^{(1/2)})/(-b*c+(-a*b*d^2-b*d^2)^{(1/2)}))+1/d^2*c*\operatorname{dilog}((-b*(c+d*x^{(1/2)})+b*c+(-a*b*d^2-b*d^2)^{(1/2)})/(b*c+(-a*b*d^2-b*d^2)^{(1/2)}))+1/d^2*c*\operatorname{dilog}((b*(c+d*x^{(1/2)})-b*c+(-a*b*d^2-b*d^2)^{(1/2)})/(-b*c+(-a*b*d^2-b*d^2)^{(1/2)}))-1/d^2*c*\ln(c+d*x^{(1/2)})*\ln((-b*(c+d*x^{(1/2)})+b*c+(-a*b*d^2+b*d^2)^{(1/2)})/(b*c+(-a*b*d^2+b*d^2)^{(1/2)}))-1/d^2*c*\ln(c+d*x^{(1/2)})*\ln((b*(c+d*x^{(1/2)})-b*c+(-a*b*d^2+b*d^2)^{(1/2)})/(-b*c+(-a*b*d^2+b*d^2)^{(1/2)}))-1/d^2*c*\operatorname{dilog}((-b*(c+d*x^{(1/2)})+b*c+(-a*b*d^2+b*d^2)^{(1/2)})/(b*c+(-a*b*d^2+b*d^2)^{(1/2)}))-1/d^2*c*\operatorname{dilog}((b*(c+d*x^{(1/2)})-b*c+(-a*b*d^2+b*d^2)^{(1/2)})/(-b*c+(-a*b*d^2+b*d^2)^{(1/2)}))+2/(a*b*d^2+b*d^2)^{(1/2)}*\arctan(1/2*(2*b*(c+d*x^{(1/2)})-2*b*c)/(a*b*d^2+b*d^2)^{(1/2)})+2/(a*b*d^2+b*d^2)^{(1/2)}*\arctan(1/2*(2*b*(c+d*x^{(1/2)})-2*b*c)/(a*b*d^2+b*d^2)^{(1/2)})*a+2/(a*b*d^2-b*d^2)^{(1/2)}*\arctan(1/2*(2*b*(c+d*x^{(1/2)})-2*b*c)/(a*b*d^2-b*d^2)^{(1/2)})-2/(a*b*d^2-b*d^2)^{(1/2)}*\arctan(1/2*(2*b*(c+d*x^{(1/2)})-2*b*c)/(a*b*d^2-b*d^2)^{(1/2)})*a$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{artanh}(bx+a)}{d\sqrt{x}+c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctanh(b*x+a)/(c+d*x^(1/2)),x, algorithm="maxima")

[Out] integrate(arctanh(b*x + a)/(d*sqrt(x) + c), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{atanh}(a+bx)}{c+d\sqrt{x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(atanh(a + b*x)/(c + d*x^(1/2)),x)

[Out] int(atanh(a + b*x)/(c + d*x^(1/2)), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(atanh(b*x+a)/(c+d*x**(1/2)),x)

[Out] Timed out

$$3.60 \quad \int \frac{\tanh^{-1}(a+bx)}{c + \frac{d}{\sqrt{x}}} dx$$

Optimal. Leaf size=661

$$\frac{d^2 \operatorname{Li}_2\left(-\frac{\sqrt{b}(\sqrt{xc+d})}{\sqrt{-a-1}c-\sqrt{bd}}\right)}{c^3} + \frac{d^2 \operatorname{Li}_2\left(-\frac{\sqrt{b}(\sqrt{xc+d})}{\sqrt{1-a}c-\sqrt{bd}}\right)}{c^3} - \frac{d^2 \operatorname{Li}_2\left(\frac{\sqrt{b}(\sqrt{xc+d})}{\sqrt{-a-1}c+\sqrt{bd}}\right)}{c^3} + \frac{d^2 \operatorname{Li}_2\left(\frac{\sqrt{b}(\sqrt{xc+d})}{\sqrt{1-a}c+\sqrt{bd}}\right)}{c^3} - \frac{d^2 \log(c\sqrt{x}+d) \log(\dots)}{c^3}$$

[Out] $\frac{1}{2}*(-b*x-a+1)*\ln(-b*x-a+1)/b/c+1/2*(b*x+a+1)*\ln(b*x+a+1)/b/c-d^2*\ln(-b*x-a+1)*\ln(d+c*x^{(1/2)})/c^3+d^2*\ln(b*x+a+1)*\ln(d+c*x^{(1/2)})/c^3-d^2*\ln(d+c*x^{(1/2)})*\ln(c*((-1-a)^{(1/2)}-b^{(1/2)}*x^{(1/2)})/(c*(-1-a)^{(1/2)}+d*b^{(1/2)}))/c^3+d^2*\ln(d+c*x^{(1/2)})*\ln(c*((1-a)^{(1/2)}-b^{(1/2)}*x^{(1/2)})/(c*(1-a)^{(1/2)}+d*b^{(1/2)}))/c^3-d^2*\ln(d+c*x^{(1/2)})*\ln(c*((-1-a)^{(1/2)}+b^{(1/2)}*x^{(1/2)})/(c*(-1-a)^{(1/2)}-d*b^{(1/2)}))/c^3+d^2*\ln(d+c*x^{(1/2)})*\ln(c*((1-a)^{(1/2)}+b^{(1/2)}*x^{(1/2)})/(c*(1-a)^{(1/2)}-d*b^{(1/2)}))/c^3-d^2*\operatorname{polylog}(2,-b^{(1/2)}*(d+c*x^{(1/2)})/(c*(-1-a)^{(1/2)}-d*b^{(1/2)}))/c^3+d^2*\operatorname{polylog}(2,-b^{(1/2)}*(d+c*x^{(1/2)})/(c*(1-a)^{(1/2)}-d*b^{(1/2)}))/c^3-d^2*\operatorname{polylog}(2,b^{(1/2)}*(d+c*x^{(1/2)})/(c*(-1-a)^{(1/2)}+d*b^{(1/2)}))/c^3+d^2*\operatorname{polylog}(2,b^{(1/2)}*(d+c*x^{(1/2)})/(c*(1-a)^{(1/2)}+d*b^{(1/2)}))/c^3+2*d*\operatorname{arctanh}(b^{(1/2)}*x^{(1/2)/(1-a)^{(1/2)}}*(1-a)^{(1/2)}/c^2/b^{(1/2)}-2*d*\operatorname{arctan}(b^{(1/2)}*x^{(1/2)/(1+a)^{(1/2)}}*(1+a)^{(1/2)}/c^2/b^{(1/2)}+d*\ln(-b*x-a+1)*x^{(1/2)}/c^2-d*\ln(b*x+a+1)*x^{(1/2)}/c^2$

Rubi [A] time = 1.07, antiderivative size = 661, normalized size of antiderivative = 1.00, number of steps used = 37, number of rules used = 16, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.889$, Rules used = {6115, 2408, 2476, 2448, 321, 205, 2454, 2389, 2295, 2462, 260, 2416, 2394, 2393, 2391, 208}

$$\frac{d^2 \operatorname{PolyLog}\left(2, -\frac{\sqrt{b}(c\sqrt{x}+d)}{\sqrt{-a-1}c-\sqrt{bd}}\right)}{c^3} + \frac{d^2 \operatorname{PolyLog}\left(2, -\frac{\sqrt{b}(c\sqrt{x}+d)}{\sqrt{1-a}c-\sqrt{bd}}\right)}{c^3} - \frac{d^2 \operatorname{PolyLog}\left(2, \frac{\sqrt{b}(c\sqrt{x}+d)}{\sqrt{-a-1}c+\sqrt{bd}}\right)}{c^3} + \frac{d^2 \operatorname{PolyLog}\left(2, \frac{\sqrt{b}(c\sqrt{x}+d)}{\sqrt{1-a}c+\sqrt{bd}}\right)}{c^3}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\operatorname{ArcTanh}[a + b*x]/(c + d/\operatorname{Sqrt}[x]), x]$

[Out] $(-2*\operatorname{Sqrt}[1 + a]*d*\operatorname{ArcTan}[(\operatorname{Sqrt}[b]*\operatorname{Sqrt}[x])/ \operatorname{Sqrt}[1 + a]])/(\operatorname{Sqrt}[b]*c^2) + (2*\operatorname{Sqrt}[1 - a]*d*\operatorname{ArcTanh}[(\operatorname{Sqrt}[b]*\operatorname{Sqrt}[x])/ \operatorname{Sqrt}[1 - a]])/(\operatorname{Sqrt}[b]*c^2) - (d^2*\operatorname{Log}[(c*(\operatorname{Sqrt}[-1 - a] - \operatorname{Sqrt}[b]*\operatorname{Sqrt}[x]))/(\operatorname{Sqrt}[-1 - a]*c + \operatorname{Sqrt}[b]*d)]*\operatorname{Log}[d + c*\operatorname{Sqrt}[x]])/c^3 + (d^2*\operatorname{Log}[(c*(\operatorname{Sqrt}[1 - a] - \operatorname{Sqrt}[b]*\operatorname{Sqrt}[x]))/(\operatorname{Sqrt}[1 - a]*c + \operatorname{Sqrt}[b]*d)]*\operatorname{Log}[d + c*\operatorname{Sqrt}[x]])/c^3 - (d^2*\operatorname{Log}[(c*(\operatorname{Sqrt}[-1 - a] + \operatorname{Sqrt}[b]*\operatorname{Sqrt}[x]))/(\operatorname{Sqrt}[-1 - a]*c - \operatorname{Sqrt}[b]*d)]*\operatorname{Log}[d + c*\operatorname{Sqrt}[x]])/c^3 + (d^2*\operatorname{Log}[(c*(\operatorname{Sqrt}[1 - a] + \operatorname{Sqrt}[b]*\operatorname{Sqrt}[x]))/(\operatorname{Sqrt}[1 - a]*c - \operatorname{Sqrt}[b]*d)]*\operatorname{Log}[d + c*\operatorname{Sqrt}[x]])/c^3$

$$\begin{aligned} & \log[d + c\sqrt{x}]/c^3 + (d\sqrt{x}\log[1 - a - b*x])/c^2 + ((1 - a - b*x)* \\ & \log[1 - a - b*x])/(2*b*c) - (d^2*\log[d + c\sqrt{x}]*\log[1 - a - b*x])/c^3 - \\ & (d*\sqrt{x}*\log[1 + a + b*x])/c^2 + ((1 + a + b*x)*\log[1 + a + b*x])/(2*b*c) \\ & + (d^2*\log[d + c\sqrt{x}]*\log[1 + a + b*x])/c^3 - (d^2*\text{PolyLog}[2, -((\sqrt{b}*(d + c\sqrt{x}))/(\sqrt{-1 - a}*c - \sqrt{b}*d))])/c^3 + (d^2*\text{PolyLog}[2, -((\sqrt{b}*(d + c\sqrt{x}))/(\sqrt{1 - a}*c - \sqrt{b}*d))])/c^3 - (d^2*\text{PolyLog}[2, (\sqrt{b}*(d + c\sqrt{x}))/(\sqrt{-1 - a}*c + \sqrt{b}*d))])/c^3 + (d^2*\text{PolyLog}[2, (\sqrt{b}*(d + c\sqrt{x}))/(\sqrt{1 - a}*c + \sqrt{b}*d))])/c^3 \end{aligned}$$
Rule 205

$$\text{Int}[(a + (b*x)^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(\text{Rt}[a/b, 2]*\text{ArcTan}[x/\text{Rt}[a/b, 2]])/a, x] \text{ ; FreeQ}\{a, b, x\} \ \&\& \ \text{PosQ}[a/b]$$
Rule 208

$$\text{Int}[(a + (b*x)^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(\text{Rt}[-(a/b), 2]*\text{ArcTanh}[x/\text{Rt}[-(a/b), 2]])/a, x] \text{ ; FreeQ}\{a, b, x\} \ \&\& \ \text{NegQ}[a/b]$$
Rule 260

$$\text{Int}[(x^m)/(a + (b*x)^n), x_Symbol] \rightarrow \text{Simp}[\text{Log}[\text{RemoveContent}[a + b*x^n, x]]/(b*n), x] \text{ ; FreeQ}\{a, b, m, n, x\} \ \&\& \ \text{EqQ}[m, n - 1]$$
Rule 321

$$\text{Int}[(c*x)^m*(a + (b*x)^n)^p, x_Symbol] \rightarrow \text{Simp}[(c^{n-1}*(c*x)^{m-n+1}*(a + b*x^n)^{p+1})/(b*(m+n*p+1)), x] - \text{Dist}[(a*c^{n-1})/(b*(m+n*p+1)), \text{Int}[(c*x)^{m-n}*(a + b*x^n)^p, x], x] \text{ ; FreeQ}\{a, b, c, p, x\} \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ \text{GtQ}[m, n - 1] \ \&\& \ \text{NeQ}[m + n*p + 1, 0] \ \&\& \ \text{IntBinomialQ}[a, b, c, n, m, p, x]$$
Rule 2295

$$\text{Int}[\text{Log}[c*x^n], x_Symbol] \rightarrow \text{Simp}[x*\text{Log}[c*x^n], x] - \text{Simp}[n*x, x] \text{ ; FreeQ}\{c, n, x\}$$
Rule 2389

$$\text{Int}[(a + \text{Log}[c*x^n])*(d + (e*x)^n)^p, x_Symbol] \rightarrow \text{Dist}[1/e, \text{Subst}[\text{Int}[(a + b*\text{Log}[c*x^n])^p, x], x, d + e*x], x] \text{ ; FreeQ}\{a, b, c, d, e, n, p, x\}$$
Rule 2391

Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] := -Simp[PolyLog[2, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]

Rule 2393

Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_Symbol] := Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*(e*f - d*g), 0]

Rule 2394

Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]*(b_.))/((f_.) + (g_.)*(x_)), x_Symbol] := Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x)^n])/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]

Rule 2408

Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]*(b_.))^(p_.)*((f_.) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] := With[{k = Denominator[r]}, Dist[k, Subst[Int[x^(k - 1)*(f + g*x^(k*r))^q*(a + b*Log[c*(d + e*x^k)^n]^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && FractionQ[r] && IGtQ[p, 0]

Rule 2416

Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]*(b_.))^(p_.)*((h_.)*(x_)^(m_.))*((f_.) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] := Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)^n]^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]

Rule 2448

Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))^(p_.)], x_Symbol] := Simp[x*Log[c*(d + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d, e, n, p}, x]

Rule 2454

Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))^(p_.))^(q_.)*(x_)^(m_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Log[c*(d + e*x)^p]^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&

!(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])

Rule 2462

Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.)*(x_)), x_Symbol] := Simp[(Log[f + g*x]*(a + b*Log[c*(d + e*x^n)^p]))/g, x] - Dist[(b*e*n*p)/g, Int[(x^(n - 1)*Log[f + g*x])/(d + e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]

Rule 2476

Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m_.)*((f_.) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] := Int[ExpandIntegrand[(a + b*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] && IntegerQ[s]

Rule 6115

Int[ArcTanh[(c_) + (d_.)*(x_)]/((e_) + (f_.)*(x_)^(n_.)), x_Symbol] := Dist[1/2, Int[Log[1 + c + d*x]/(e + f*x^n), x], x] - Dist[1/2, Int[Log[1 - c - d*x]/(e + f*x^n), x], x] /; FreeQ[{c, d, e, f}, x] && RationalQ[n]

Rubi steps

$$\begin{aligned}
\int \frac{\tanh^{-1}(a+bx)}{c+\frac{d}{\sqrt{x}}} dx &= -\left(\frac{1}{2} \int \frac{\log(1-a-bx)}{c+\frac{d}{\sqrt{x}}} dx\right) + \frac{1}{2} \int \frac{\log(1+a+bx)}{c+\frac{d}{\sqrt{x}}} dx \\
&= -\text{Subst}\left(\int \frac{x \log(1-a-bx^2)}{c+\frac{d}{x}} dx, x, \sqrt{x}\right) + \text{Subst}\left(\int \frac{x \log(1+a+bx^2)}{c+\frac{d}{x}} dx, x, \sqrt{x}\right) \\
&= -\text{Subst}\left(\int \left(-\frac{d \log(1-a-bx^2)}{c^2} + \frac{x \log(1-a-bx^2)}{c} + \frac{d^2 \log(1-a-bx^2)}{c^2(d+cx)}\right) dx, x, \sqrt{x}\right) \\
&= -\frac{\text{Subst}\left(\int x \log(1-a-bx^2) dx, x, \sqrt{x}\right)}{c} + \frac{\text{Subst}\left(\int x \log(1+a+bx^2) dx, x, \sqrt{x}\right)}{c} + \\
&= \frac{d\sqrt{x} \log(1-a-bx)}{c^2} - \frac{d^2 \log(d+c\sqrt{x}) \log(1-a-bx)}{c^3} - \frac{d\sqrt{x} \log(1+a+bx)}{c^2} + \frac{d^2 \log(d+c\sqrt{x}) \log(1+a+bx)}{c^3} \\
&= \frac{d\sqrt{x} \log(1-a-bx)}{c^2} - \frac{d^2 \log(d+c\sqrt{x}) \log(1-a-bx)}{c^3} - \frac{d\sqrt{x} \log(1+a+bx)}{c^2} + \frac{d^2 \log(d+c\sqrt{x}) \log(1+a+bx)}{c^3} \\
&= -\frac{2\sqrt{1+a} d \tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{1+a}}\right)}{\sqrt{b}c^2} + \frac{2\sqrt{1-a} d \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{1-a}}\right)}{\sqrt{b}c^2} + \frac{d\sqrt{x} \log(1-a-bx)}{c^2} + \frac{(1-a)d\sqrt{x} \log(1+a+bx)}{c^2} \\
&= -\frac{2\sqrt{1+a} d \tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{1+a}}\right)}{\sqrt{b}c^2} + \frac{2\sqrt{1-a} d \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{1-a}}\right)}{\sqrt{b}c^2} - \frac{d^2 \log\left(\frac{c(\sqrt{-1-a}-\sqrt{b}\sqrt{x})}{\sqrt{-1-a}c+\sqrt{b}d}\right) \log\left(\frac{c(\sqrt{-1-a}+\sqrt{b}\sqrt{x})}{\sqrt{-1-a}c+\sqrt{b}d}\right)}{c^3} \\
&= -\frac{2\sqrt{1+a} d \tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{1+a}}\right)}{\sqrt{b}c^2} + \frac{2\sqrt{1-a} d \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{1-a}}\right)}{\sqrt{b}c^2} - \frac{d^2 \log\left(\frac{c(\sqrt{-1-a}-\sqrt{b}\sqrt{x})}{\sqrt{-1-a}c+\sqrt{b}d}\right) \log\left(\frac{c(\sqrt{-1-a}+\sqrt{b}\sqrt{x})}{\sqrt{-1-a}c+\sqrt{b}d}\right)}{c^3} \\
&= -\frac{2\sqrt{1+a} d \tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{1+a}}\right)}{\sqrt{b}c^2} + \frac{2\sqrt{1-a} d \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{1-a}}\right)}{\sqrt{b}c^2} - \frac{d^2 \log\left(\frac{c(\sqrt{-1-a}-\sqrt{b}\sqrt{x})}{\sqrt{-1-a}c+\sqrt{b}d}\right) \log\left(\frac{c(\sqrt{-1-a}+\sqrt{b}\sqrt{x})}{\sqrt{-1-a}c+\sqrt{b}d}\right)}{c^3}
\end{aligned}$$

Mathematica [A] time = 0.64, size = 598, normalized size = 0.90

$$-\frac{c^2(a+bx-1)\log(-a-bx+1)}{b} + \frac{c^2(a+bx+1)\log(a+bx+1)}{b} - 2d^2 \left(\text{Li}_2\left(\frac{\sqrt{b}(\sqrt{x}c+d)}{\sqrt{b}d-\sqrt{-a-1}c}\right) + \text{Li}_2\left(\frac{\sqrt{b}(\sqrt{x}c+d)}{\sqrt{-a-1}c+\sqrt{b}d}\right) + \log(c\sqrt{x}+d) \left(\log\left(\frac{c(\sqrt{-1-a}-\sqrt{b}\sqrt{x})}{\sqrt{-1-a}c+\sqrt{b}d}\right) + \log\left(\frac{c(\sqrt{-1-a}+\sqrt{b}\sqrt{x})}{\sqrt{-1-a}c+\sqrt{b}d}\right) \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[ArcTanh[a + b*x]/(c + d/Sqrt[x]),x]

[Out] (4*c*d*(Sqrt[x] - (Sqrt[1 + a]*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[1 + a]])/Sqrt[b]) - 4*c*d*(Sqrt[x] - (Sqrt[1 - a]*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[1 - a]])/Sqrt[b]) + 2*c*d*Sqrt[x]*Log[1 - a - b*x] - (c^2*(-1 + a + b*x)*Log[1 - a - b*x])/b - 2*d^2*Log[d + c*Sqrt[x]]*Log[1 - a - b*x] - 2*c*d*Sqrt[x]*Log[1 + a + b*x] + (c^2*(1 + a + b*x)*Log[1 + a + b*x])/b + 2*d^2*Log[d + c*Sqrt[x]]*Log[1 + a + b*x] - 2*d^2*((Log[(c*(Sqrt[-1 - a] - Sqrt[b]*Sqrt[x]))/(Sqrt[-1 - a]*c + Sqrt[b]*d)] + Log[(c*(Sqrt[-1 - a] + Sqrt[b]*Sqrt[x]))/(Sqrt[-1 - a]*c - Sqrt[b]*d)])*Log[d + c*Sqrt[x]] + PolyLog[2, (Sqrt[b]*(d + c*Sqrt[x]))/(-(Sqrt[-1 - a]*c) + Sqrt[b]*d)] + PolyLog[2, (Sqrt[b]*(d + c*Sqrt[x]))/(Sqrt[-1 - a]*c + Sqrt[b]*d)]) + 2*d^2*((Log[(c*(Sqrt[1 - a] - Sqrt[b]*Sqrt[x]))/(Sqrt[1 - a]*c + Sqrt[b]*d)] + Log[(c*(Sqrt[1 - a] + Sqrt[b]*Sqrt[x]))/(Sqrt[1 - a]*c - Sqrt[b]*d)])*Log[d + c*Sqrt[x]] + PolyLog[2, (Sqrt[b]*(d + c*Sqrt[x]))/(-(Sqrt[1 - a]*c) + Sqrt[b]*d)] + PolyLog[2, (Sqrt[b]*(d + c*Sqrt[x]))/(Sqrt[1 - a]*c + Sqrt[b]*d)))]/(2*c^3)

fricas [F] time = 0.73, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{cx \operatorname{artanh}(bx + a) - d\sqrt{x} \operatorname{artanh}(bx + a)}{c^2x - d^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctanh(b*x+a)/(c+d/x^(1/2)),x, algorithm="fricas")

[Out] integral((c*x*arctanh(b*x + a) - d*sqrt(x)*arctanh(b*x + a))/(c^2*x - d^2), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{artanh}(bx + a)}{c + \frac{d}{\sqrt{x}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctanh(b*x+a)/(c+d/x^(1/2)),x, algorithm="giac")

[Out] integrate(arctanh(b*x + a)/(c + d/sqrt(x)), x)

maple [A] time = 0.10, size = 970, normalized size = 1.47

$$\frac{\operatorname{arctanh}(bx + a)x}{c} - \frac{2 \operatorname{arctanh}(bx + a)d\sqrt{x}}{c^2} + \frac{2 \operatorname{arctanh}(bx + a)d^2 \ln(d + c\sqrt{x})}{c^3} + \frac{\ln\left(b(d + c\sqrt{x})^2 - 2(d + c\sqrt{x})\right)}{2bc}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(arctanh(b*x+a)/(c+d/x^(1/2)),x)`

[Out]
$$\begin{aligned} & \arctanh(b*x+a)/c*x-2*\arctanh(b*x+a)/c^2*d*x^(1/2)+2*\arctanh(b*x+a)*d^2/c^3* \\ & \ln(d+c*x^(1/2))+1/2/b/c*\ln(b*(d+c*x^(1/2))^2-2*(d+c*x^(1/2))*b*d+a*c^2+b*d^2+c^2)-2/c*d/(a*b*c^2+b*c^2)^(1/2)*\arctan(1/2*(2*b*(d+c*x^(1/2))-2*b*d)/(a* \\ & b*c^2+b*c^2)^(1/2))+1/2/b/c*a*\ln(b*(d+c*x^(1/2))^2-2*(d+c*x^(1/2))*b*d+a*c^2+b*d^2+c^2)-2/c*a*d/(a*b*c^2+b*c^2)^(1/2)*\arctan(1/2*(2*b*(d+c*x^(1/2))-2* \\ & b*d)/(a*b*c^2+b*c^2)^(1/2))-1/2/b/c*a*\ln(b*(d+c*x^(1/2))^2-2*(d+c*x^(1/2))* \\ & b*d+a*c^2+b*d^2-c^2)+2/c*a*d/(a*b*c^2-b*c^2)^(1/2)*\arctan(1/2*(2*b*(d+c*x^(1/2))-2*b*d)/(a*b*c^2-b*c^2)^(1/2))+1/2/b/c*\ln(b*(d+c*x^(1/2))^2-2*(d+c*x^(1/2))* \\ & b*d+a*c^2+b*d^2-c^2)-2/c*d/(a*b*c^2-b*c^2)^(1/2)*\arctan(1/2*(2*b*(d+c*x^(1/2))-2*b*d)/(a*b*c^2-b*c^2)^(1/2))-1/c^3*d^2*\ln(d+c*x^(1/2))*\ln((-b*(d+c*x^(1/2))+b*d+(-a*b*c^2-b*c^2)^(1/2))/(b*d+(-a*b*c^2-b*c^2)^(1/2)))-1/c^3*d^2*\ln(d+c*x^(1/2))*\ln((b*(d+c*x^(1/2))-b*d+(-a*b*c^2-b*c^2)^(1/2))/(-b*d+(-a*b*c^2-b*c^2)^(1/2)))-1/c^3*d^2*dilog((-b*(d+c*x^(1/2))+b*d+(-a*b*c^2-b*c^2)^(1/2))/(b*d+(-a*b*c^2-b*c^2)^(1/2)))-1/c^3*d^2*dilog((b*(d+c*x^(1/2))-b*d+(-a*b*c^2-b*c^2)^(1/2))/(-b*d+(-a*b*c^2-b*c^2)^(1/2)))+1/c^3*d^2*\ln(d+c*x^(1/2))*\ln((-b*(d+c*x^(1/2))+b*d+(-a*b*c^2+b*c^2)^(1/2))/(b*d+(-a*b*c^2+b*c^2)^(1/2)))+1/c^3*d^2*\ln(d+c*x^(1/2))*\ln((b*(d+c*x^(1/2))-b*d+(-a*b*c^2+b*c^2)^(1/2))/(-b*d+(-a*b*c^2+b*c^2)^(1/2)))+1/c^3*d^2*dilog((-b*(d+c*x^(1/2))+b*d+(-a*b*c^2+b*c^2)^(1/2))/(b*d+(-a*b*c^2+b*c^2)^(1/2)))+1/c^3*d^2*dilog((b*(d+c*x^(1/2))-b*d+(-a*b*c^2+b*c^2)^(1/2))/(-b*d+(-a*b*c^2+b*c^2)^(1/2))) \end{aligned}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{artanh}(bx+a)}{c + \frac{d}{\sqrt{x}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arctanh(b*x+a)/(c+d/x^(1/2)),x, algorithm="maxima")`

[Out] `integrate(arctanh(b*x + a)/(c + d/sqrt(x)), x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{atanh}(a + bx)}{c + \frac{d}{\sqrt{x}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(atanh(a + b*x)/(c + d/x^(1/2)),x)`

```
[Out] int(atanh(a + b*x)/(c + d/x^(1/2)), x)
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(b*x+a)/(c+d/x**(1/2)),x)
```

```
[Out] Timed out
```


$$3.61 \quad \int \frac{\tanh^{-1}(d+ex)}{a+bx+cx^2} dx$$

Optimal. Leaf size=335

$$\frac{\operatorname{Li}_2\left(\frac{2(2cd-(b-\sqrt{b^2-4ac})e^{-2c(d+ex)})}{(-2dc+2c+be-\sqrt{b^2-4ac}e)(d+ex+1)}+1\right)}{2\sqrt{b^2-4ac}} + \frac{\operatorname{Li}_2\left(\frac{2(2cd-(b+\sqrt{b^2-4ac})e^{-2c(d+ex)})}{(2c(1-d)+(b+\sqrt{b^2-4ac})e)(d+ex+1)}+1\right)}{2\sqrt{b^2-4ac}} + \frac{\tanh^{-1}(d+ex)\log\left(\frac{2e(-\sqrt{b^2-4ac})}{(d+ex+1)(e(b-\sqrt{b^2-4ac}))}\right)}{\sqrt{b^2-4ac}}$$

[Out] $\operatorname{arctanh}(e*x+d)*\ln(2*e*(b+2*c*x-(-4*a*c+b^2)^{(1/2)})/(e*x+d+1)/(2*c*(1-d)+e*(b-(-4*a*c+b^2)^{(1/2)})))/(-4*a*c+b^2)^{(1/2)}-\operatorname{arctanh}(e*x+d)*\ln(2*e*(b+2*c*x+(-4*a*c+b^2)^{(1/2)})/(e*x+d+1)/(2*c*(1-d)+e*(b+(-4*a*c+b^2)^{(1/2)})))/(-4*a*c+b^2)^{(1/2)}-1/2*\operatorname{polylog}(2,1+2*(2*c*d-2*c*(e*x+d)-e*(b-(-4*a*c+b^2)^{(1/2)})))/(e*x+d+1)/(2*c-2*c*d+b*e-e*(-4*a*c+b^2)^{(1/2)})/(-4*a*c+b^2)^{(1/2)}+1/2*\operatorname{polylog}(2,1+2*(2*c*d-2*c*(e*x+d)-e*(b+(-4*a*c+b^2)^{(1/2)})))/(e*x+d+1)/(2*c*(1-d)+e*(b+(-4*a*c+b^2)^{(1/2)})))/(-4*a*c+b^2)^{(1/2)}$

Rubi [A] time = 0.72, antiderivative size = 335, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {618, 206, 6728, 6111, 5920, 2402, 2315, 2447}

$$\frac{\operatorname{PolyLog}\left(2, \frac{2(-e(b-\sqrt{b^2-4ac})-2c(d+ex)+2cd)}{(d+ex+1)(-e\sqrt{b^2-4ac}+be-2cd+2c)}+1\right)}{2\sqrt{b^2-4ac}} + \frac{\operatorname{PolyLog}\left(2, \frac{2(-e(\sqrt{b^2-4ac}+b)-2c(d+ex)+2cd)}{(d+ex+1)(e(\sqrt{b^2-4ac}+b)+2c(1-d))}+1\right)}{2\sqrt{b^2-4ac}} + \frac{\tanh^{-1}(d+ex)}{\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\operatorname{ArcTanh}[d + e*x]/(a + b*x + c*x^2), x]$

[Out] $(\operatorname{ArcTanh}[d + e*x]*\operatorname{Log}[(2*e*(b - \operatorname{Sqrt}[b^2 - 4*a*c] + 2*c*x))/((2*c*(1 - d) + (b - \operatorname{Sqrt}[b^2 - 4*a*c])*e)*(1 + d + e*x))])/ \operatorname{Sqrt}[b^2 - 4*a*c] - (\operatorname{ArcTanh}[d + e*x]*\operatorname{Log}[(2*e*(b + \operatorname{Sqrt}[b^2 - 4*a*c] + 2*c*x))/((2*c*(1 - d) + (b + \operatorname{Sqrt}[b^2 - 4*a*c])*e)*(1 + d + e*x))])/ \operatorname{Sqrt}[b^2 - 4*a*c] - \operatorname{PolyLog}[2, 1 + (2*(2*c*d - (b - \operatorname{Sqrt}[b^2 - 4*a*c])*e - 2*c*(d + e*x)))/((2*c - 2*c*d + b*e - \operatorname{Sqrt}[b^2 - 4*a*c])*e)*(1 + d + e*x))]/(2*\operatorname{Sqrt}[b^2 - 4*a*c]) + \operatorname{PolyLog}[2, 1 + (2*(2*c*d - (b + \operatorname{Sqrt}[b^2 - 4*a*c])*e - 2*c*(d + e*x)))/((2*c*(1 - d) + (b + \operatorname{Sqrt}[b^2 - 4*a*c])*e)*(1 + d + e*x))]/(2*\operatorname{Sqrt}[b^2 - 4*a*c])$

Rule 206

$\operatorname{Int}[(a_+ + (b_+)*(x_+)^2)^{-1}, x_Symbol] \rightarrow \operatorname{Simp}[(1*\operatorname{ArcTanh}[(\operatorname{Rt}[-b, 2]*x)/\operatorname{Rt}[a, 2]])/(\operatorname{Rt}[a, 2]*\operatorname{Rt}[-b, 2]), x] /; \operatorname{FreeQ}\{a, b\}, x] \ \&\& \operatorname{NegQ}[a/b] \ \&\& (\operatorname{GtQ}[a, 0] \ || \operatorname{LtQ}[b, 0])$

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] := -Simp[PolyLog[2, 1 - c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] := -Dist[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] := With[{C = FullSimplify[(Pq^m*(1 - u))/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x] /; IntegerQ[m] && PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u, x][[2]], Expon[Pq, x]]
```

Rule 5920

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] := -Simp[((a + b*ArcTanh[c*x])*Log[2/(1 + c*x)])/e, x] + (Dist[(b*c)/e, Int[Log[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x))/(c*d + e*(1 + c*x))]/(1 - c^2*x^2), x], x] + Simp[((a + b*ArcTanh[c*x])*Log[(2*c*(d + e*x))/(c*d + e*(1 + c*x))])/e, x]) /; FreeQ[{a, b, c, d, e}, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rule 6111

```
Int[((a_.) + ArcTanh[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m_.), x_Symbol] := Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*ArcTanh[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] := With[{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
```

mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]

Rubi steps

$$\begin{aligned}
 \int \frac{\tanh^{-1}(d+ex)}{a+bx+cx^2} dx &= \int \left(\frac{2c \tanh^{-1}(d+ex)}{\sqrt{b^2-4ac} (b-\sqrt{b^2-4ac}+2cx)} - \frac{2c \tanh^{-1}(d+ex)}{\sqrt{b^2-4ac} (b+\sqrt{b^2-4ac}+2cx)} \right) dx \\
 &= \frac{(2c) \int \frac{\tanh^{-1}(d+ex)}{b-\sqrt{b^2-4ac}+2cx} dx}{\sqrt{b^2-4ac}} - \frac{(2c) \int \frac{\tanh^{-1}(d+ex)}{b+\sqrt{b^2-4ac}+2cx} dx}{\sqrt{b^2-4ac}} \\
 &= \frac{(2c) \text{Subst} \left(\int \frac{\tanh^{-1}(x)}{\frac{-2cd+(b-\sqrt{b^2-4ac})e}{e} + \frac{2cx}{e}} dx, x, d+ex \right)}{\sqrt{b^2-4ac} e} - \frac{(2c) \text{Subst} \left(\int \frac{\tanh^{-1}(x)}{\frac{-2cd+(b+\sqrt{b^2-4ac})e}{e} + \frac{2cx}{e}} dx, x, d+ex \right)}{\sqrt{b^2-4ac} e} \\
 &= \frac{\tanh^{-1}(d+ex) \log \left(\frac{2e(b-\sqrt{b^2-4ac}+2cx)}{(2c-2cd+be-\sqrt{b^2-4ac}e)(1+d+ex)} \right)}{\sqrt{b^2-4ac}} - \frac{\tanh^{-1}(d+ex) \log \left(\frac{2e(b+\sqrt{b^2-4ac}+2cx)}{(2c(1-d)+(b+\sqrt{b^2-4ac})e)} \right)}{\sqrt{b^2-4ac}} \\
 &= \frac{\tanh^{-1}(d+ex) \log \left(\frac{2e(b-\sqrt{b^2-4ac}+2cx)}{(2c-2cd+be-\sqrt{b^2-4ac}e)(1+d+ex)} \right)}{\sqrt{b^2-4ac}} - \frac{\tanh^{-1}(d+ex) \log \left(\frac{2e(b+\sqrt{b^2-4ac}+2cx)}{(2c(1-d)+(b+\sqrt{b^2-4ac})e)} \right)}{\sqrt{b^2-4ac}}
 \end{aligned}$$

Mathematica [A] time = 0.57, size = 403, normalized size = 1.20

$$-\text{Li}_2 \left(\frac{2c(d+ex-1)}{2c(d-1)+(\sqrt{b^2-4ac}-b)e} \right) + \text{Li}_2 \left(\frac{2c(d+ex-1)}{2c(d-1)-(b+\sqrt{b^2-4ac})e} \right) + \text{Li}_2 \left(\frac{2c(d+ex+1)}{2c(d+1)+(\sqrt{b^2-4ac}-b)e} \right) - \text{Li}_2 \left(\frac{2c(d+ex+1)}{2c(d+1)-(b+\sqrt{b^2-4ac})e} \right) +$$

Warning: Unable to verify antiderivative.

[In] Integrate[ArcTanh[d + e*x]/(a + b*x + c*x^2), x]

```
[Out] (-Log[(e*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x))/(2*c*(-1 + d) + (-b + Sqrt[b^2 - 4*a*c])*e)]*Log[1 - d - e*x] + Log[(e*(b + Sqrt[b^2 - 4*a*c] + 2*c*x))/(-2*c*(-1 + d) + (b + Sqrt[b^2 - 4*a*c])*e)]*Log[1 - d - e*x] + Log[(e*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x))/(2*c*(1 + d) + (-b + Sqrt[b^2 - 4*a*c])*e)]*Log[1 + d + e*x] - Log[(e*(b + Sqrt[b^2 - 4*a*c] + 2*c*x))/(-2*c*(1 + d) + (b + Sqrt[b^2 - 4*a*c])*e)]*Log[1 + d + e*x] - PolyLog[2, (2*c*(-1 + d + e*x))/(2*c*(-1 + d) + (-b + Sqrt[b^2 - 4*a*c])*e)] + PolyLog[2, (2*c*(-1 + d + e*x))/(2*c*(-1 + d) - (b + Sqrt[b^2 - 4*a*c])*e)] + PolyLog[2, (2*c*(1 + d + e*x))/(2*c*(1 + d) + (-b + Sqrt[b^2 - 4*a*c])*e)] - PolyLog[2, (2*c*(1 + d + e*x))/(2*c*(1 + d) - (b + Sqrt[b^2 - 4*a*c])*e)])/(2*Sqrt[b^2 - 4*a*c])
```

fricas [F] time = 0.61, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\text{artanh}(ex + d)}{cx^2 + bx + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(e*x+d)/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] integral(arctanh(e*x + d)/(c*x^2 + b*x + a), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(e*x+d)/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.82, size = 2140, normalized size = 6.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(e*x+d)/(c*x^2+b*x+a),x)
```

```
[Out] -1/e*(e^2*(-4*a*c+b^2))^(1/2)/(4*a*c-b^2)*arctanh(e*x+d)*ln(1-(a*e^2-b*d*e+c*d^2+b*e-2*c*d+c)*(e*x+d+1)^2/(-(e*x+d)^2+1)/(-a*e^2+b*e*d-c*d^2+(-e^2*(4*a*c-b^2))^(1/2)+c))+1/e*(e^2*(-4*a*c+b^2))^(1/2)/(4*a*c-b^2)*arctanh(e*x+d)^2-1/2/e*(e^2*(-4*a*c+b^2))^(1/2)/(4*a*c-b^2)*polylog(2,(a*e^2-b*d*e+c*d^2+b*e-2*c*d+c)*(e*x+d+1)^2/(-(e*x+d)^2+1)/(-a*e^2+b*e*d-c*d^2+(-e^2*(4*a*c-b^2))^(1/2)+c))+e*(e^2*(-4*a*c+b^2))^(1/2)/(4*a*c-b^2)*ln(1-(a*e^2-b*d*e+c*d^2+b*e-2*c*d+c)*(e*x+d+1)^2/(-(e*x+d)^2+1)/(-a*e^2+b*e*d-c*d^2-(-e^2*(4*a*c-
```

$$\begin{aligned}
& b^2)^{(1/2)+c)} * \operatorname{arctanh}(e*x+d) * a / (a*e^2 - b*e*d + c*d^2 + (e^2*(-4*a*c + b^2))^{(1/2)} \\
&) - c) - (e^2*(-4*a*c + b^2))^{(1/2)} / (4*a*c - b^2) * \ln(1 - (a*e^2 - b*d*e + c*d^2 + b*e - 2*c*d \\
& + c) * (e*x+d+1)^2 / (- (e*x+d)^2 + 1) / (-a*e^2 + b*e*d - c*d^2 - (-e^2*(4*a*c - b^2))^{(1/2)} \\
& + c)) * \operatorname{arctanh}(e*x+d) * b*d / (a*e^2 - b*e*d + c*d^2 + (e^2*(-4*a*c + b^2))^{(1/2)} - c) + 1/e * \\
& (e^2*(-4*a*c + b^2))^{(1/2)} / (4*a*c - b^2) * \ln(1 - (a*e^2 - b*d*e + c*d^2 + b*e - 2*c*d + c) * (\\
& e*x+d+1)^2 / (- (e*x+d)^2 + 1) / (-a*e^2 + b*e*d - c*d^2 - (-e^2*(4*a*c - b^2))^{(1/2)} + c)) * \\
& \operatorname{arctanh}(e*x+d) * c*d^2 / (a*e^2 - b*e*d + c*d^2 + (e^2*(-4*a*c + b^2))^{(1/2)} - c) - e * (e^2 * \\
& (-4*a*c + b^2))^{(1/2)} / (4*a*c - b^2) * \operatorname{arctanh}(e*x+d)^2 * a / (a*e^2 - b*e*d + c*d^2 + (e^2 * \\
& (-4*a*c + b^2))^{(1/2)} - c) + (e^2 * (-4*a*c + b^2))^{(1/2)} / (4*a*c - b^2) * \operatorname{arctanh}(e*x+d)^2 * b*d / \\
& (a*e^2 - b*e*d + c*d^2 + (e^2 * (-4*a*c + b^2))^{(1/2)} - c) - 1/e * (e^2 * (-4*a*c + b^2)) \\
& ^{(1/2)} / (4*a*c - b^2) * \operatorname{arctanh}(e*x+d)^2 * c*d^2 / (a*e^2 - b*e*d + c*d^2 + (e^2 * (-4*a*c + b \\
& ^2))^{(1/2)} - c) + 1/2 * e * (e^2 * (-4*a*c + b^2))^{(1/2)} / (4*a*c - b^2) * \operatorname{polylog}(2, (a*e^2 - b \\
& *d*e + c*d^2 + b*e - 2*c*d + c) * (e*x+d+1)^2 / (- (e*x+d)^2 + 1) / (-a*e^2 + b*e*d - c*d^2 - (-e^2 * \\
& (4*a*c - b^2))^{(1/2)} + c)) * a / (a*e^2 - b*e*d + c*d^2 + (e^2 * (-4*a*c + b^2))^{(1/2)} - c) - 1 \\
& / 2 * (e^2 * (-4*a*c + b^2))^{(1/2)} / (4*a*c - b^2) * \operatorname{polylog}(2, (a*e^2 - b*d*e + c*d^2 + b*e - 2 * \\
& c*d + c) * (e*x+d+1)^2 / (- (e*x+d)^2 + 1) / (-a*e^2 + b*e*d - c*d^2 - (-e^2 * (4*a*c - b^2))^{(1 \\
& / 2) + c)) * b*d / (a*e^2 - b*e*d + c*d^2 + (e^2 * (-4*a*c + b^2))^{(1/2)} - c) + 1/2 * e * (e^2 * (-4*a \\
& *c + b^2))^{(1/2)} / (4*a*c - b^2) * \operatorname{polylog}(2, (a*e^2 - b*d*e + c*d^2 + b*e - 2*c*d + c) * (e*x+d \\
& + 1)^2 / (- (e*x+d)^2 + 1) / (-a*e^2 + b*e*d - c*d^2 - (-e^2 * (4*a*c - b^2))^{(1/2)} + c)) * c*d^2 \\
& / (a*e^2 - b*e*d + c*d^2 + (e^2 * (-4*a*c + b^2))^{(1/2)} - c) - e / (a*e^2 - b*e*d + c*d^2 + (e^2 * (\\
& -4*a*c + b^2))^{(1/2)} - c) * \ln(1 - (a*e^2 - b*d*e + c*d^2 + b*e - 2*c*d + c) * (e*x+d+1)^2 / (- (e \\
& *x+d)^2 + 1) / (-a*e^2 + b*e*d - c*d^2 - (-e^2 * (4*a*c - b^2))^{(1/2)} + c)) * \operatorname{arctanh}(e*x+d) - \\
& 1/e * (e^2 * (-4*a*c + b^2))^{(1/2)} / (4*a*c - b^2) * \ln(1 - (a*e^2 - b*d*e + c*d^2 + b*e - 2*c*d + \\
& c) * (e*x+d+1)^2 / (- (e*x+d)^2 + 1) / (-a*e^2 + b*e*d - c*d^2 - (-e^2 * (4*a*c - b^2))^{(1/2)} + \\
& c)) * \operatorname{arctanh}(e*x+d) * c / (a*e^2 - b*e*d + c*d^2 + (e^2 * (-4*a*c + b^2))^{(1/2)} - c) + e / (a*e^2 \\
& - b*e*d + c*d^2 + (e^2 * (-4*a*c + b^2))^{(1/2)} - c) * \operatorname{arctanh}(e*x+d)^2 + 1/e * (e^2 * (-4*a*c \\
& + b^2))^{(1/2)} / (4*a*c - b^2) * \operatorname{arctanh}(e*x+d)^2 * c / (a*e^2 - b*e*d + c*d^2 + (e^2 * (-4*a*c \\
& + b^2))^{(1/2)} - c) - 1/2 * e / (a*e^2 - b*e*d + c*d^2 + (e^2 * (-4*a*c + b^2))^{(1/2)} - c) * \operatorname{polylo \\
& g}(2, (a*e^2 - b*d*e + c*d^2 + b*e - 2*c*d + c) * (e*x+d+1)^2 / (- (e*x+d)^2 + 1) / (-a*e^2 + b*e * \\
& d - c*d^2 - (-e^2 * (4*a*c - b^2))^{(1/2)} + c)) - 1/2 * e * (e^2 * (-4*a*c + b^2))^{(1/2)} / (4*a*c - \\
& b^2) * \operatorname{polylog}(2, (a*e^2 - b*d*e + c*d^2 + b*e - 2*c*d + c) * (e*x+d+1)^2 / (- (e*x+d)^2 + 1) / (\\
& -a*e^2 + b*e*d - c*d^2 - (-e^2 * (4*a*c - b^2))^{(1/2)} + c)) * c / (a*e^2 - b*e*d + c*d^2 + (e^2 * (\\
& -4*a*c + b^2))^{(1/2)} - c)
\end{aligned}$$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctanh(e*x+d)/(c*x^2+b*x+a),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a dditional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for more details)Is 4*a*c-b^2 positive or negative?

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{atanh}(d + ex)}{cx^2 + bx + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(atanh(d + e*x)/(a + b*x + c*x^2), x)`

[Out] `int(atanh(d + e*x)/(a + b*x + c*x^2), x)`

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(atanh(e*x+d)/(c*x**2+b*x+a), x)`

[Out] Timed out

$$3.62 \quad \int \frac{(ce+dex)(a+b \tanh^{-1}(c+dx))}{1-(c+dx)^2} dx$$

Optimal. Leaf size=83

$$-\frac{e(a+b \tanh^{-1}(c+dx))^2}{2bd} + \frac{e \log\left(\frac{2}{-c-dx+1}\right)(a+b \tanh^{-1}(c+dx))}{d} + \frac{be\text{Li}_2\left(-\frac{c+dx+1}{-c-dx+1}\right)}{2d}$$

[Out] $-1/2*e*(a+b*\text{arctanh}(d*x+c))^2/b/d+e*(a+b*\text{arctanh}(d*x+c))*\ln(2/(-d*x-c+1))/d+1/2*b*e*\text{polylog}(2,(-d*x-c-1)/(-d*x-c+1))/d$

Rubi [A] time = 0.26, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {12, 5984, 5918, 2402, 2315}

$$\frac{be\text{PolyLog}\left(2, -\frac{c+dx+1}{-c-dx+1}\right)}{2d} - \frac{e(a+b \tanh^{-1}(c+dx))^2}{2bd} + \frac{e \log\left(\frac{2}{-c-dx+1}\right)(a+b \tanh^{-1}(c+dx))}{d}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(c*e + d*e*x)*(a + b*\text{ArcTanh}[c + d*x])]/(1 - (c + d*x)^2), x]$

[Out] $-(e*(a + b*\text{ArcTanh}[c + d*x])^2)/(2*b*d) + (e*(a + b*\text{ArcTanh}[c + d*x])*Log[2/(1 - c - d*x)])/d + (b*e*\text{PolyLog}[2, -((1 + c + d*x)/(1 - c - d*x))])/(2*d)$

Rule 12

$\text{Int}[(a_*)(u_), x_Symbol] \rightarrow \text{Dist}[a, \text{Int}[u, x], x] /; \text{FreeQ}[a, x] \ \&\& \ !\text{Match}[Q[u, (b_)*(v_)] /; \text{FreeQ}[b, x]]$

Rule 2315

$\text{Int}[\text{Log}[(c_*)(x_)]/((d_*) + (e_*)(x_)), x_Symbol] \rightarrow -\text{Simp}[\text{PolyLog}[2, 1 - c*x]/e, x] /; \text{FreeQ}[\{c, d, e\}, x] \ \&\& \ \text{EqQ}[e + c*d, 0]$

Rule 2402

$\text{Int}[\text{Log}[(c_)]/((d_*) + (e_*)(x_))]/((f_*) + (g_*)(x_)^2), x_Symbol] \rightarrow -\text{Dist}[e/g, \text{Subst}[\text{Int}[\text{Log}[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; \text{FreeQ}[\{c, d, e, f, g\}, x] \ \&\& \ \text{EqQ}[c, 2*d] \ \&\& \ \text{EqQ}[e^2*f + d^2*g, 0]$

Rule 5918

$\text{Int}[(a_*) + \text{ArcTanh}[(c_*)(x_)]*(b_*)]^{(p_*)}/((d_*) + (e_*)(x_)), x_Symbol] \rightarrow -\text{Simp}[(a + b*\text{ArcTanh}[c*x])^p*\text{Log}[2/(1 + (e*x)/d)]/e, x] + \text{Dist}[(b*c*$

p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)]/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0]

Rule 5984

Int[(((a_.) + ArcTanh[(c_.)*(x_.)]*(b_.))^(p_.)*(x_.))/((d_.) + (e_.)*(x_.)^2), x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]

Rubi steps

$$\begin{aligned}
 \int \frac{(ce + dex)(a + b \tanh^{-1}(c + dx))}{1 - (c + dx)^2} dx &= \frac{\text{Subst}\left(\int \frac{ex^{(a+b \tanh^{-1}(x))}}{1-x^2} dx, x, c + dx\right)}{d} \\
 &= \frac{e \text{Subst}\left(\int \frac{x^{(a+b \tanh^{-1}(x))}}{1-x^2} dx, x, c + dx\right)}{d} \\
 &= -\frac{e(a + b \tanh^{-1}(c + dx))^2}{2bd} + \frac{e \text{Subst}\left(\int \frac{a+b \tanh^{-1}(x)}{1-x} dx, x, c + dx\right)}{d} \\
 &= -\frac{e(a + b \tanh^{-1}(c + dx))^2}{2bd} + \frac{e(a + b \tanh^{-1}(c + dx)) \log\left(\frac{2}{1-c-dx}\right)}{d} \\
 &= -\frac{e(a + b \tanh^{-1}(c + dx))^2}{2bd} + \frac{e(a + b \tanh^{-1}(c + dx)) \log\left(\frac{2}{1-c-dx}\right)}{d} + \dots \\
 &= -\frac{e(a + b \tanh^{-1}(c + dx))^2}{2bd} + \frac{e(a + b \tanh^{-1}(c + dx)) \log\left(\frac{2}{1-c-dx}\right)}{d} + \dots
 \end{aligned}$$

Mathematica [A] time = 0.12, size = 114, normalized size = 1.37

$$\frac{e\left(4a \log(-c - dx + 1) + 4a \log(c + dx + 1) - 2b \text{Li}_2\left(\frac{1}{2}(-c - dx + 1)\right) + 2b \text{Li}_2\left(\frac{1}{2}(c + dx + 1)\right) - b \log^2(-c - dx + 1)\right)}{8d}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(((c*e + d*e*x)*(a + b*ArcTanh[c + d*x]))/(1 - (c + d*x)^2), x]

[Out] $-1/8*(e*(4*a*\text{Log}[1 - c - d*x] - b*\text{Log}[1 - c - d*x]^2 + b*\text{Log}[4]*\text{Log}[-1 + c + d*x] + 4*a*\text{Log}[1 + c + d*x] - b*\text{Log}[4]*\text{Log}[1 + c + d*x] + b*\text{Log}[1 + c + d*x]^2 - 2*b*\text{PolyLog}[2, (1 - c - d*x)/2] + 2*b*\text{PolyLog}[2, (1 + c + d*x)/2]))/d$

fricas [F] time = 0.79, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{adex + ace + (bdex + bce) \operatorname{artanh}(dx + c)}{d^2x^2 + 2cdx + c^2 - 1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*e*x+c*e)*(a+b*arctanh(d*x+c))/(1-(d*x+c)^2),x, algorithm="fricas")`

[Out] `integral(-(a*d*e*x + a*c*e + (b*d*e*x + b*c*e)*arctanh(d*x + c))/(d^2*x^2 + 2*c*d*x + c^2 - 1), x)`

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*e*x+c*e)*(a+b*arctanh(d*x+c))/(1-(d*x+c)^2),x, algorithm="giac")`

[Out] Timed out

maple [B] time = 0.06, size = 194, normalized size = 2.34

$$\frac{ae \ln(dx + c - 1)}{2d} - \frac{ae \ln(dx + c + 1)}{2d} - \frac{be \operatorname{arctanh}(dx + c) \ln(dx + c - 1)}{2d} - \frac{be \operatorname{arctanh}(dx + c) \ln(dx + c + 1)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d*e*x+c*e)*(a+b*arctanh(d*x+c))/(1-(d*x+c)^2),x)`

[Out] $-1/2/d*a*e*\ln(d*x+c-1)-1/2/d*a*e*\ln(d*x+c+1)-1/2/d*b*e*\operatorname{arctanh}(d*x+c)*\ln(d*x+c-1)-1/2/d*b*e*\operatorname{arctanh}(d*x+c)*\ln(d*x+c+1)-1/8/d*b*e*\ln(d*x+c-1)^2+1/2/d*b*e*\operatorname{dilog}(1/2+1/2*d*x+1/2*c)+1/4/d*b*e*\ln(d*x+c-1)*\ln(1/2+1/2*d*x+1/2*c)+1/8/d*b*e*\ln(d*x+c+1)^2-1/4/d*b*e*\ln(-1/2*d*x-1/2*c+1/2)*\ln(d*x+c+1)+1/4/d*b*e*\ln(-1/2*d*x-1/2*c+1/2)*\ln(1/2+1/2*d*x+1/2*c)$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2} bce \left(\frac{\log(dx + c + 1)}{d} - \frac{\log(dx + c - 1)}{d} \right) \operatorname{artanh}(dx + c) - \frac{1}{2} ade \left(\frac{(c + 1) \log(dx + c + 1)}{d^2} - \frac{(c - 1) \log(dx + c - 1)}{d^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*e*x+c*e)*(a+b*arctanh(d*x+c))/(1-(d*x+c)^2),x, algorithm="maxima")

[Out] 1/2*b*c*e*(log(d*x + c + 1)/d - log(d*x + c - 1)/d)*arctanh(d*x + c) - 1/2*a*d*e*((c + 1)*log(d*x + c + 1)/d^2 - (c - 1)*log(d*x + c - 1)/d^2) + 1/2*a*c*e*(log(d*x + c + 1)/d - log(d*x + c - 1)/d) + 1/8*b*d*e*((2*(c + 1)*log(d*x + c + 1)*log(-d*x - c + 1) - (c - 1)*log(-d*x - c + 1)^2)/d^2 - 4*integrate(1/2*(c^2 + (c*d + 3*d)*x + 2*c + 1)*log(d*x + c + 1)/(d^3*x^2 + 2*c*d^2*x + c^2*d - d), x)) - 1/8*(log(d*x + c + 1)^2 - 2*log(d*x + c + 1)*log(d*x + c - 1) + log(d*x + c - 1)^2)*b*c*e/d

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{(ce + dex)(a + b \operatorname{atanh}(c + dx))}{(c + dx)^2 - 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-((c*e + d*e*x)*(a + b*atanh(c + d*x)))/((c + d*x)^2 - 1),x)

[Out] int(-((c*e + d*e*x)*(a + b*atanh(c + d*x)))/((c + d*x)^2 - 1), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-e \left(\int \frac{ac}{c^2 + 2cdx + d^2x^2 - 1} dx + \int \frac{adx}{c^2 + 2cdx + d^2x^2 - 1} dx + \int \frac{bc \operatorname{atanh}(c + dx)}{c^2 + 2cdx + d^2x^2 - 1} dx + \int \frac{bdx \operatorname{atanh}(c + dx)}{c^2 + 2cdx + d^2x^2 - 1} dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*e*x+c*e)*(a+b*atanh(d*x+c))/(1-(d*x+c)**2),x)

[Out] -e*(Integral(a*c/(c**2 + 2*c*d*x + d**2*x**2 - 1), x) + Integral(a*d*x/(c**2 + 2*c*d*x + d**2*x**2 - 1), x) + Integral(b*c*atanh(c + d*x)/(c**2 + 2*c*d*x + d**2*x**2 - 1), x) + Integral(b*d*x*atanh(c + d*x)/(c**2 + 2*c*d*x + d**2*x**2 - 1), x))

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)

(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*      GradeAntiderivative[result,optimal] returns*)
(* "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(* "C" if result involves higher level functions than necessary*)
(* "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(* "A" if result can be considered optimal*)

GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
```

```

If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
  If[LeafCount[result]<=2*LeafCount[optimal],
    "A",
    "B"],
  "C"],
If[FreeQ[result,Integrate] && FreeQ[result,Int],
  "C",
"F"]]

```

```

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)

```

```

ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
        Max[ExpnType[expn[[1]],2]],
      Max[ExpnType[expn[[1]],ExpnType[expn[[2]],3]],
    If[Head[expn]===Plus || Head[expn]===Times,
      Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
    If[ElementaryFunctionQ[Head[expn]],
      Max[3,ExpnType[expn[[1]]],
    If[SpecialFunctionQ[Head[expn]],
      Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
    If[HypergeometricFunctionQ[Head[expn]],
      Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
    If[AppellFunctionQ[Head[expn]],
      Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],

```

```

If[Head[expn]===RootSum,
  Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
If[Head[expn]===Integrate || Head[expn]===Int,
  Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
9]]]]]]]]]]

ElementaryFunctionQ[func_] :=
MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]

SpecialFunctionQ[func_] :=
MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]

HypergeometricFunctionQ[func_] :=
MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]

AppellFunctionQ[func_] :=
MemberQ[{AppellF1},func]

```

4.0.2 Maple grading function

```

# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017 Use Maple leaf count instead since buildin
#Nasser 03/23/2017 missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017 corrected the check for complex result
#Nasser 10/27/2017 check for leafsize and do not call ExpnType()
#
# if leaf size is "too large". Set at 500,000

```

```

#Nasser 12/22/2019 Added debug flag, added 'dilog' to special functions
# see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
    debug:=false;

    leaf_count_result:=leafcount(result);
    #do NOT call ExpnType() if leaf size is too large. Recursion problem
    if leaf_count_result > 500000 then
        return "B";
    fi;

    leaf_count_optimal:=leafcount(optimal);

    ExpnType_result:=ExpnType(result);
    ExpnType_optimal:=ExpnType(optimal);

    if debug then
        print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
ExpnType_optimal);
    fi;

# If result and optimal are mathematical expressions,
# GradeAntiderivative[result,optimal] returns
# "F" if the result fails to integrate an expression that
# is integrable
# "C" if result involves higher level functions than necessary
# "B" if result is more than twice the size of the optimal
# antiderivative
# "A" if result can be considered optimal

#This check below actually is not needed, since I only
#call this grading only for passed integrals. i.e. I check
#for "F" before calling this. But no harm of keeping it here.
#just in case.

if not type(result,freeof('int')) then
    return "F";
end if;

if ExpnType_result<=ExpnType_optimal then
    if debug then
        print("ExpnType_result<=ExpnType_optimal");
    fi;

```

```

if is_contains_complex(result) then
  if is_contains_complex(optimal) then
    if debug then
      print("both result and optimal complex");
    fi;
    #both result and optimal complex
    if leaf_count_result<=2*leaf_count_optimal then
      return "A";
    else
      return "B";
    end if
  else #result contains complex but optimal is not
    if debug then
      print("result contains complex but optimal is not");
    fi;
    return "C";
  end if
else # result do not contain complex
  # this assumes optimal do not as well
  if debug then
    print("result do not contain complex, this assumes optimal do not
as well");
  fi;
  if leaf_count_result<=2*leaf_count_optimal then
    if debug then
      print("leaf_count_result<=2*leaf_count_optimal");
    fi;
    return "A";
  else
    if debug then
      print("leaf_count_result>2*leaf_count_optimal");
    fi;
    return "B";
  end if
end if
else #ExpnType(result) > ExpnType(optimal)
  if debug then
    print("ExpnType(result) > ExpnType(optimal)");
  fi;
  return "C";
end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false

```

```

#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
      1
    else
      max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'^^') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
        1
      else
        max(2,ExpnType(op(1,expn)))
      end if
    else
      max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  end if
end proc:

```



```

elif HypergeometricFunctionQ(op(0,expn)) then
  max(5,apply(max,map(ExpnType,[op(expn)])))
elif AppellFunctionQ(op(0,expn)) then
  max(6,apply(max,map(ExpnType,[op(expn)])))
elif op(0,expn)='int' then
  max(8,apply(max,map(ExpnType,[op(expn)]))) else
9
end if
end proc:

ElementaryFunctionQ := proc(func)
  member(func,[
    exp,log,ln,
    sin,cos,tan,cot,sec,csc,
    arcsin,arccos,arctan,arccot,arcsec,arccsc,
    sinh,cosh,tanh,coth,sech,csch,
    arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
    erf,erfc,erfi,
    FresnelS,FresnelC,
    Ei,Ei,Li,Si,Ci,Shi,Chi,
    GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
    EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product. rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
    op(2,u)
  else
    apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

```

```
#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17 Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
    MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#           Port of original Maple grading function by
#           Albert Rich to use with Sympy/Python
#Dec 27, 2019 Nasser. Added `RootSum`. See problem 177, Timofeev file
#           added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
        asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
        asinh,acosh,atanh,acoth,asech,acsch
    ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
        fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
        gamma,loggamma,digamma,zeta,polylog,LambertW,
        elliptic_f,elliptic_e,elliptic_pi,exp_polar
    ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]
```

```

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
            return True
        else:
            return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
        return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn)) #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0])) #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow): #type(expn,'`^`')
        if isinstance(expn.args[1],Integer): #type(op(2,expn),'integer')
            return expnType(expn.args[0]) #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational): #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0])) #max(2,ExpnType(op(1,expn)
    ))
    else:
        return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
    ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type
    (expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2) #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func): #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0])) #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1) #max(4,apply(max,map(ExpnType,[op(expn)])))

```

```

elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
expn))
    m1 = max(map(expnType, list(expn.args)))
    return max(5,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
elif is_appell_function(expn.func):
    m1 = max(map(expnType, list(expn.args)))
    return max(6,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
elif isinstance(expn,RootSum):
    m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,
Apply[List,expn]],7]],
    return max(7,m1)
elif str(expn).find("Integral") != -1:
    m1 = max(map(expnType, list(expn.args)))
    return max(8,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
else:
    return 9

#main function
def grade_antiderivative(result,optimal):

    leaf_count_result = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"

```

4.0.4 SageMath grading function

```

#Dec 24, 2019. Nasser: Ported original Maple grading function by
#       Albert Rich to use with Sagemath. This is used to
#       grade Fracas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#       'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow: #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug=False
    m = func.name() in ['exp','log','ln',
        'sin','cos','tan','cot','sec','csc',
        'arcsin','arccos','arctan','arccot','arcsec','arccsc',
        'sinh','cosh','tanh','coth','sech','csch',
        'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
    ]
    if debug:
        if m:

```

```

        print ("func ", func , " is elementary_function")
    else:
        print ("func ", func , " is NOT elementary_function")

    return m

def is_special_function(func):
    debug=False
    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
        'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
sinh_integral'
        'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
        'polylog','lambert_w','elliptic_f','elliptic_e',
        'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")

    return m

def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U
']

def is_appell_function(func):
    return func.name() in ['hypergeometric']    #[appellf1] can't find this in
sagemath

def is_atom(expn):

    debug=False
    if debug: print ("Enter is_atom")

    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:

```

```

        return expn.operator() is None
    if expn.parent() in (ZZ, QQ, AA, QQbar):
        return expn in expn.parent() # Should always return True
    if hasattr(expn.parent(), "base_ring") and hasattr(expn.parent(), "gens"):
        return expn in expn.parent().base_ring() or expn in expn.parent().
gens()
    return False

except AttributeError as error:
    return False

def expnType(expn):

    if debug:
        print(">>>>Enter expnType, expn=", expn)
        print(">>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list: #instance(expn,list):
        return max(map(expnType, expn)) #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if type(expn.operands()[0])==Rational: #type(instance(expn.args[0],
Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0])) #max(2,expnType(expn.
args[0]))
    elif expn.operator() == operator.pow: #instance(expn,Pow)
        if type(expn.operands()[1])==Integer: #instance(expn.args[1],Integer)
            return expnType(expn.operands()[0]) #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational: #instance(expn.args[1],
Rational)
            if type(expn.operands()[0])==Rational: #instance(expn.args[0],
Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0])) #max(2,expnType(expn.
args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()
[1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
instance(expn,Add) or instance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))

```

```

    return max(m1,m2) #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
elif is_elementary_function(expn.operator()): #is_elementary_function(expn.
func)
    return max(3,expnType(expn.operands()[0]))
elif is_special_function(expn.operator()): #is_special_function(expn.func)
    m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))
    return max(4,m1) #max(4,m1)
elif is_hypergeometric_function(expn.operator()): #
is_hypergeometric_function(expn.func)
    m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))
    return max(5,m1) #max(5,m1)
elif is_appell_function(expn.operator()):
    m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))
    return max(6,m1) #max(6,m1)
elif str(expn).find("Integral") != -1: #this will never happen, since it
#is checked before calling the grading function that is passed.
#but kept it here.
    m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))
    return max(8,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
else:
    return 9

```

```
#main function
```

```
def grade_antiderivative(result,optimal):
```

```

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "
leaf_count_optimal=",leaf_count_optimal)

```

```

    expnType_result = expnType(result)
    expnType_optimal = expnType(optimal)

```

```

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
expnType_optimal)

```

```

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex

```



```
        if leaf_count_result <= 2*leaf_count_optimal:
            return "A"
        else:
            return "B"
    else: #result contains complex but optimal is not
        return "C"
else: # result do not contain complex, this assumes optimal do not as
well
    if leaf_count_result <= 2*leaf_count_optimal:
        return "A"
    else:
        return "B"
else:
    return "C"
```